Mix-Nets

Lecture 19

Some tools for electronic-voting (and other things)

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- Corruption model: Active adversary can corrupt a limited number of servers

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- Active adversary can corrupt a limited number of servers
- Ideal: Same as for SIM-CPA, but with servers also getting the message (if the receiver decides to get it); if number of corrupted servers above threshold, adversary can block (but not substitute) output to others

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  - Proof using an Honest-Verifier ZK proof
    - Using a special purpose proof (Chaum-Pederson), rather than ZK for general NP statements

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  - Can amplify soundness using parallel repetition: still 3 rounds

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      - Removes need for interaction!

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- Special soundness: given answers for $v \neq v'$ either $v_1 \neq v_1'$ or $v_2 \neq v_2'$. By special soundness, extract witness for $stmnt_1$ or $stmnt_2$

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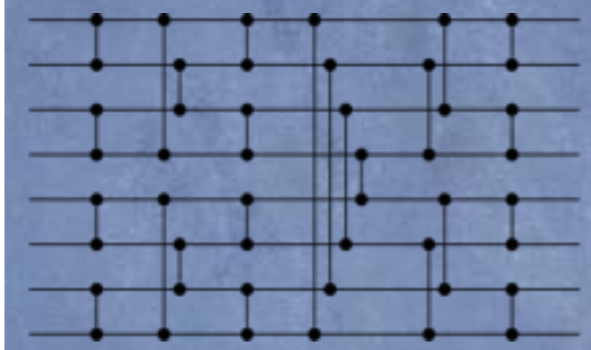

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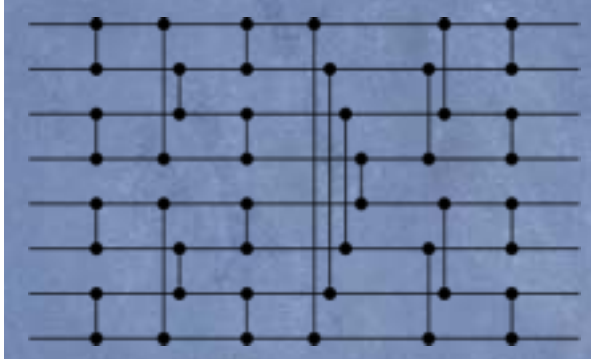

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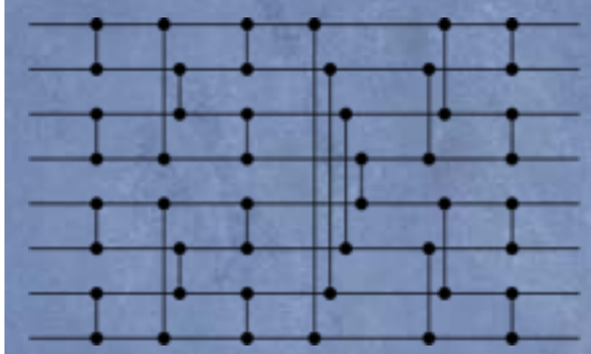

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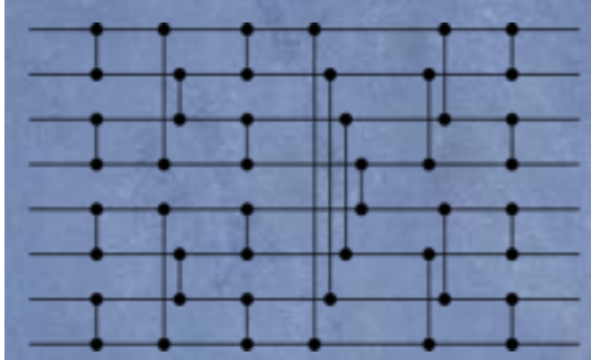

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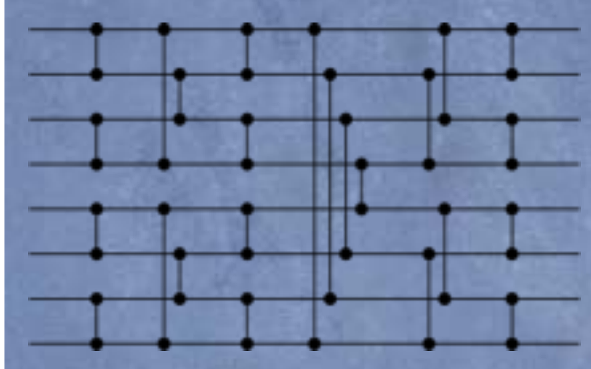

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  - 3 rounds: Parallel composition of HVZK proofs

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  - 7 rounds, using homomorphic commitments
    - Possible with sub-linear communication for the proof

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- Homomorphism: com(x;r) * com(x′;r′) = com(x+x′;r+r′)
  - (Operations in respective groups)

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  - Binding, because of collision resistance when K picked at random

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- HVZK PoK of (x,r): Send $Com_{g,h}(u_1;u_2)$, and on challenge v, send $(xv+u_1)$ and $(rv+u_2)$

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- HVZK PoK of (x,r): Send $Com_{g,h}(u_1;u_2)$, and on challenge v, send $(xv+u_1)$ and $(rv+u_2)$
- Improved efficiency: $H_{g1,..,gn,h}(x_1,\ldots,x_n,r) = g_1^{x1}\ldots g_n^{xn}\, h^r$

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- Use homomorphic commitments to carry out the polynomial evaluation and check equality (details omitted)

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    - Use homomorphic properties of the commitments to carry out equality proofs w.r.t committed permutation (omitted)

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