

Signatures

Lecture 22

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- Definitions sometimes have subtleties (not all of them have ideal functionality specifications)

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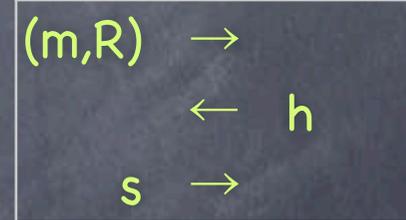
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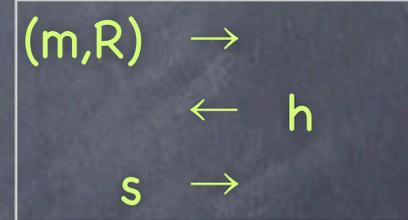
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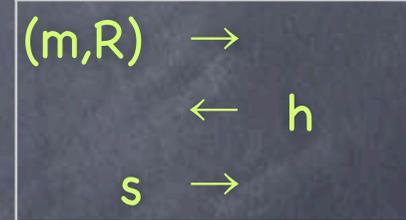
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- Extended to a multi-signature scheme [BN'06] →



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 - Then, sequentially $s_i = s_{i-1} + r_i + h_i X_i$ (starting with $s_0 = 0$)
 - So that final signature $s_n = r + h_1 X_1 + \dots + h_n X_n$ where $R = g^r$

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- General aggregation: signatures can be created independently and then aggregated in arbitrary order

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 - **AddSign(mⁱ, (R', S'); x_i, yⁱ₀, yⁱ₁, ..., yⁱ_k) = ReRand(R'', S'')**, where $R'' = R'$ and $S'' = S' \cdot g^{x_i} \cdot (R')^{h_i}$ where h_i s.t. $g^{h_i} = H_i$

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 - $\text{ReRand}(R'', S'') = (R, S)$, where $R = R'' g^t$ and $S = S'' (H_1 \dots H_i)^t$

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 - Aggregate signatures saves on bandwidth and verification time, but needs coordination among signers and does not allow un-aggregating the signatures

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 - e.g. Waters' signature: $e(S,g)=e(R,H).X$ (g same for all signers)
 - Can save on number of pairing operations using $\prod_i e(S_i,g)^{w_i} = \prod_i e(S_i^{w_i},g) = e(\prod_i S_i^{w_i},g)$

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 - Signature shows that message was signed by some member of a group
 - But a group manager can “trace” the signer
 - However, the group manager or other group members “cannot frame” a member

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 - **Implies unlinkability** (can't link signatures from same user)
- **Full-Traceability**: If a set of group members collude and create a valid signature, the tracing algorithm will trace at least one member of the set. This holds even if the group manager is passively corrupt.

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- **Full-Anonymity**: Adversary gives (m, ID_0, ID_1) and gets back $\text{Sign}(m; ID_b)$ for a random bit b . Advantage of the adversary in finding b should be negligible.
 - Adversary knows secret keys of all group-members, and has oracle access to the “tracing algorithm” (but not allowed to query it on the challenge)
 - **Implies unlinkability** (can't link signatures from same user)
- **Full-Traceability**: If a set of group members collude and create a valid signature, the tracing algorithm will trace at least one member of the set. This holds even if the group manager is passively corrupt.
 - **Implies unforgeability** (i.e., with no group members colluding with it, adversary cannot produce a valid signature) and **framing-resistance** (even colluding with the group manager)

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- Tracing algorithm decrypts C to find SK^*_i and hence ID_i

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- Unwitting collaborators: F_i 's could be the verification keys for a standard signature scheme

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- Security requirements: Unforgeability and Hiding

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- c.f. Mesh signatures: here, instead of multiple parties signing a message, a single party with multiple attributes

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- Zero-knowledge verification: A verifier cannot transfer a signature that it verified
- Note: Still allows multiple (mutually distrusting) verifiers to be convinced if they run a secure MPC protocol to implement a virtual verifier

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 - e.g. a ring signature with a ring of size 2, containing the signer and the designated verifier

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 - Using Blind signatures and ρ -signatures