Hash Functions in Action

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Lecture 11

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- Today: CRHF construction. Domain Extension.
  Applications of hash functions

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    - Will see how to extend the domain to arbitrarily long strings (without increasing output size)

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  - All candidates use mathematical structures that are considered computationally expensive

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  - Hash halves the size of the input

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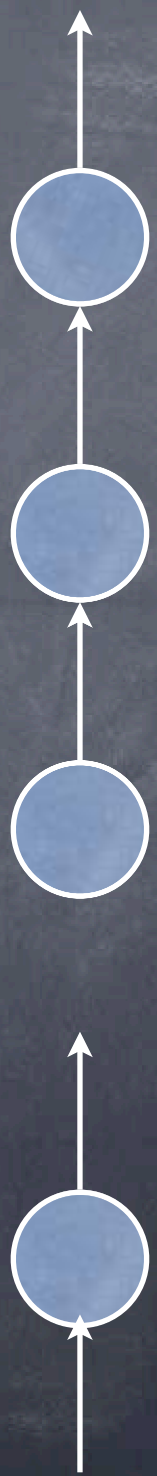

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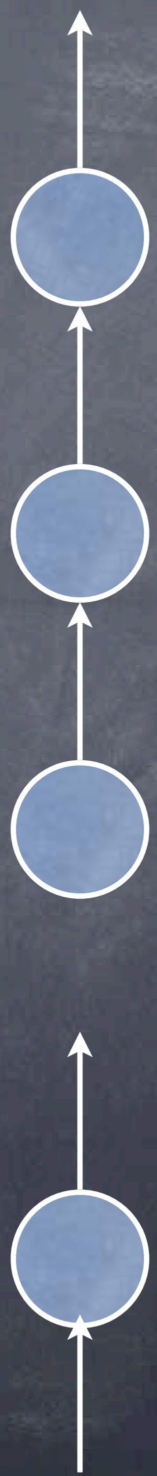

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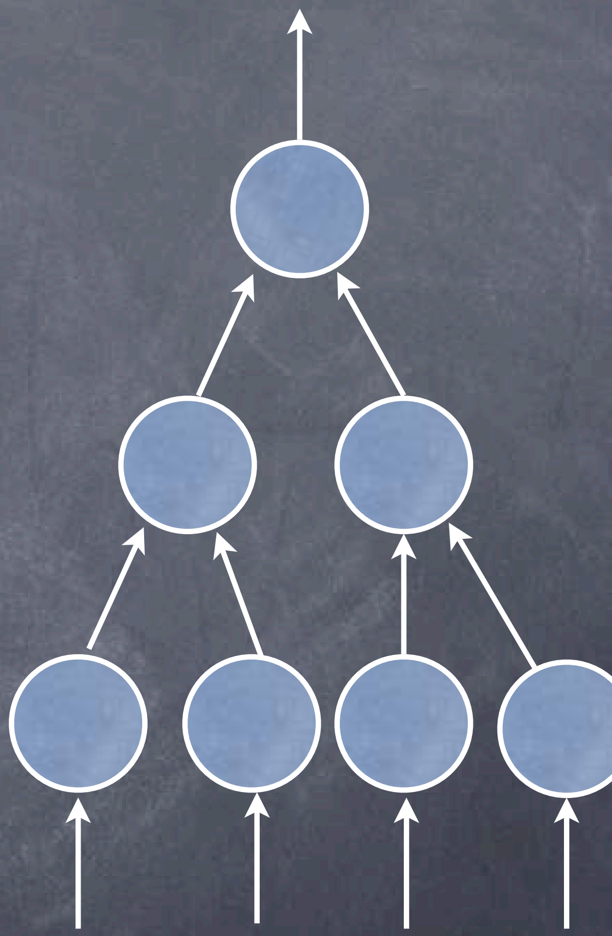

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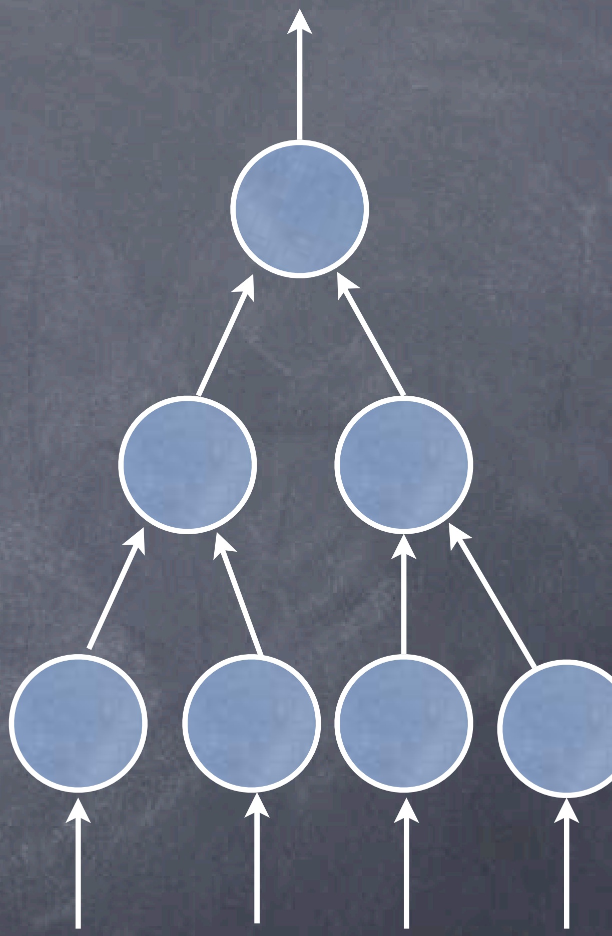

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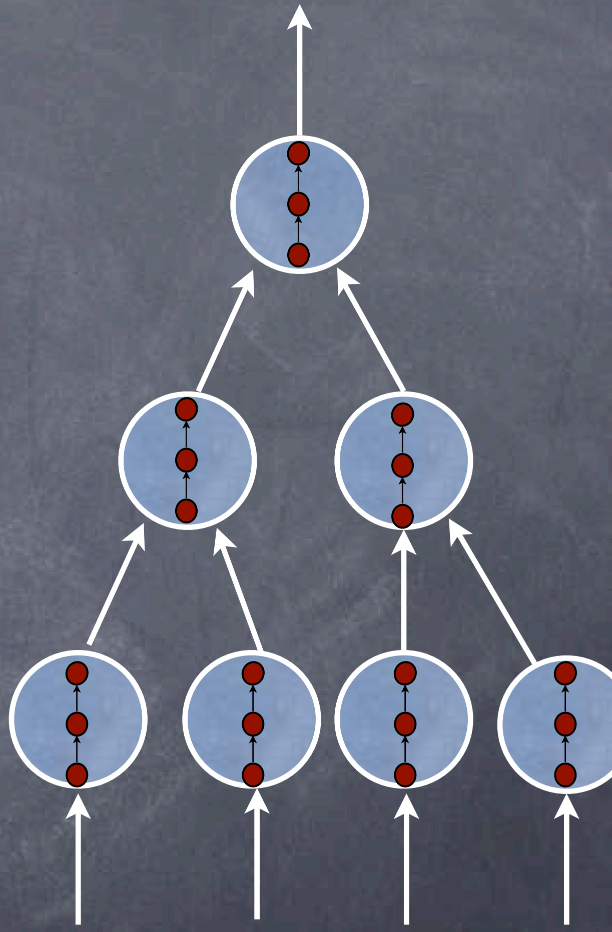

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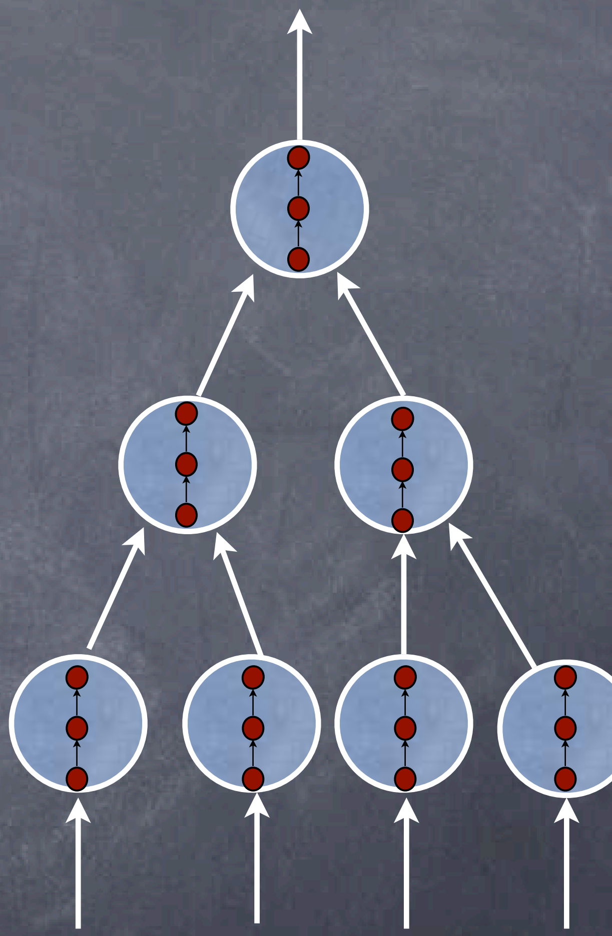

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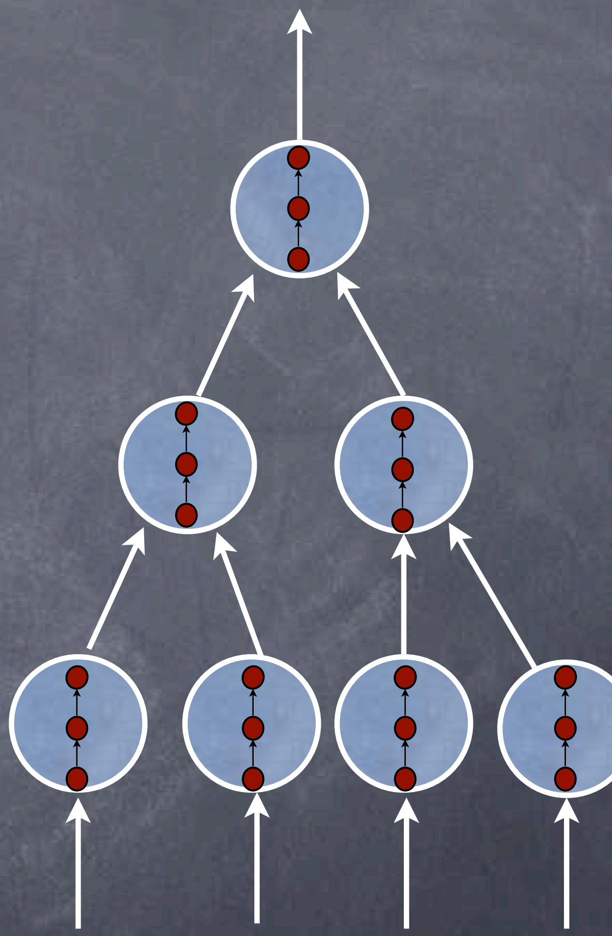

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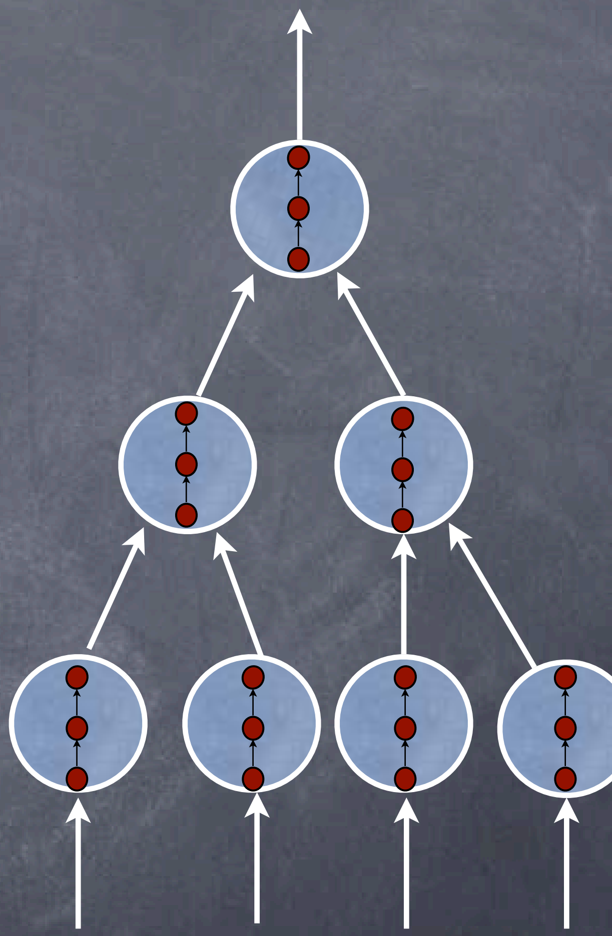
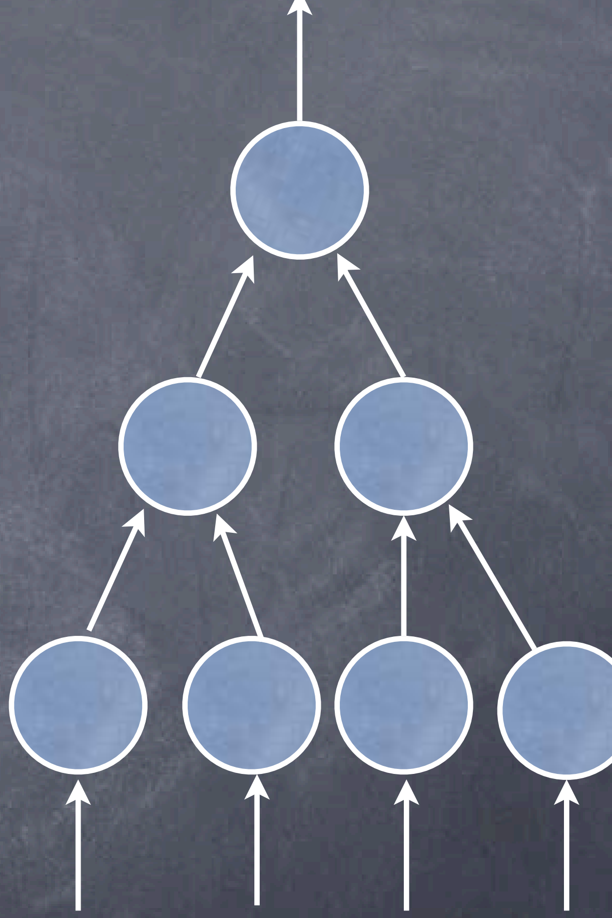


Domain Extension for CRHF



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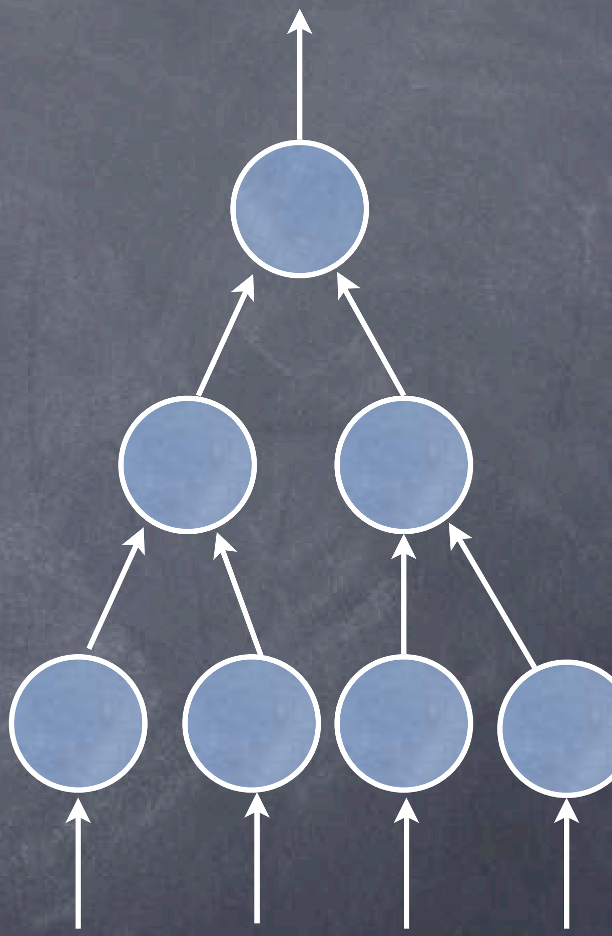

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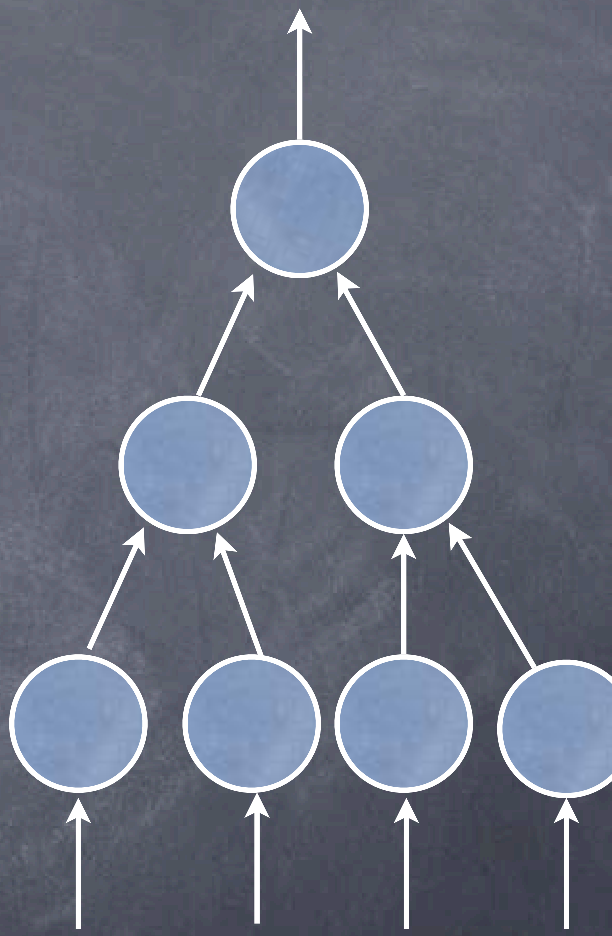

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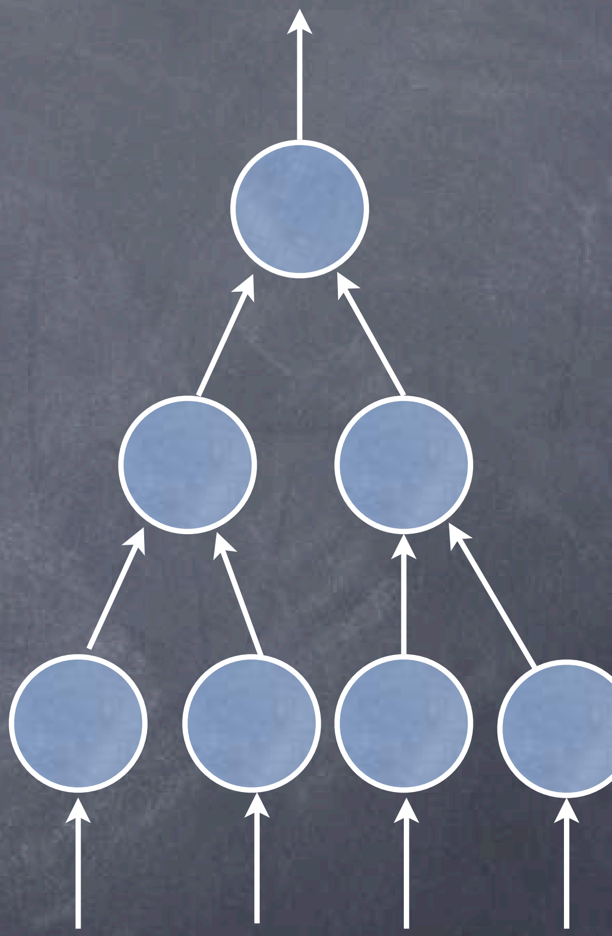
    - Consider moving a "frontline" from bottom to top



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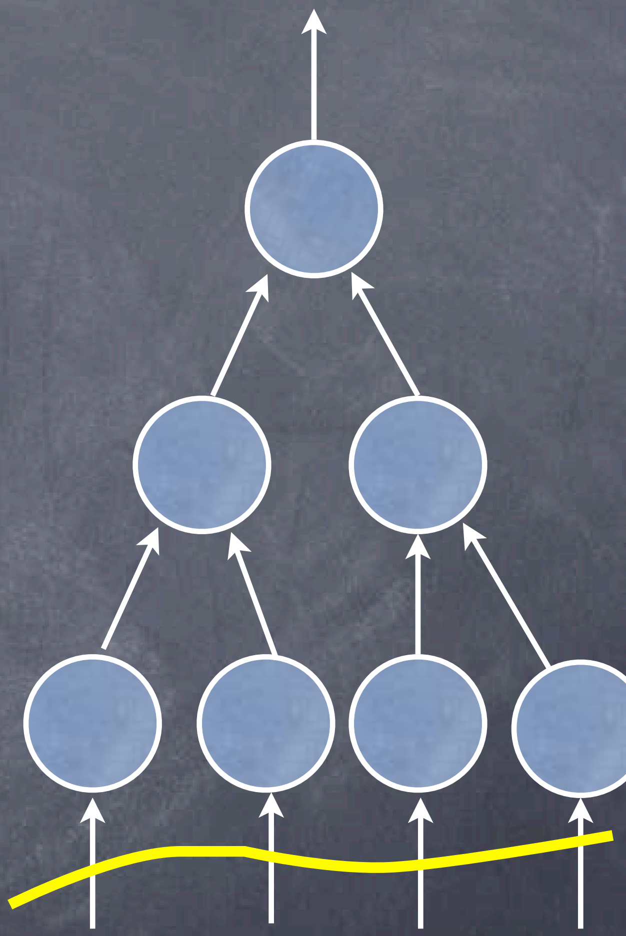

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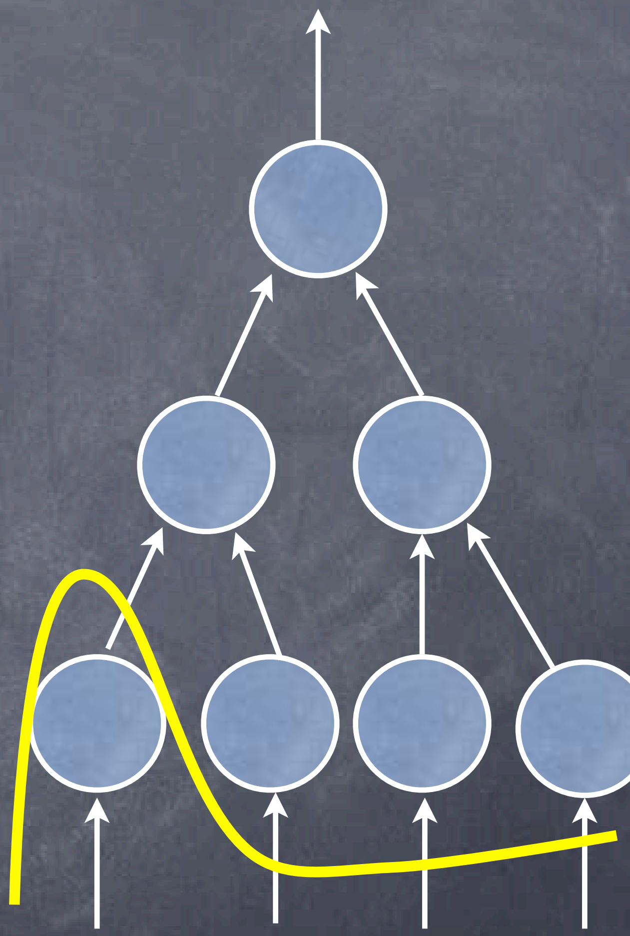

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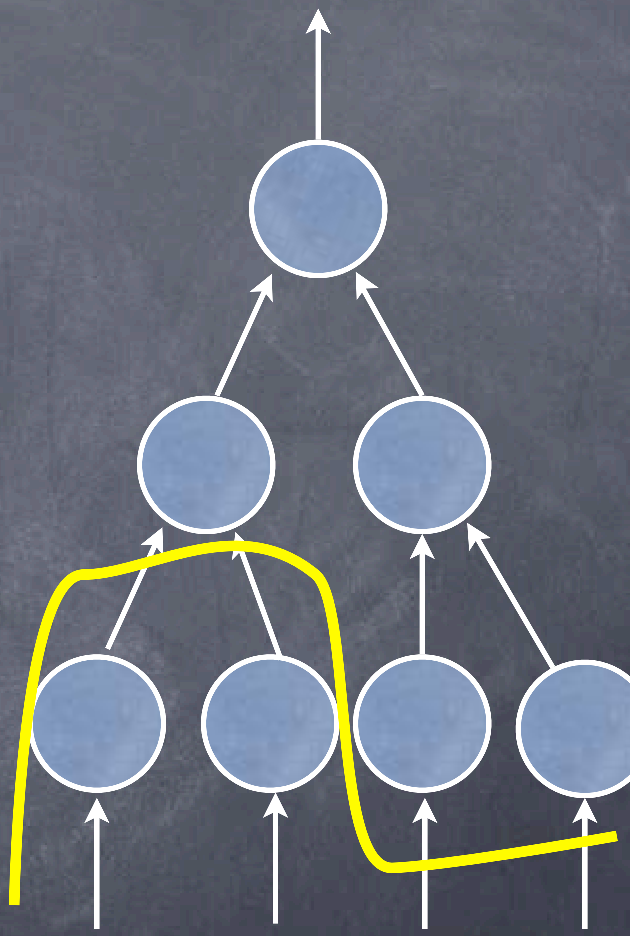

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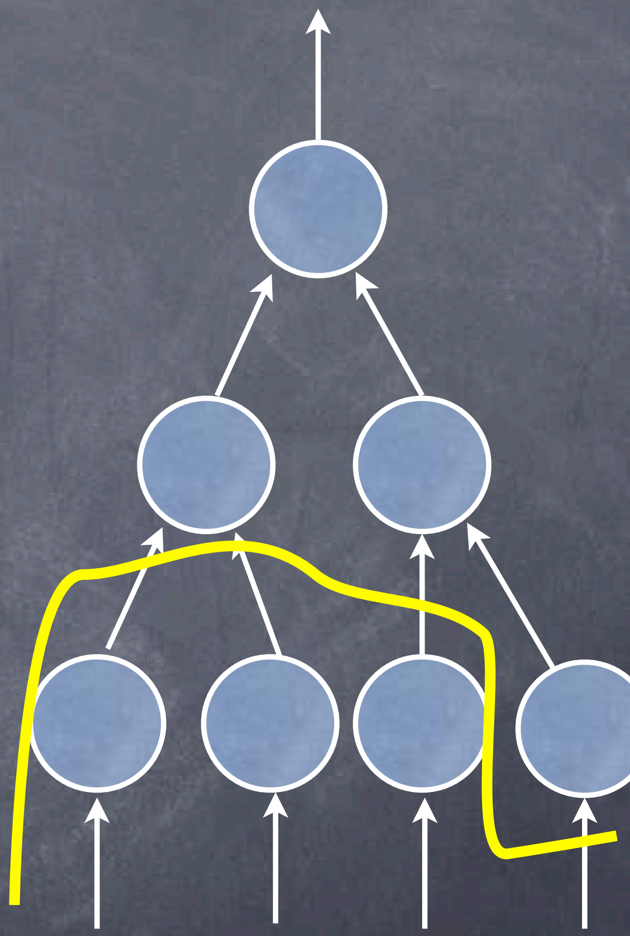

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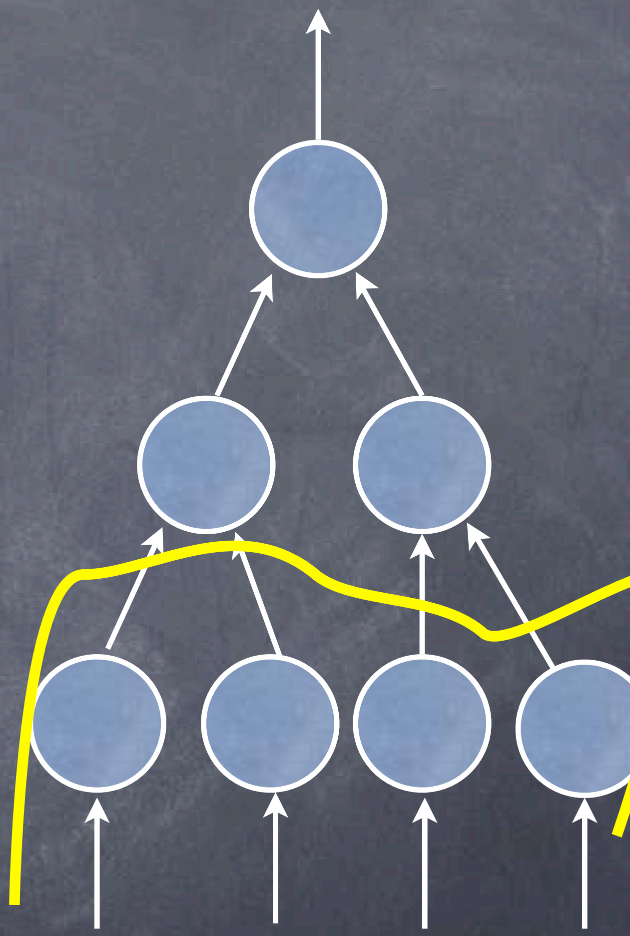

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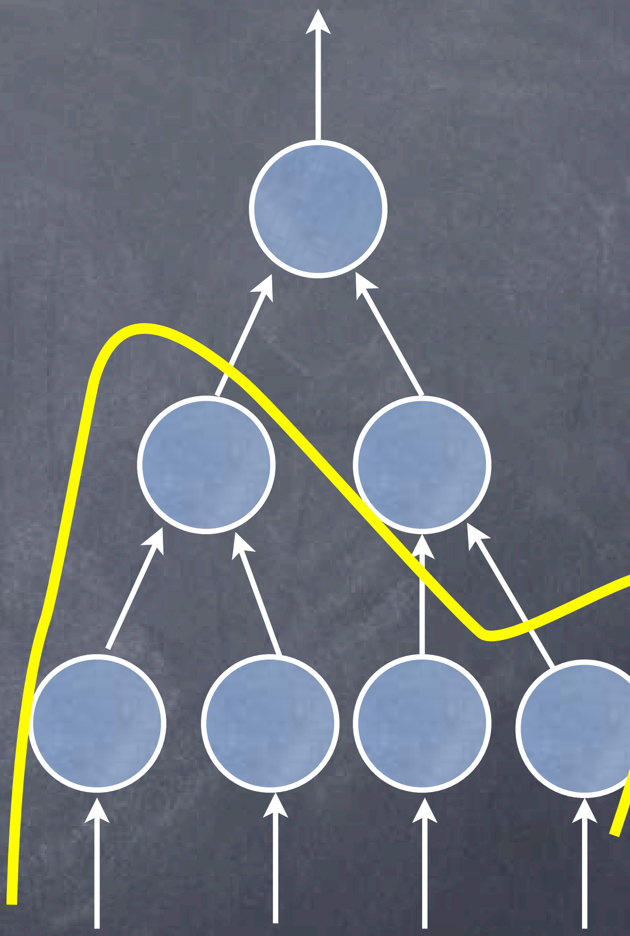

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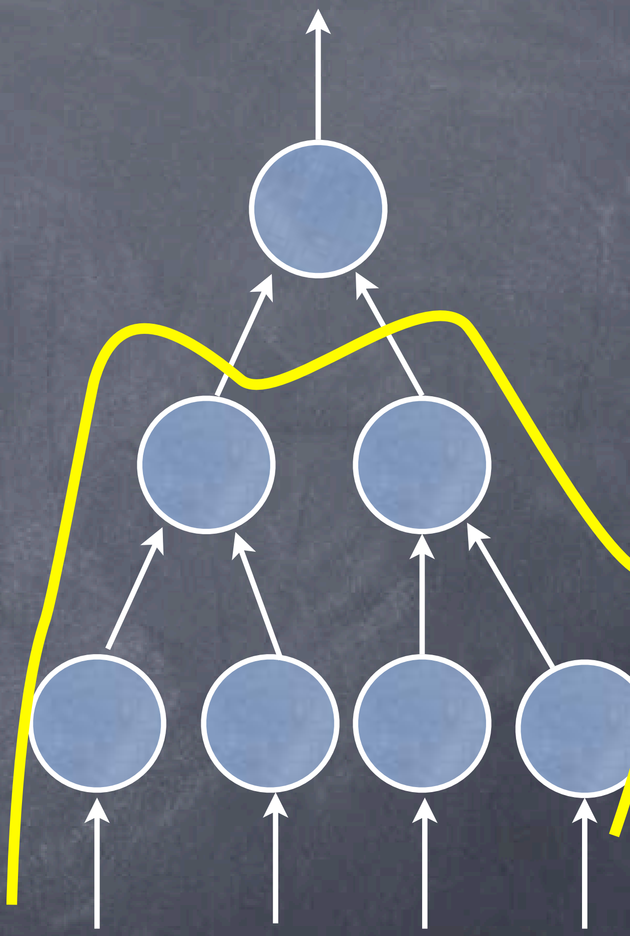

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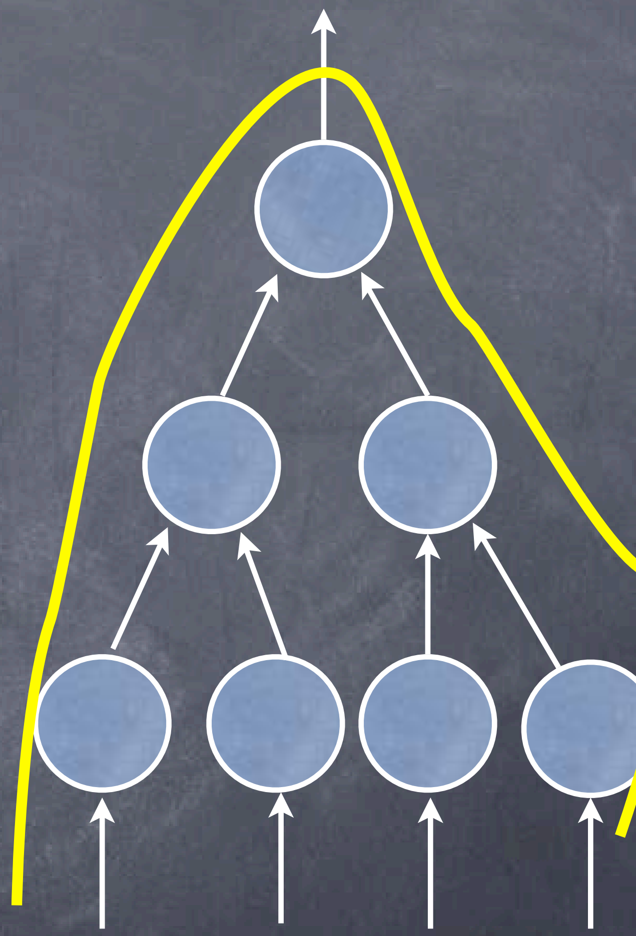

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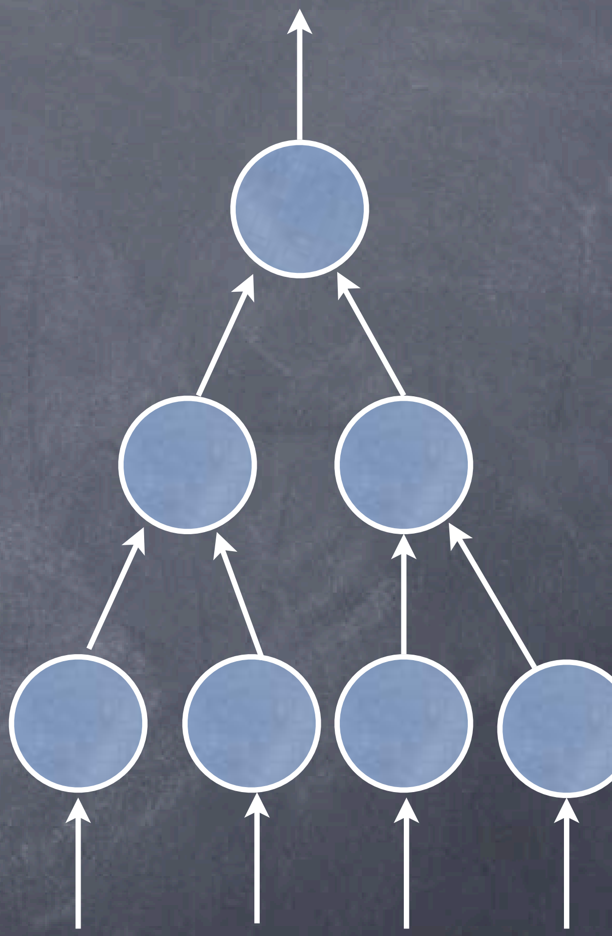

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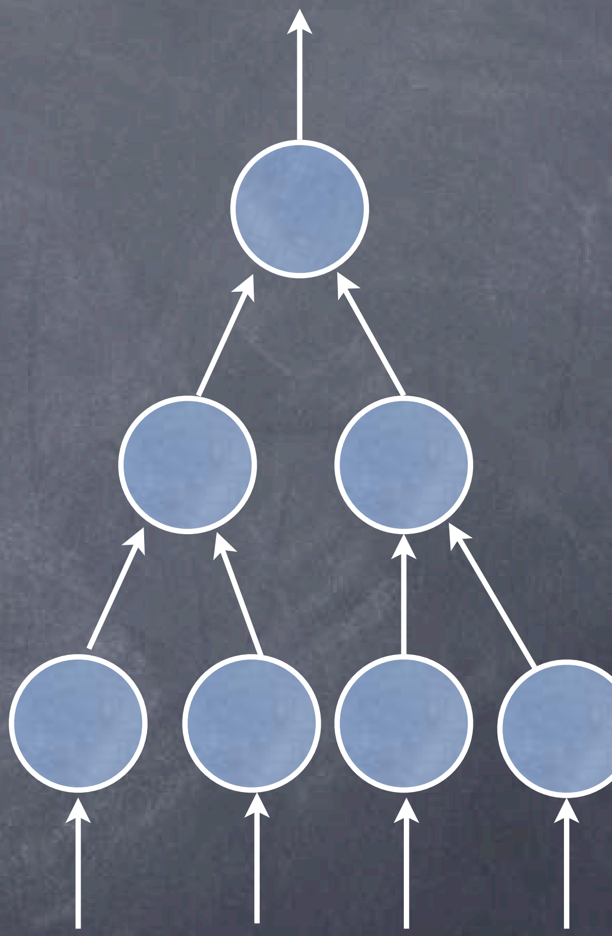

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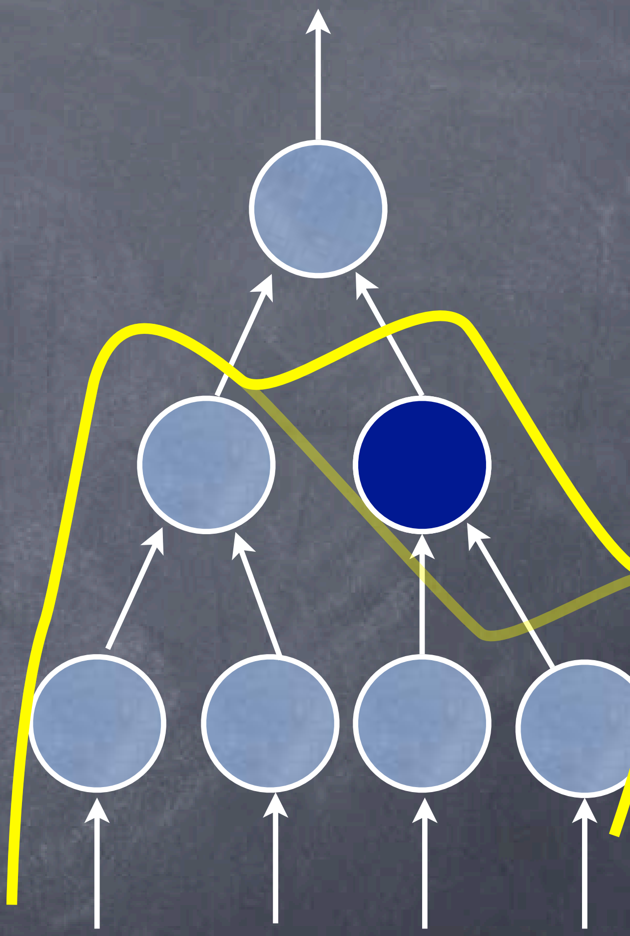

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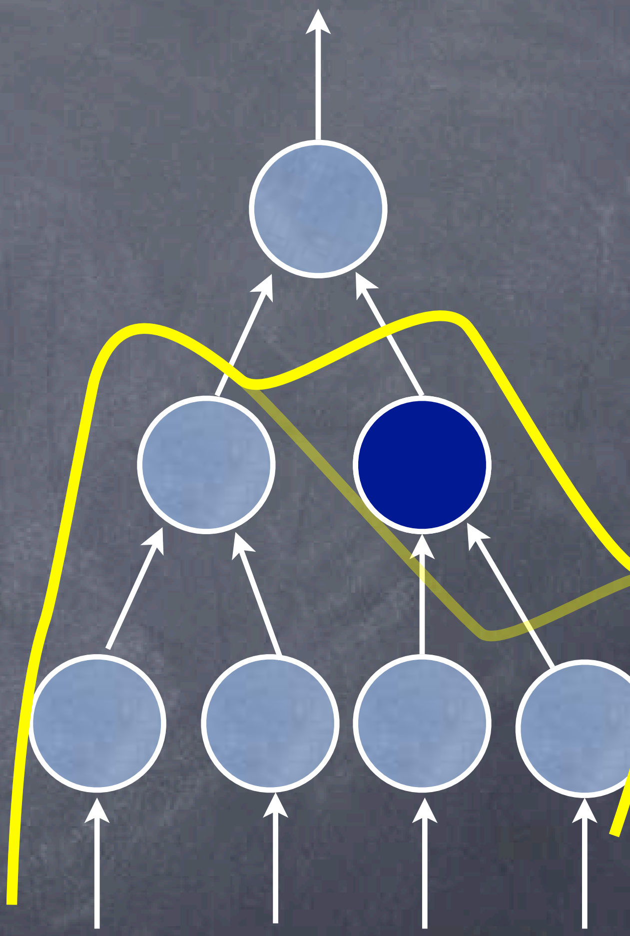

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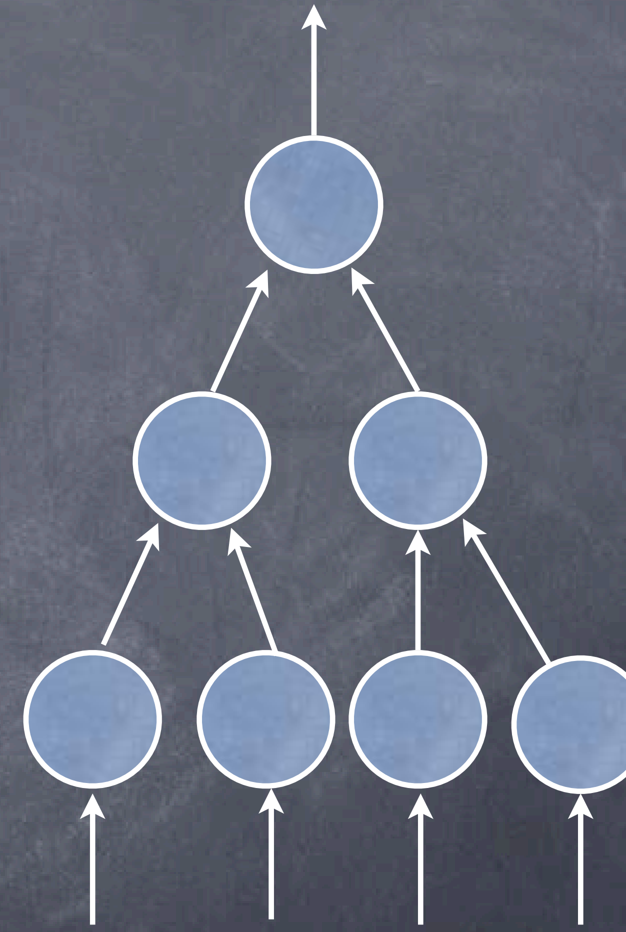

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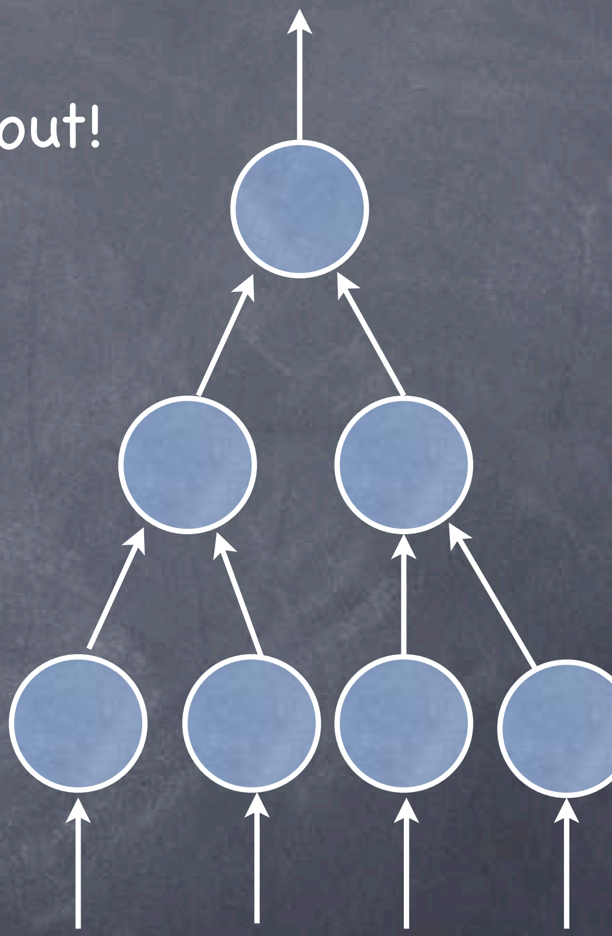

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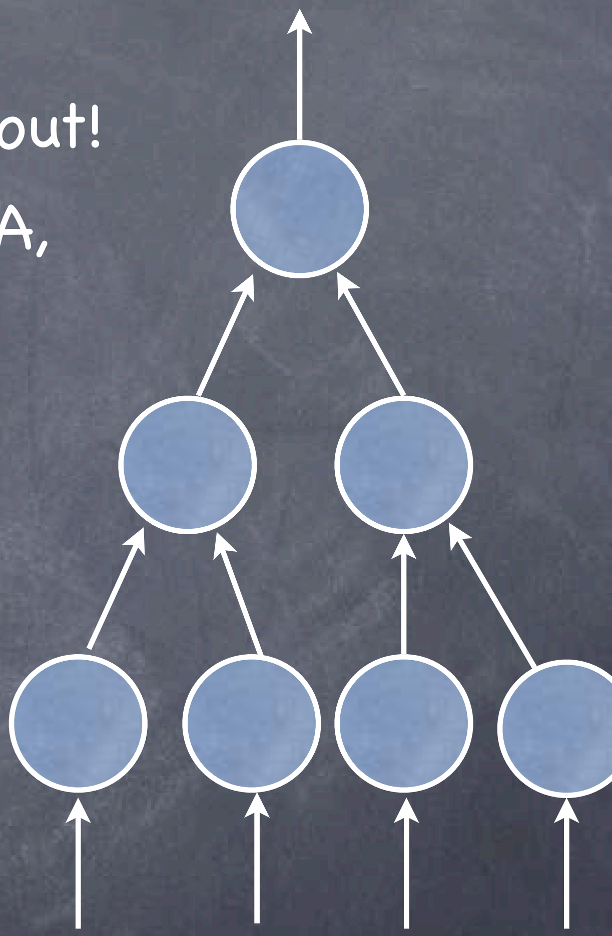

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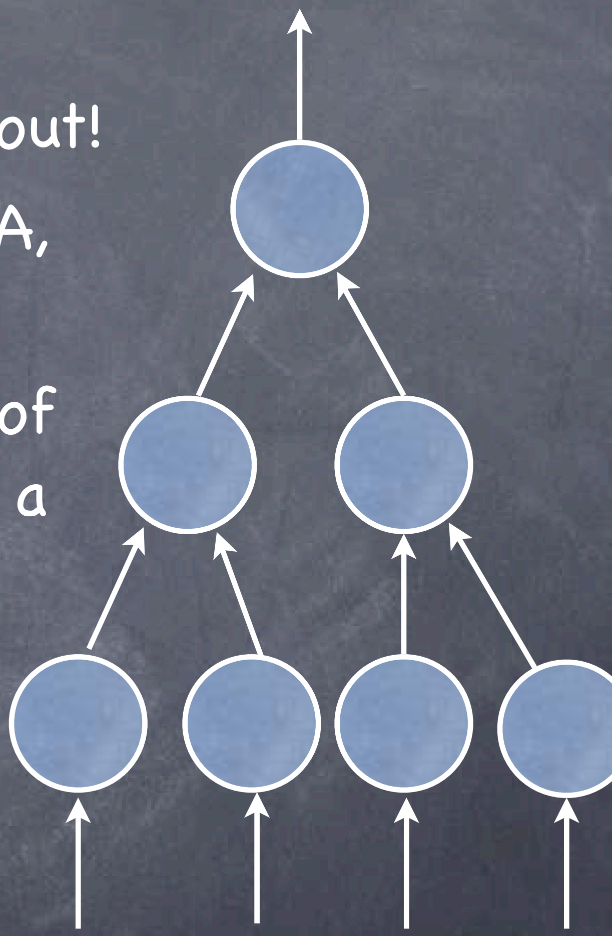

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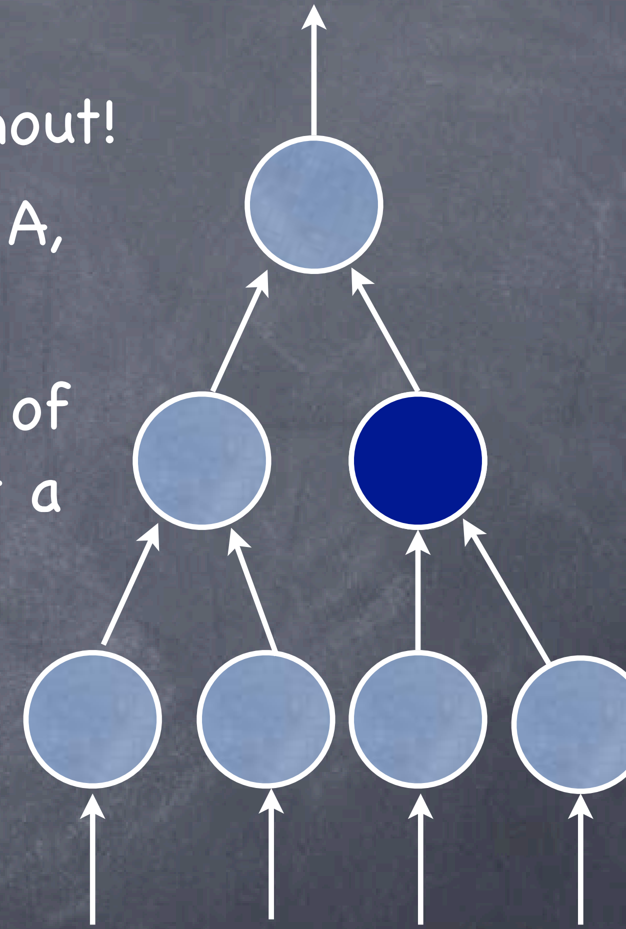

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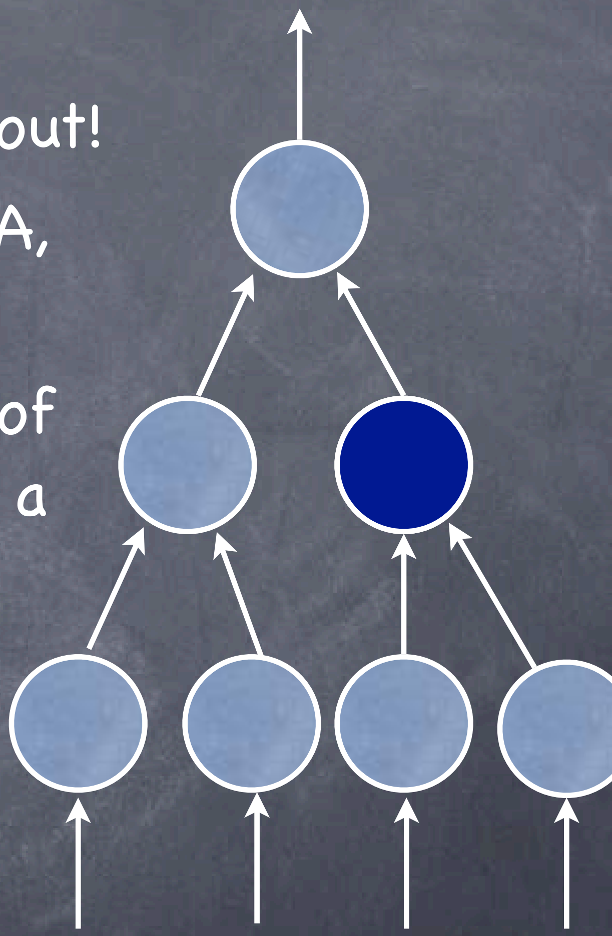

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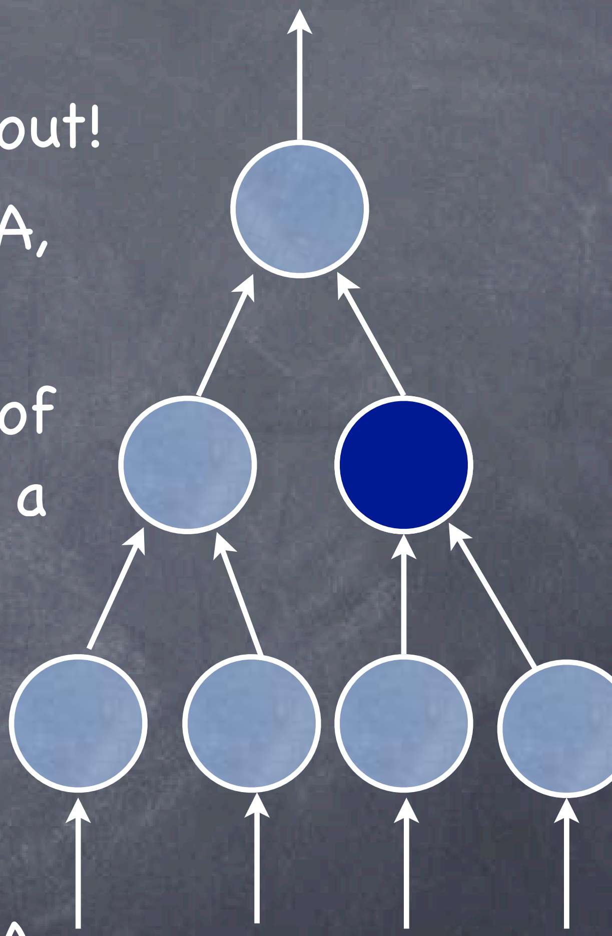

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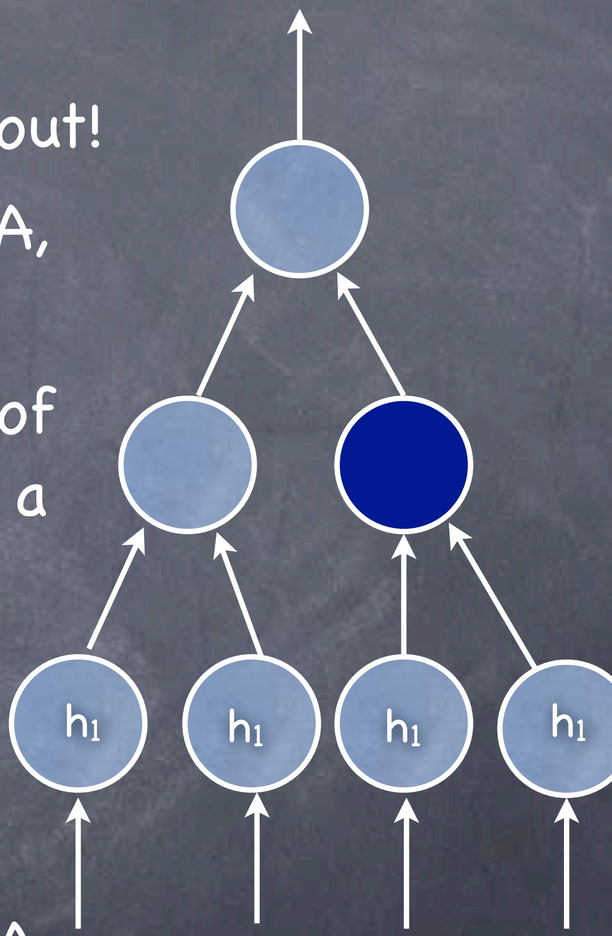

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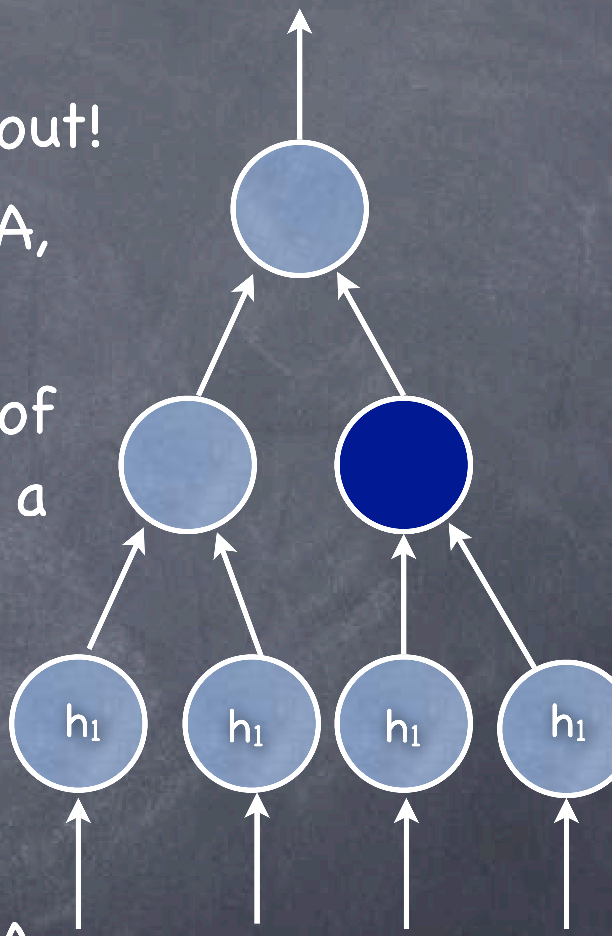

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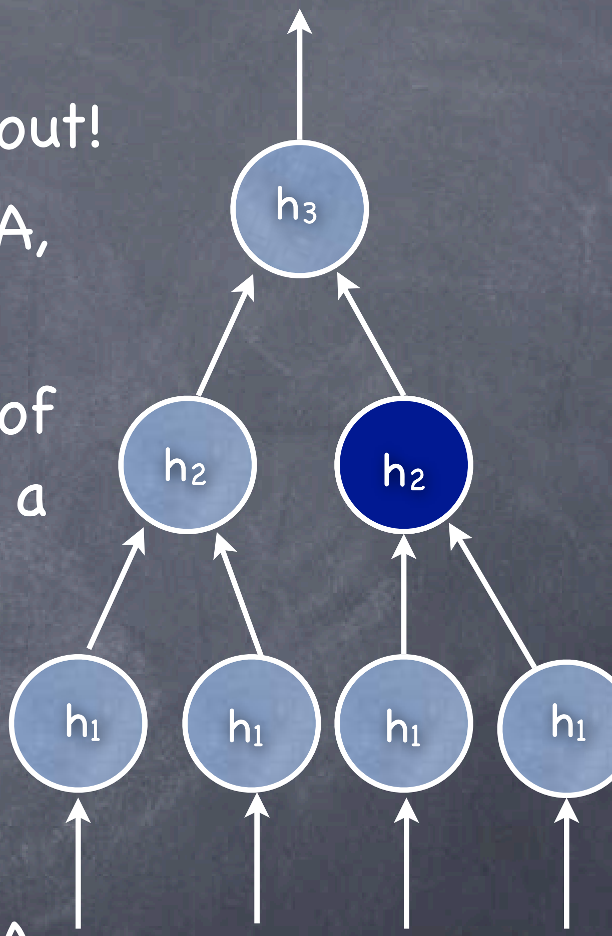

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- Current practice: much less paranoid; faith on efficient, ad hoc (and unkeyed) constructions (though increasingly under attack)

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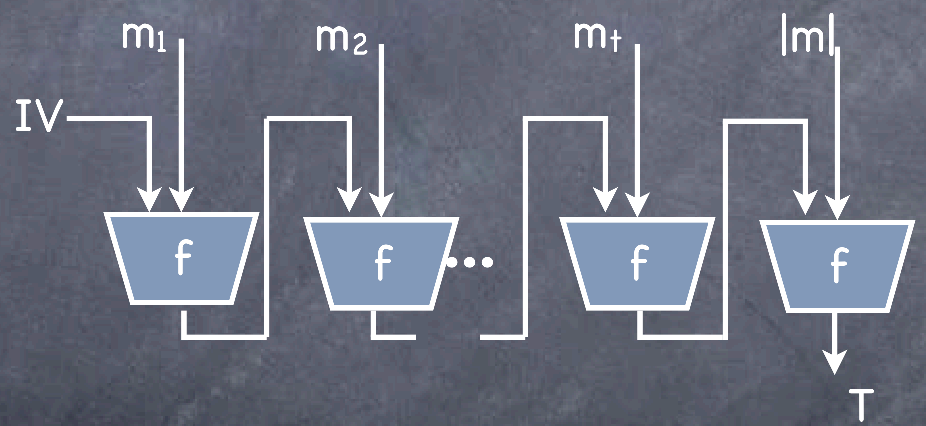

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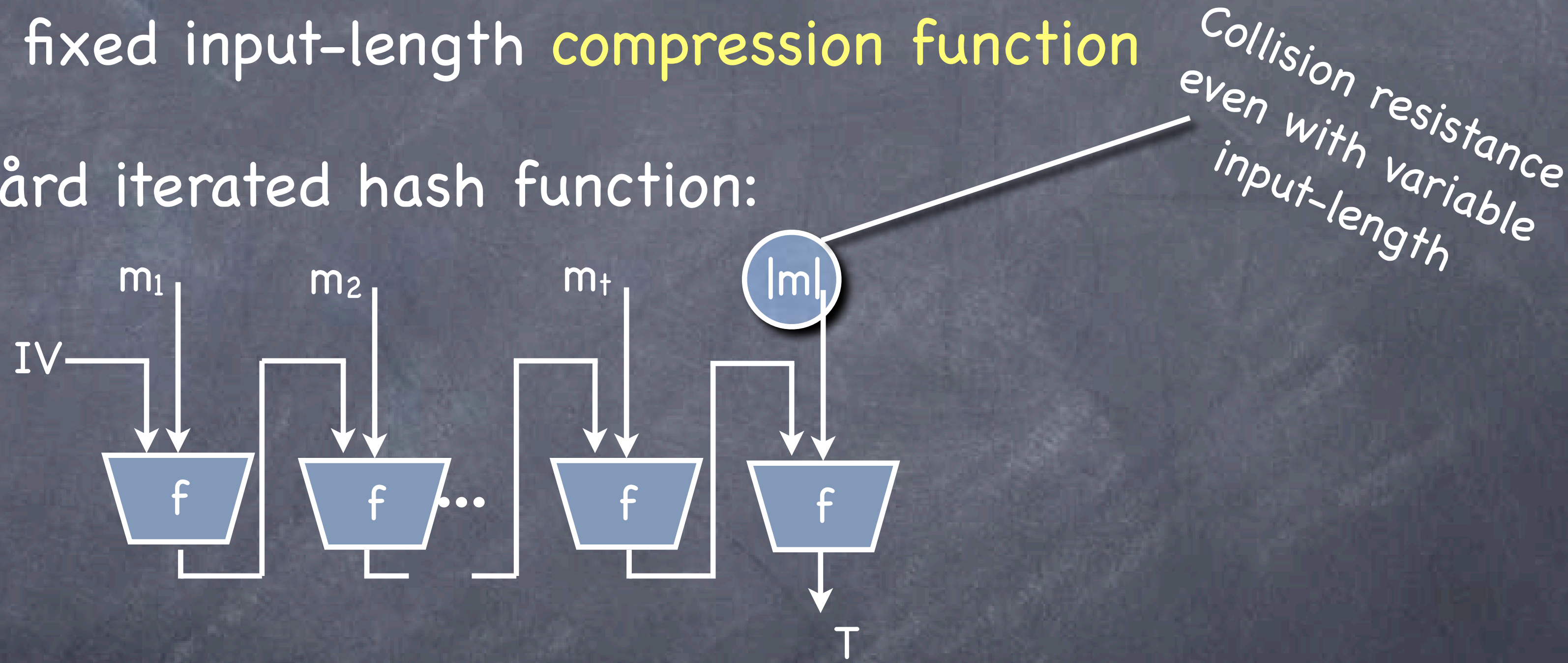

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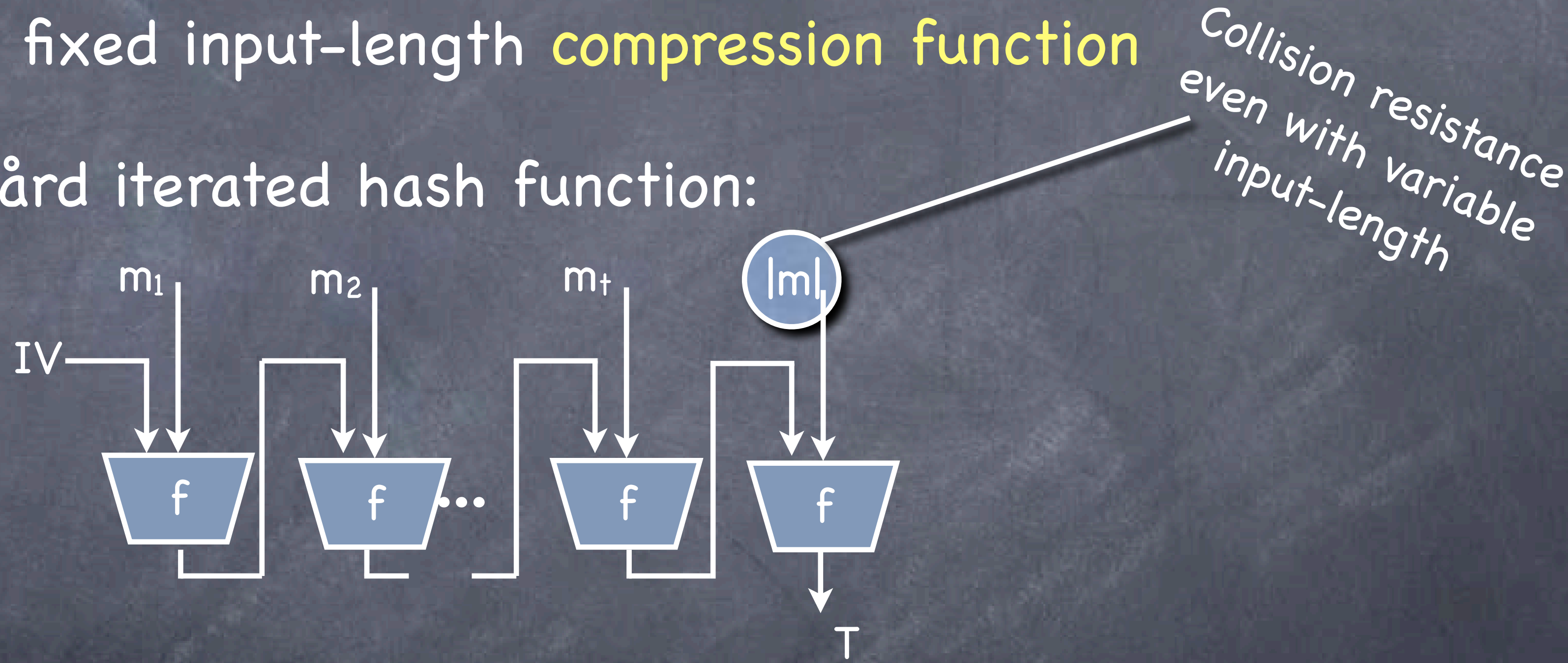

- If f collision resistant (not as "keyed" hash, but "concretely"), then so is the Merkle-Damgård iterated hash-function (for any IV)

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With 2-Universal Hash Functions

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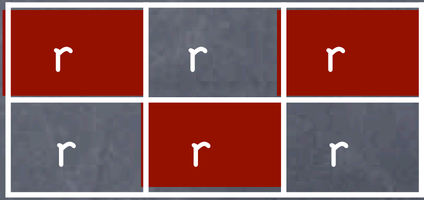

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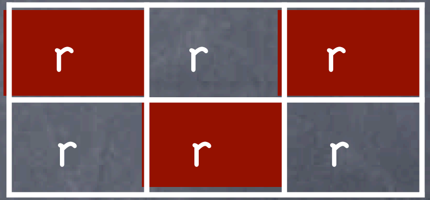

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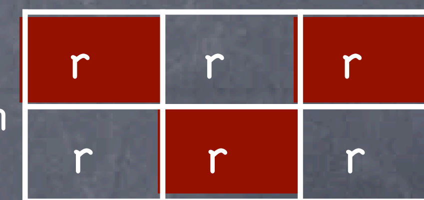

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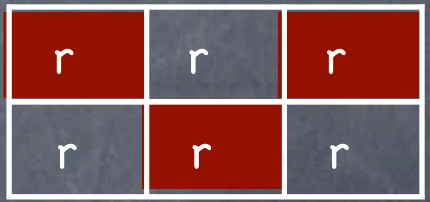

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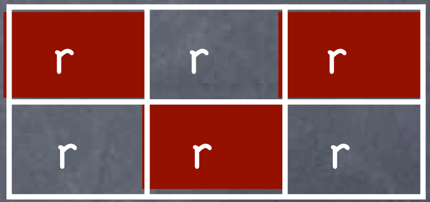

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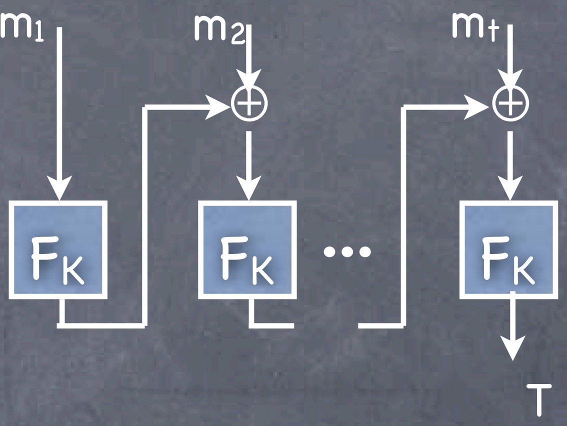

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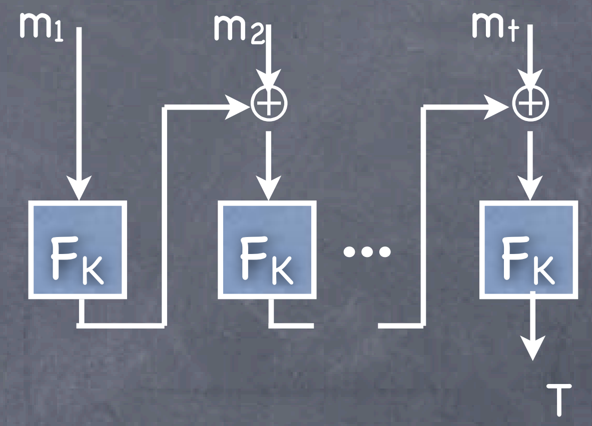

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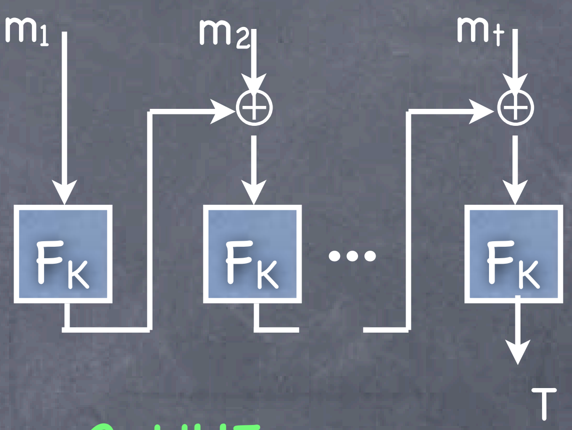

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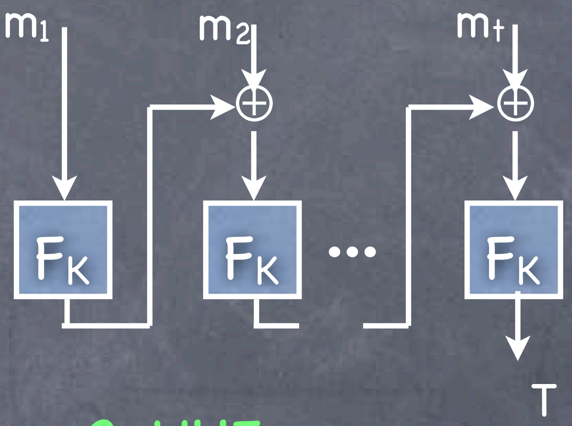
  - $MAC_{K,h}^*(M) = PRF_K(h(M))$ where $h \leftarrow \mathcal{H}$, and $\mathcal{H}$ a 2-UHF
- A proper MAC must work on inputs of variable length
- Making CBC-MAC variable input-length (can be proven secure):
  - Derive K as $F_{K'}(t)$, where t is the number of blocks
  - Or, Use first block to specify number of blocks

MAC

With Combinatorial Hash Functions and PRF



- Recall: PRF is a MAC (on one-block messages)
- CBC-MAC: Extends to any fixed length domain
- Alternate approach (for fixed length domains):
  - $MAC_{K,h}^*(M) = PRF_K(h(M))$ where $h \leftarrow \mathcal{H}$, and $\mathcal{H}$ a 2-UHF
- A proper MAC must work on inputs of variable length
- Making CBC-MAC variable input-length (can be proven secure):
  - Derive K as $F_{K'}(t)$, where t is the number of blocks
  - Or, Use first block to specify number of blocks
  - Or, output not the last tag T, but $F_{K'}(T)$, where K' is an independent key (EMAC)

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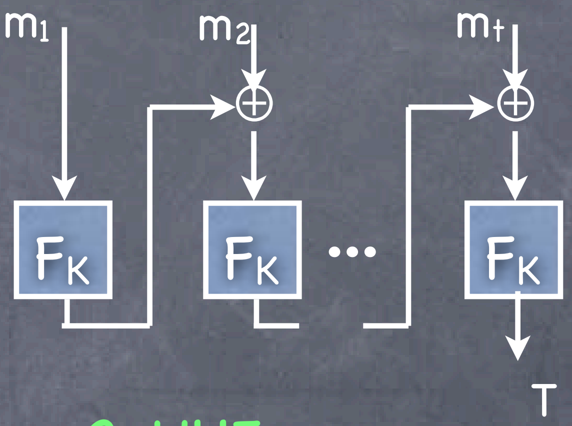

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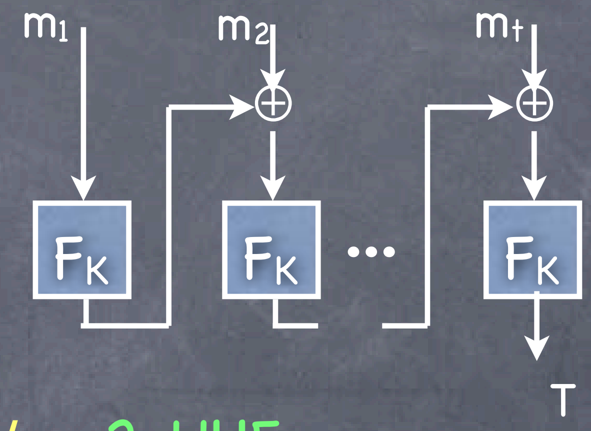

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- Leave variable input-lengths to the hash? (But 2-UHF won't work)

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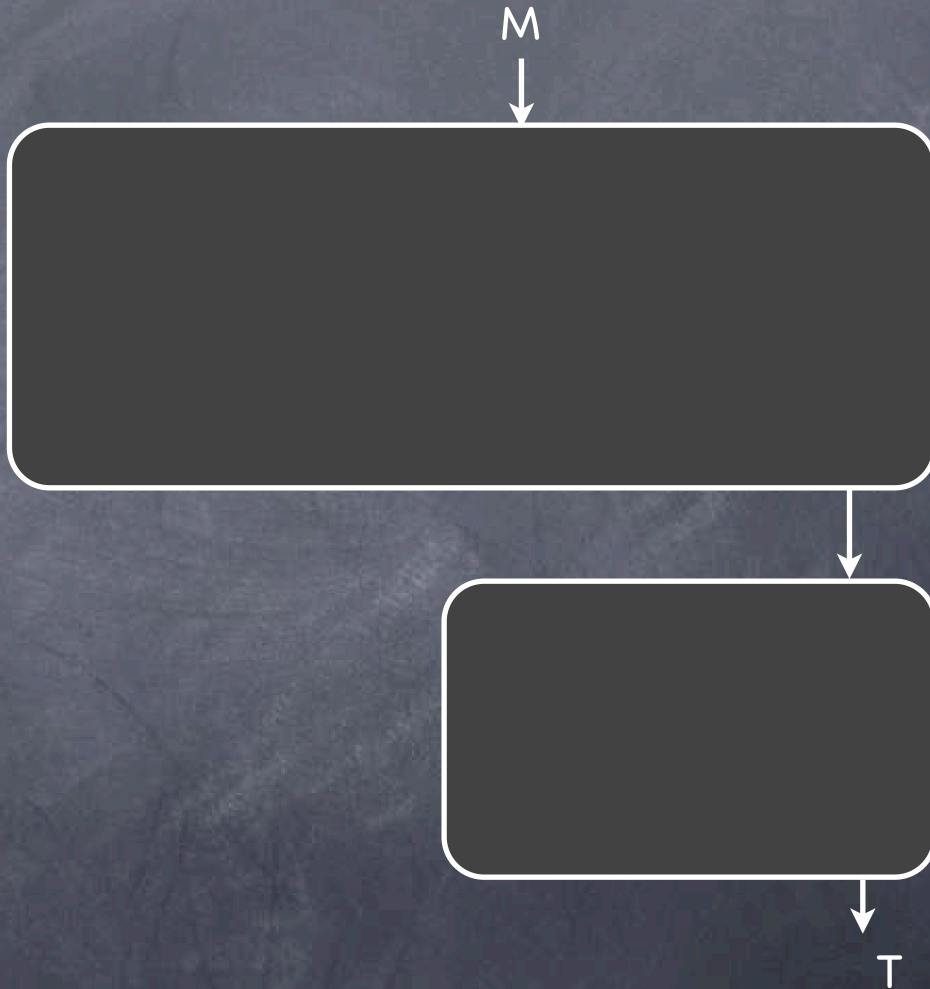
  - Weak-CRHFs can be based on OWF. Can be efficiently constructed from fixed input-length MACs.

HMAC



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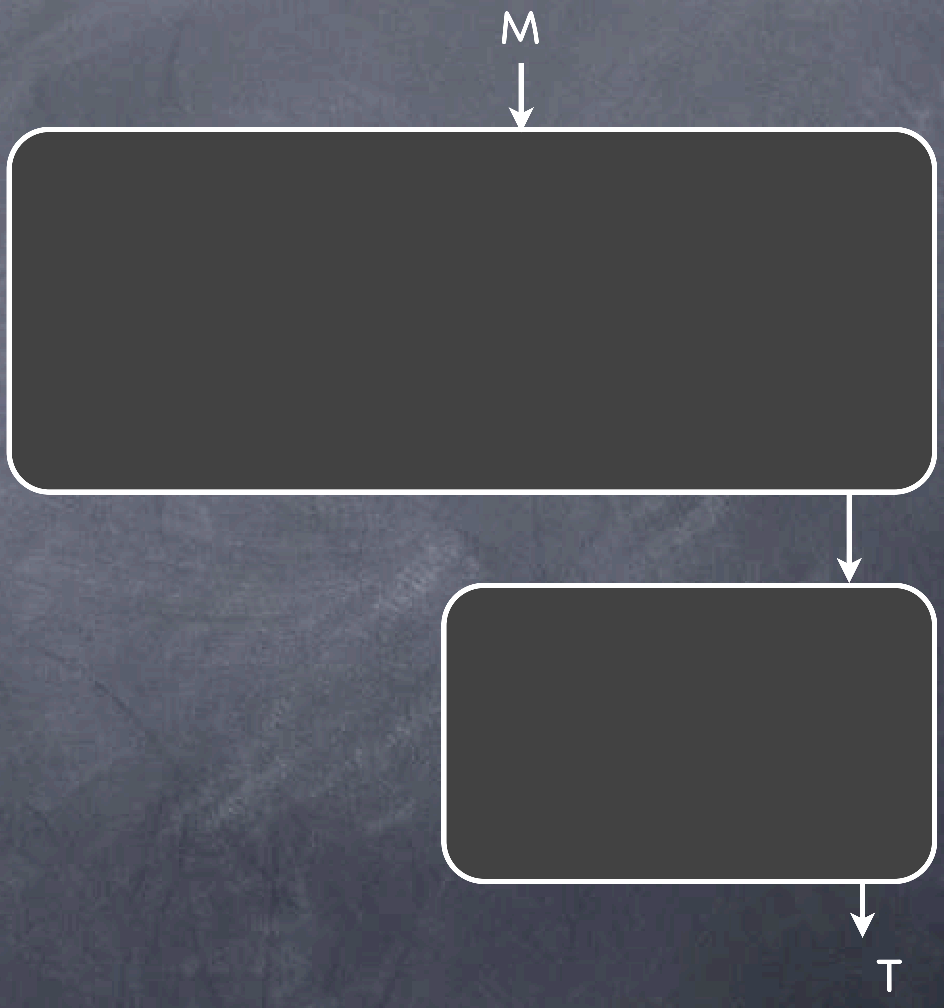

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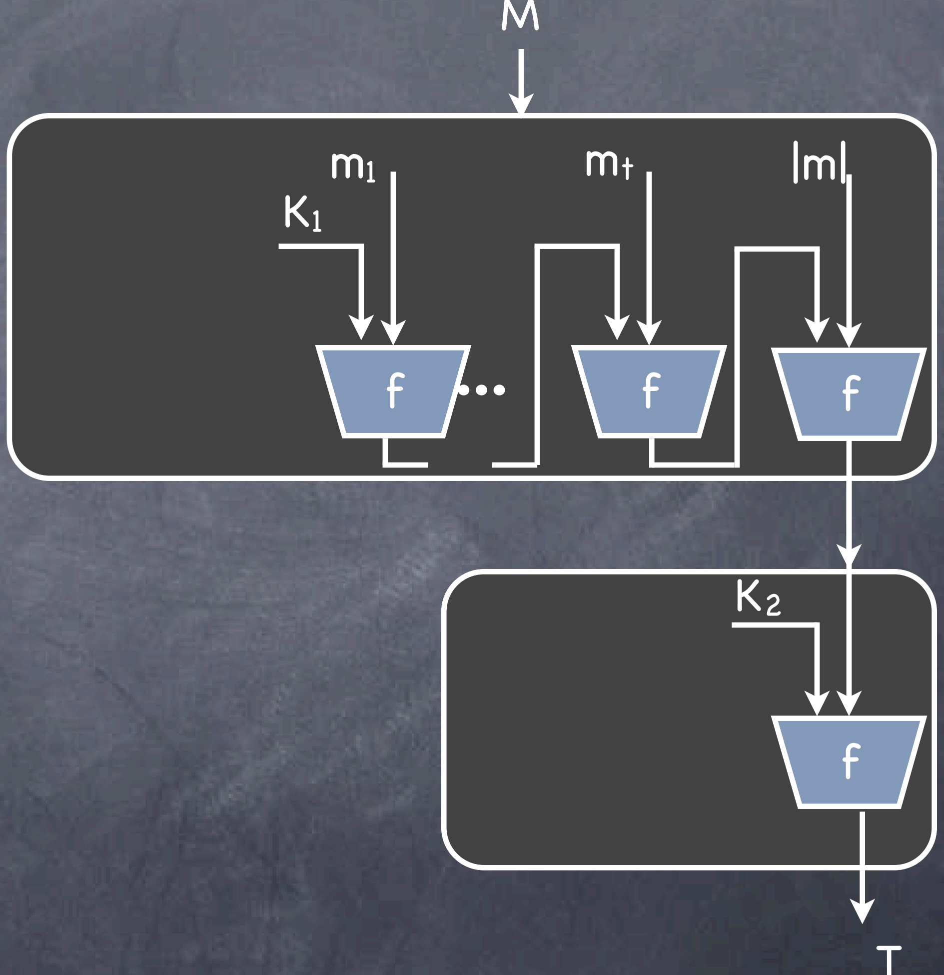

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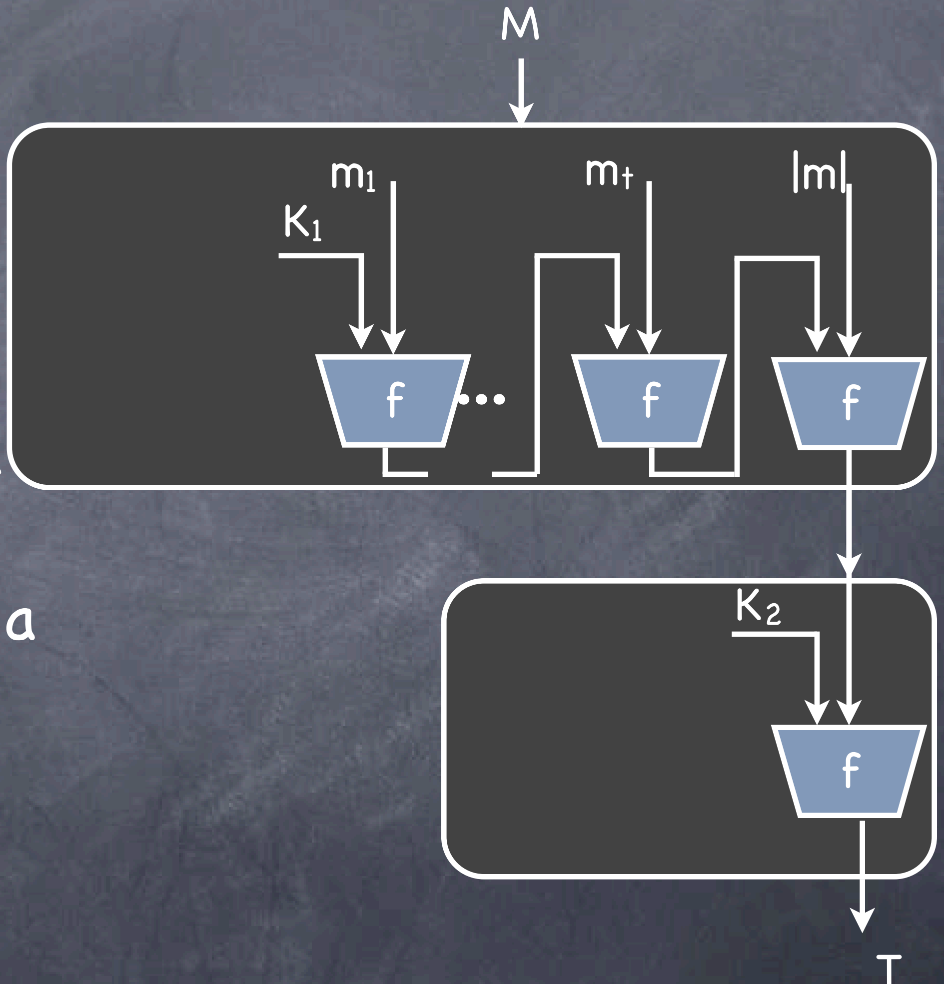

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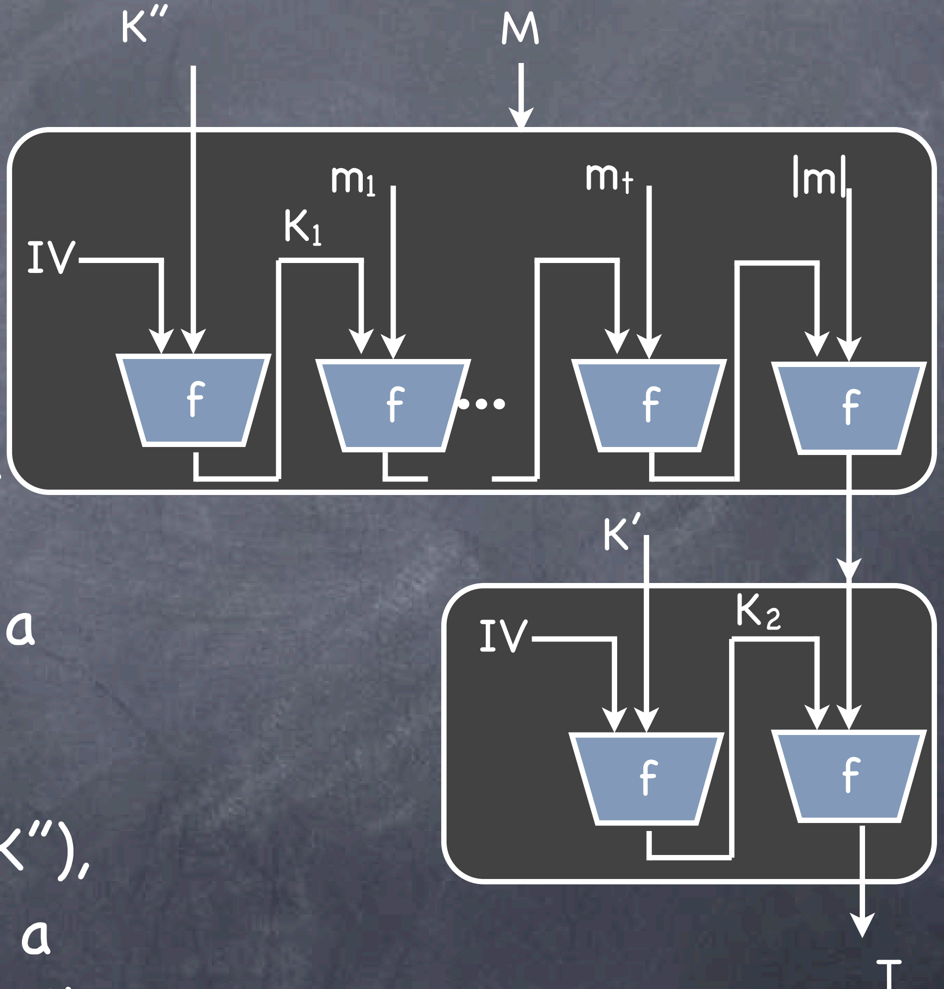
  - In HMAC (K_1,K_2) derived from (K',K''), in turn heuristically derived from a single key K. If f is a (weak kind of) PRF K_1, K_2 can be considered independent



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  - But if H is a Merkle-Damgård iterated-hash function, then there is a simple length-extension attack for forgery
    - (That attack can be fixed by preventing extension: prefix-free encoding)
  - Other suggestions like SHA1(M||K), SHA1(K||M||K) all turned out to be flawed too

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- Next: Digital Signatures