

# Signatures

Lecture 22

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- Definitions sometimes have subtleties (not all of them have ideal functionality specifications)



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- Resulting signature must be "compact": size independent of the number of signers
- Security requirement: Unforgeability (chosen message security)
  - Adversary can collude with all but one signer



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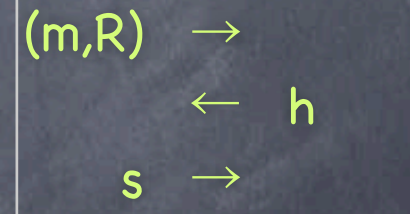
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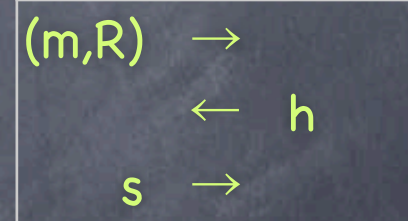
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$$\begin{array}{ccc} (m,R) & \rightarrow & \\ & & \leftarrow h \\ s & \rightarrow & \end{array}$$
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- Extended to a multi-signature scheme [BN'06]  $\rightarrow$

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  - So that final signature  $s_n = r + h_1 X_1 + \dots + h_n X_n$  where  $R = g^r$

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- General aggregation: signatures can be created independently and then aggregated in arbitrary order

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- Extended to a sequential aggregate scheme [LOSSW'06] →

# A Sequential Aggregate Signature Scheme



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- Signing done sequentially by individual signers. Initially set  $R=1$  and  $S = 1$  (identity in the group). Then:
  - **AddSign(m<sup>i</sup>, (R', S'); x<sub>i</sub>, y<sup>i</sup><sub>0</sub>, y<sup>i</sup><sub>1</sub>, ..., y<sup>i</sup><sub>k</sub>) = ReRand(R'', S'')**, where  $R'' = R'$  and  $S'' = S' \cdot g^{x_i} \cdot (R')^{h_i}$  where  $h_i$  s.t.  $g^{h_i} = H_i$

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  - **AddSign** $(m^i, (R', S'); x_i, y^i_0, y^i_1, \dots, y^i_k) = \text{ReRand}(R'', S'')$ , where  $R'' = R'$  and  $S'' = S' \cdot g^{x_i} \cdot (R')^{h_i}$  where  $h_i$  s.t.  $g^{h_i} = H_i$
  - $\text{ReRand}(R'', S'') = (R, S)$ , where  $R = R'' g^t$  and  $S = S'' (H_1 \dots H_i)^t$



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    - Aggregate signatures saves on bandwidth and verification time, but needs coordination among signers and does not allow un-aggregating the signatures



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  - e.g. Waters' signature:  $e(S,g)=e(R,H).X$  ( $g$  same for all signers)



# Batch Verification

- Idea: to verify several equations of the form  $Z_i = g^{z_i}$ , pick random weights  $w_i$  and check  $\prod_i Z_i^{w_i} = g^{\sum z_i \cdot w_i}$ 
  - If one (or more) equation is wrong, probability of verifying is at most  $1/q$ , where  $q$  is the size of the domain of  $w_i$
  - Efficiency by using a small domain for  $w_i$ . e.g., use  $w_i \in \{0,1\}$ , and repeat  $k$  times (independent of number of signatures)
- Similarly for pairing equations, but with further optimizations
  - e.g. Waters' signature:  $e(S,g)=e(R,H).X$  ( $g$  same for all signers)
    - Can save on number of pairing operations using  $\prod_i e(S_i,g)^{w_i} = \prod_i e(S_i^{w_i},g) = e(\prod_i S_i^{w_i},g)$

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  - However, the group manager or other group members “cannot frame” a member



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  - **Implies unforgeability** (i.e., with no group members colluding with it, adversary cannot produce a valid signature) and **framing-resistance** (even colluding with the group manager)



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- Tracing algorithm decrypts  $C$  to find  $SK^*_i$  and hence  $ID_i$

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- Unwitting collaborators:  $F_i$ 's could be the verification keys for a standard signature scheme

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- Security requirements: Unforgeability and Hiding



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- c.f. Mesh signatures: here, instead of multiple parties signing a message, a single party with multiple attributes



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- Note: Still allows multiple (mutually distrusting) verifiers to be convinced if they run a secure MPC protocol to implement a virtual verifier

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  - e.g. a ring signature with a ring of size 2, containing the signer and the designated verifier

Today



# Today

- Signatures

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- Signatures
  - Multi-signatures



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- Signatures
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  - Aggregate Signatures

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  - Using Blind signatures and  $\rho$ -signatures