Secure 2-Party Computation

Lecture 14
Yao’s Garbled Circuit
SIM-Secure MPC

Secure (and correct) if:

∀ \exists s.t. output of is distributed identically in REAL and IDEAL

RECALL
Passive Adversary

- Gets only read access to the internal state of the corrupted players (and can use that information in talking to environment)
- Also called “Honest-But-Curious” adversary
- Will require that simulator also corrupts passively
- Simplifies several cases
  - e.g. coin-tossing [why?], commitment [coming up]
- Oddly, sometimes security against a passive adversary is more demanding than against an active adversary
  - Active adversary: too pessimistic about what guarantee is available even in the IDEAL world
  - e.g. 2-party SFE for OR, with output going to only one party (trivial against active adversary; impossible without computational assumptions against passive adversary)
Oblivious Transfer

Pick one out of two, without revealing which

Intuitive property: transfer partial information “obliviously”

\[
\begin{align*}
&x_0, x_1 \\
&b \\
&b
\end{align*}
\]

We Predict STOCKS!!

A: up, B: down

I need just one
But can’t tell you which

Sure

All 2 of them!

IDEAL World
2-Party (Passive)
Secure Function Evaluation
2-Party (Passive) Secure Function Evaluation

Functionality takes \((X;Y)\) and outputs \(f(X;Y)\) to Alice, \(g(X;Y)\) to Bob.
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- **One-sided SFE:** only one party gets any output
  - **Symmetric SFE from one-sided SFE** (passive secure) [How?]
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  - General SFE from appropriate symmetric SFE [\textbf{How?}]
- \textbf{One-sided SFE:} only one party gets any output
  - Symmetric SFE from one-sided SFE (passive secure) [\textbf{How?}]
- So, for passive security, enough to consider one-sided SFE
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Randomized Functions: $f(X;Y;r)$
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- e.g., Noisy channel:
  - Alice's input $X$, Bob's input none
  - Bob's output: $X$ with prob $3/4$
  - $1-X$ with prob $1/4$
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Neither party should know $r$ (beyond what is revealed by output)

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For passive security, realizing **deterministic, one-sided SFE** enough for all SFE

Can we do “general” deterministic, one-sided SFE (i.e., for all functions)?

e.g., Noisy channel: Alice’s input $X$, Bob’s input none
Bob’s output: $X$, w/ prob $3/4$
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Boolean Circuits
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  - Acyclic: output well-defined
    - Note: no memory gates
Circuits and Functions
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$(x_1 \text{ AND } (\text{NOT } y_1)) \text{ OR } (\text{NOT}(x_1 \text{ XOR } y_1) \text{ AND } (x_0 \text{ AND } (\text{NOT } y_0)))$
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- e.g.: $X > Y$ for two bit inputs $X=x_1x_0$, $Y=y_1y_0$: $(x_1 \text{ AND } \neg y_1)) \text{ OR } (\neg(x_1 \text{ XOR } y_1) \text{ AND } (x_0 \text{ AND } \neg y_0))$
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In general, finding a small/smallest circuit from truth-table is notoriously hard
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- Size of circuit: number of wires (as a function of number of input wires)
- Can convert a **truth-table** into a circuit
  - Directly: circuit size exponential in input size
  - In general, finding a small/smallest circuit from truth-table is notoriously hard
- Often problems already described as succinct programs/circuits
2-Party SFE using General Circuits
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Either party maybe corrupted passively
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Alice holds $x=a$, Bob has $y=b$; Bob should get $\text{OR}(x,y)$
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Can use Oblivious Transfer
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Any ideas?
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Alice prepares 4 boxes $B_{xy}$ corresponding to 4 possible input scenarios, and 4 padlocks/keys $K_{x=0}$, $K_{x=1}$, $K_{y=0}$ and $K_{y=1}$
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What Bob sees: $K_y$ opens a lock in two boxes, $K_x$ opens a lock in two boxes; only one random box fully opens. It has the outcome.
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Formally, easy to simulate (can stuff unopenable boxes arbitrarily)
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For each gate \( G \) with input wires \( (u,v) \) and output wire \( w \), prepare 4 boxes \( B_{uv} \) and place \( K_{w=G(a,b)} \) inside box \( B_{uv=ab} \). Lock \( B_{uv=ab} \) with keys \( K_{u=a} \) and \( K_{v=b} \).
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Obliviously: one key for each of Bob's input wires.
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Larger Circuits

Idea: For each gate in the circuit Alice will prepare locked boxes, but will use it to keep keys for the next gate.

For each wire \( w \) in the circuit (i.e., input wires, or output of a gate) pick 2 keys \( K_{w=0} \) and \( K_{w=1} \).

For each gate \( G \) with input wires \( (u,v) \) and output wire \( w \), prepare 4 boxes \( B_{uv} \) and place \( K_{w=G(a,b)} \) inside box \( B_{uv=ab} \). Lock \( B_{uv=ab} \) with keys \( K_{u=a} \) and \( K_{v=b} \).

Give to Bob: Boxes for each gate, one key for each of Alice’s input wires.

Obliviously: one key for each of Bob’s input wires.

Boxes for output gates have values instead of keys.
Larger Circuits
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Evaluation: Bob gets one key for each input wire of a gate, opens one box for the gate, gets one key for the output wire, and proceeds.
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- Curious Alice sees nothing (as Bob picks up keys obliviously)
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- Evaluation: Bob gets one key for each input wire of a gate, opens one box for the gate, gets one key for the output wire, and proceeds.
- Gets output from a box in the output gate.
- Security similar to before.
- Curious Alice sees nothing (as Bob picks up keys obliviously).
- Everything is simulatable for curious Bob given final output: Bob could prepare boxes and keys (stuffing unopenable boxes arbitrarily); for an output gate, place the output bit in the box that opens.
Garbled Circuit
Garbled Circuit

That was too physical!
Garbled Circuit

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Yao’s Garbled circuit: boxes/keys replaced by IND-CPA secure SKE (i.e., using PRF, and independent randomness when key reused)
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Double lock: $\text{Enc}_{K_x}(\text{Enc}_{K_y}(m))$
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Need proof to ensure that this suffices for indistinguishability of simulation. (In fact, one-time-like security for $\text{Enc}$ suffices)
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Oblivious Transfer: We already saw for one bit (using T-OWP); with passive adversaries, just repeat bit-OT several times to transfer longer keys
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Can we really compose? Yes, for passive security.
Today
Today

- 2-Party SFE secure against passive adversaries
Today

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- Yao’s Garbled Circuit
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- 2-Party SFE secure against passive adversaries
- Yao’s Garbled Circuit
- Using OT and IND-CPA encryption
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- Composition (implicitly)
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  - Using OT and IND-CPA encryption
    - OT using TOWP
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- Coming up: Zero-Knowledge proofs and general multi-party computation, more protocols (for different settings).
  - Universal Composition