Defining Encryption Lecture 2

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Lecture 2

Towards Defining Secrecy against the Computationally Bounded

First, Symmetric Key Encryption

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 Defining the problem
 We'll do it elaborately, so that it will be easy to see different levels of security

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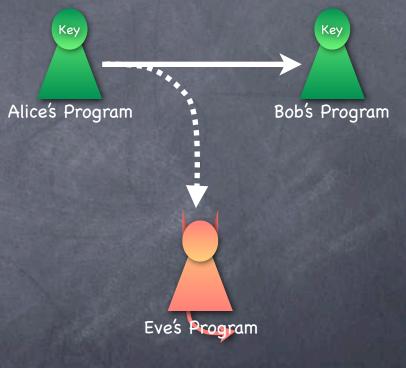
Alice wants Bob to learn a message,
 "without Eve learning it"



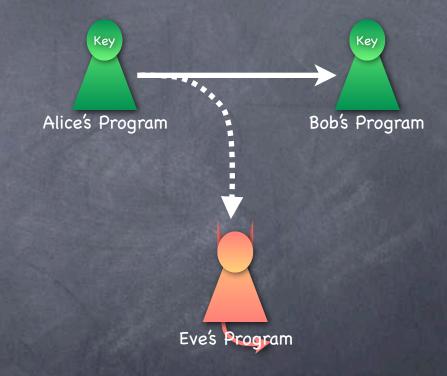




- Alice, Bob and Eve. Alice and Bob share a key (a bit string)
- Alice wants Bob to learn a message, "without Eve learning it"
- Alice can send out a bit string on the channel. Bob and Eve both get it



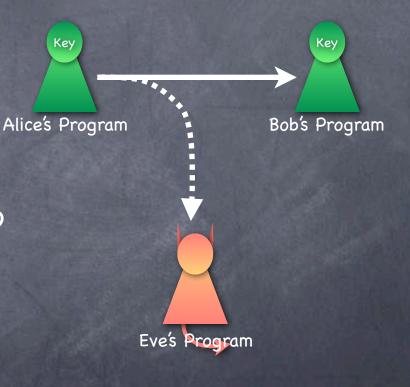
Encryption: Syntax



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Three algorithms

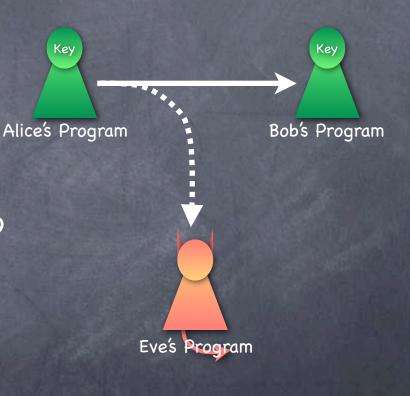
- Key Generation: What Alice and Bob do a priori, for creating the shared secret key
- Encryption: What Alice does with the message and the key to obtain a "ciphertext"
- Decryption: What Bob does with the ciphertext and the key to get the message out of it

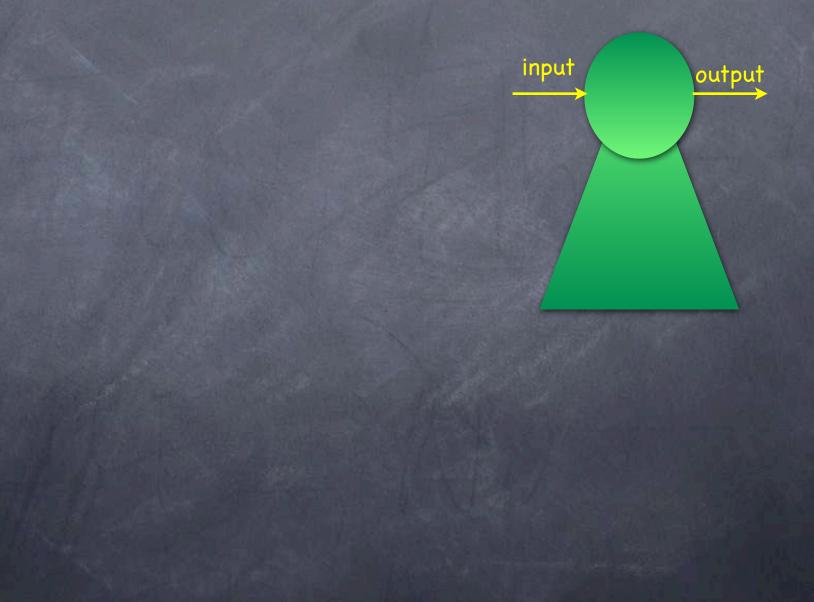


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- Decryption: What Bob does with the ciphertext and the key to get the message out of it
- All of these are (probabilistic) computations





input

output

 In our model (standard model) parties are programs (computations, say Turing Machines)

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Can be probabilistic

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coin

flips

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coin

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output

input

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 - Can be probabilistic
 - Sometimes stateful

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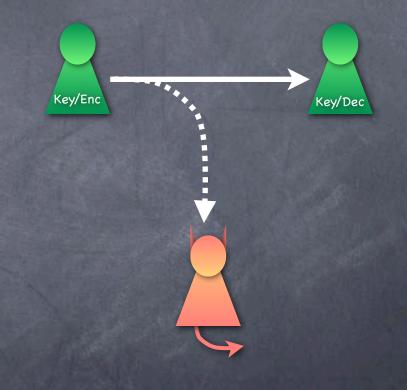
state

coin

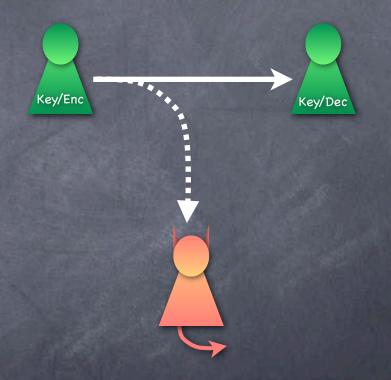
flips

output

input

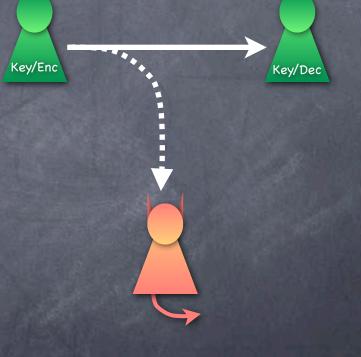


Where does the message come from?



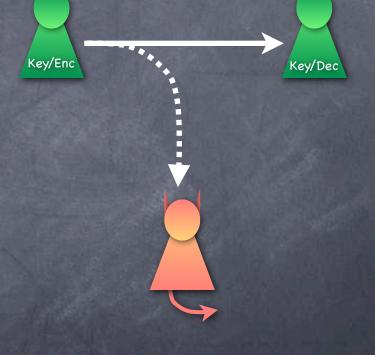
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 Eve might already have partial information about the message, or might receive such information later



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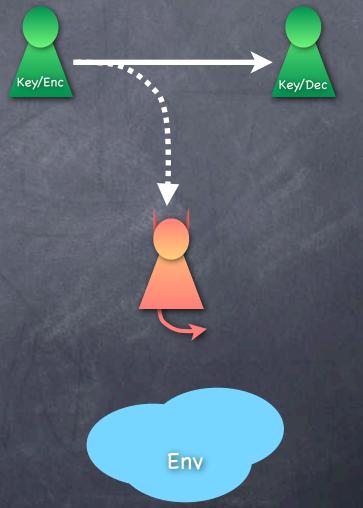
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 Includes the operating systems and other programs run by the participants, as well as other parties, if in a network



Env

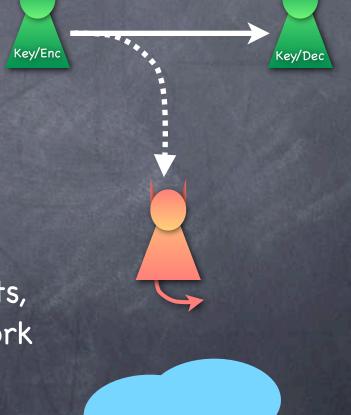
Key/Enc

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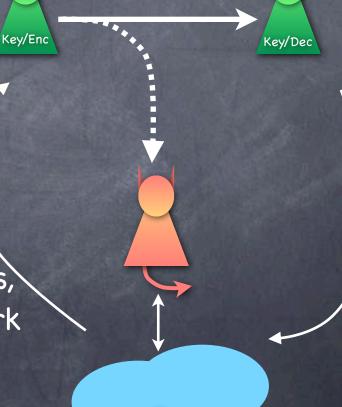
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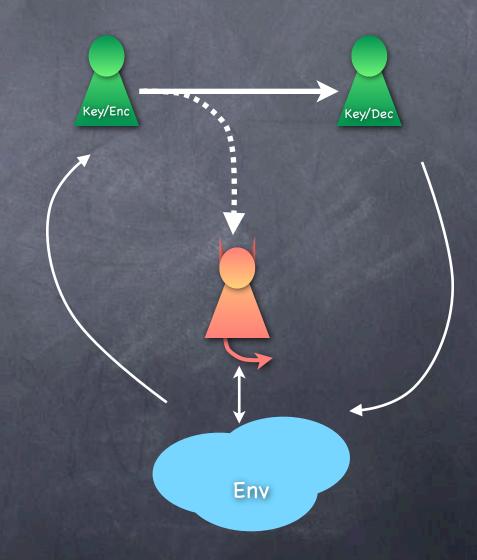
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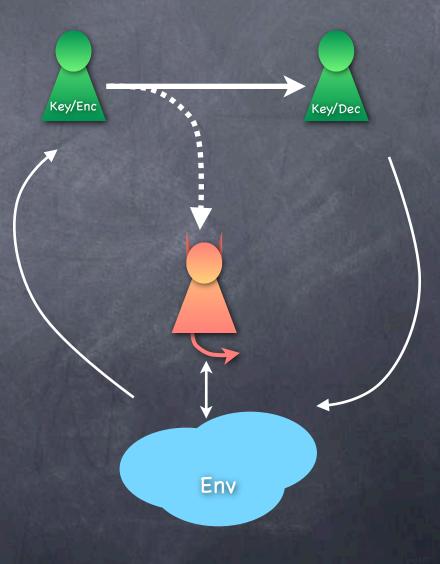
- Includes the operating systems and other programs run by the participants, as well as other parties, if in a network
- Abstract entity from which the input comes and to which the output goes.
 Arbitrarily influenced by Eve



Env

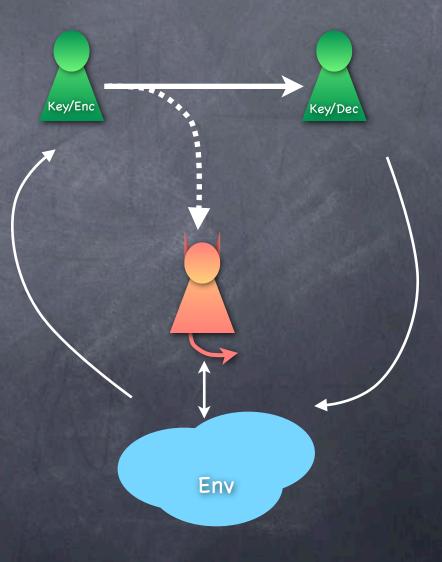


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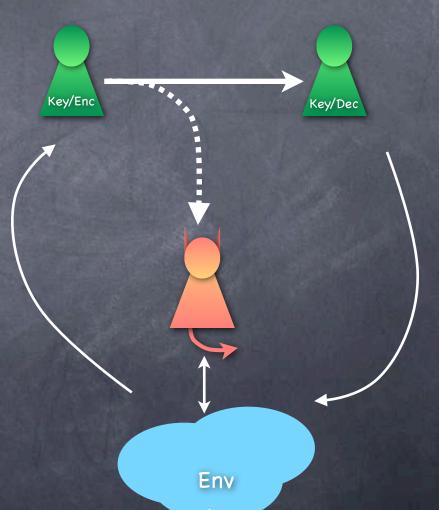
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 Effects in the environment: modeled as a bit in the environment (called the output bit)



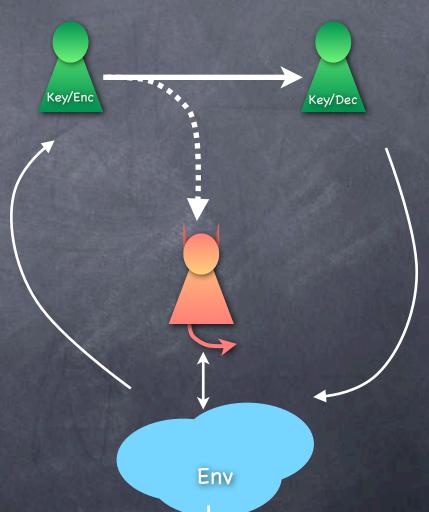
Defining Security

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Key/Enc

Key/Dec

Env

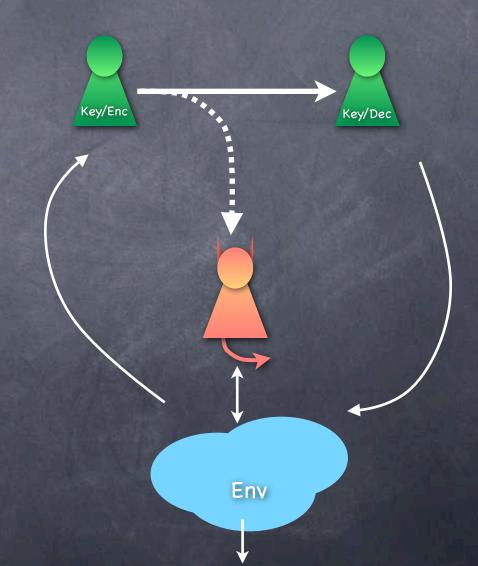
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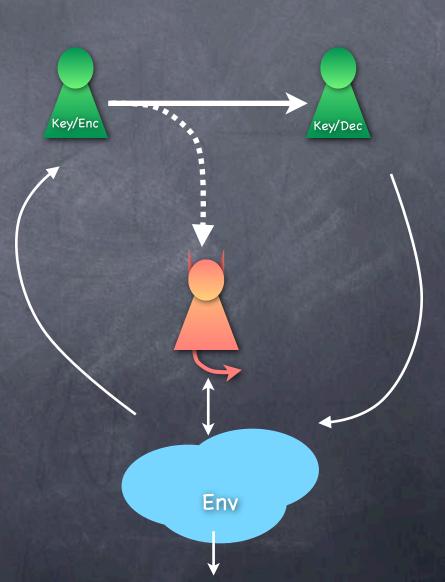
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What is bad?

Anything that Eve couldn't have caused if an "ideal channel" was used

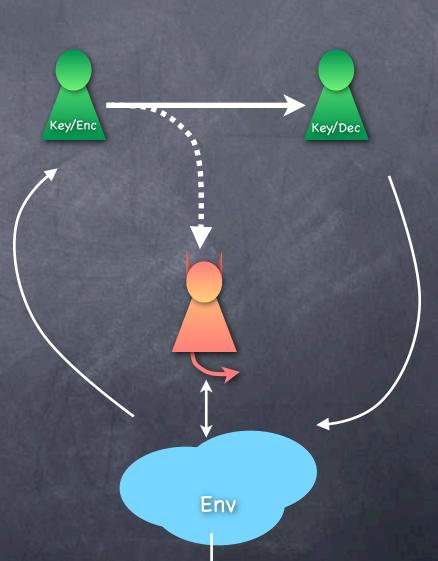


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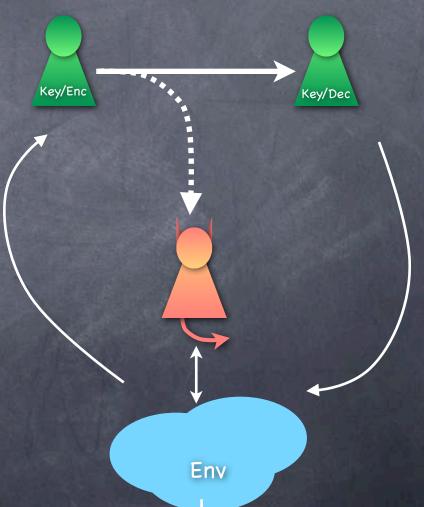


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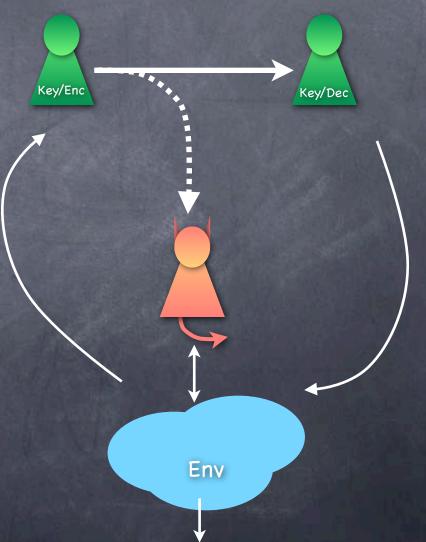
 IDEAL world: Message sent over a (physically) secure channel. No encryption in this world.

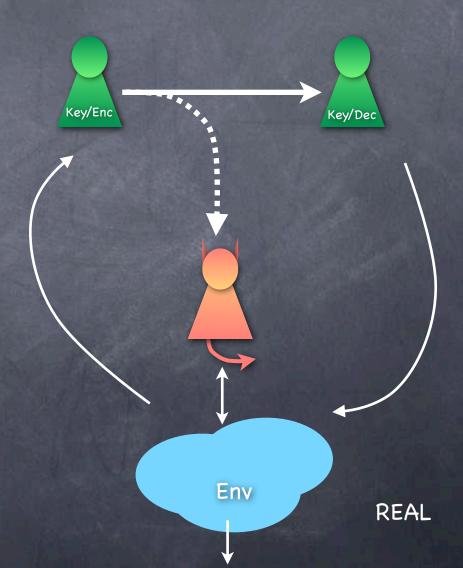


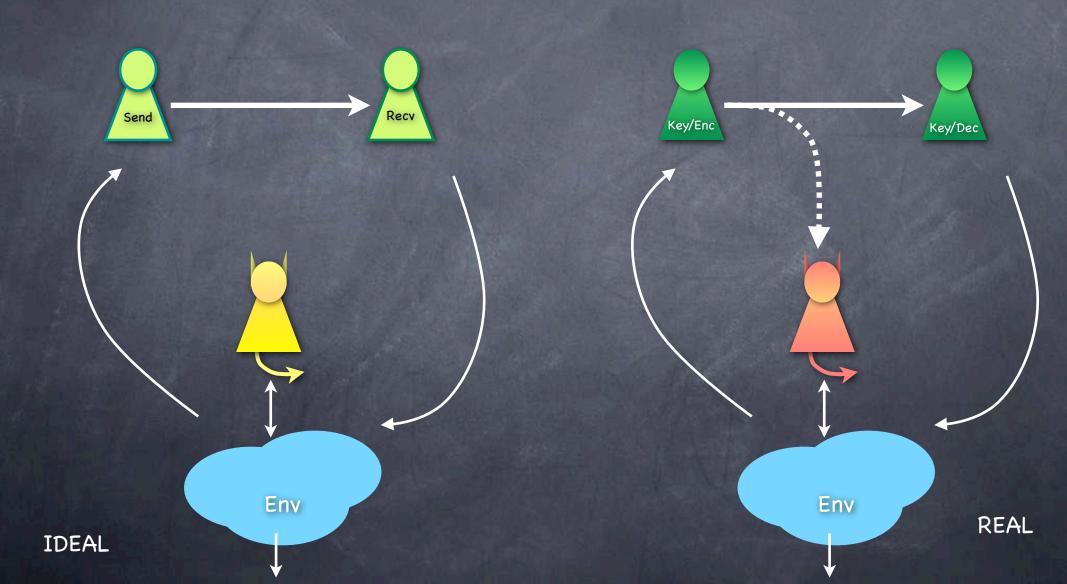
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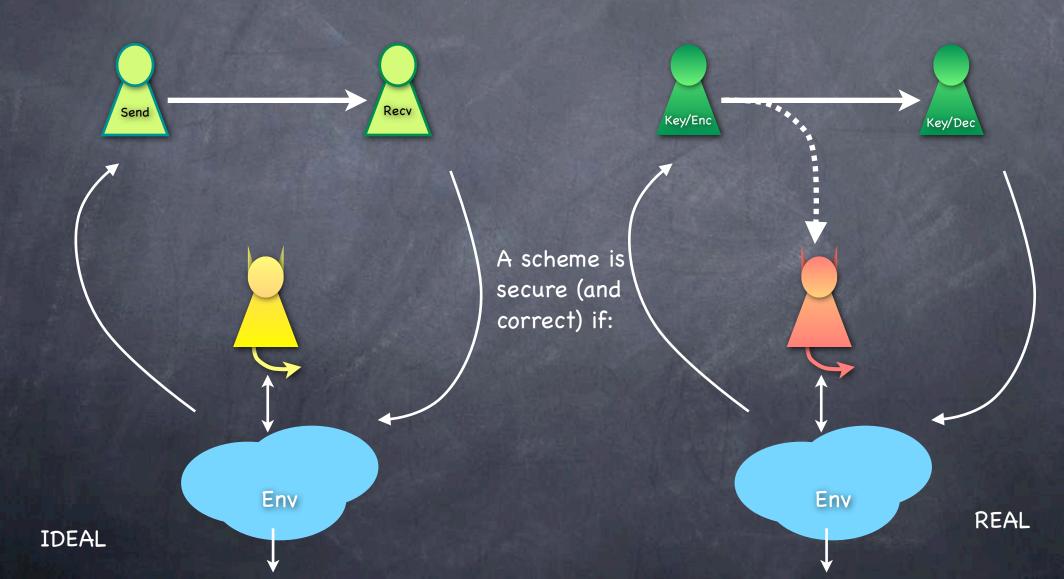


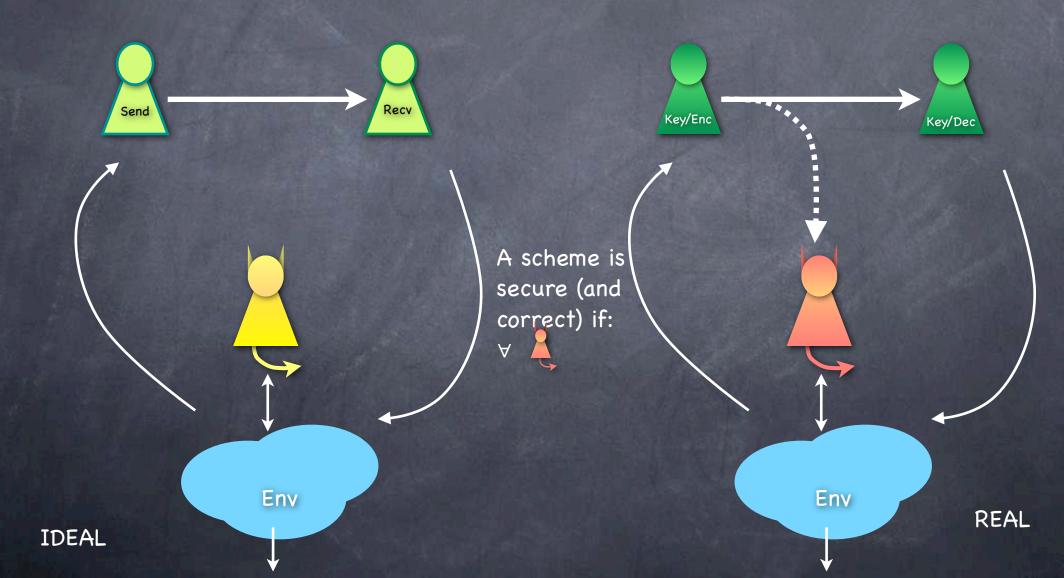
- Eve shouldn't produce any more effects than she could have in the ideal world
 - IDEAL world: Message sent over a (physically) secure channel. No encryption in this world.
 - REAL world: Using encryption
 - Encryption is secure if whatever Eve can do in the REAL world (using some strategy), she can do in the IDEAL world too (using an appropriate strategy)

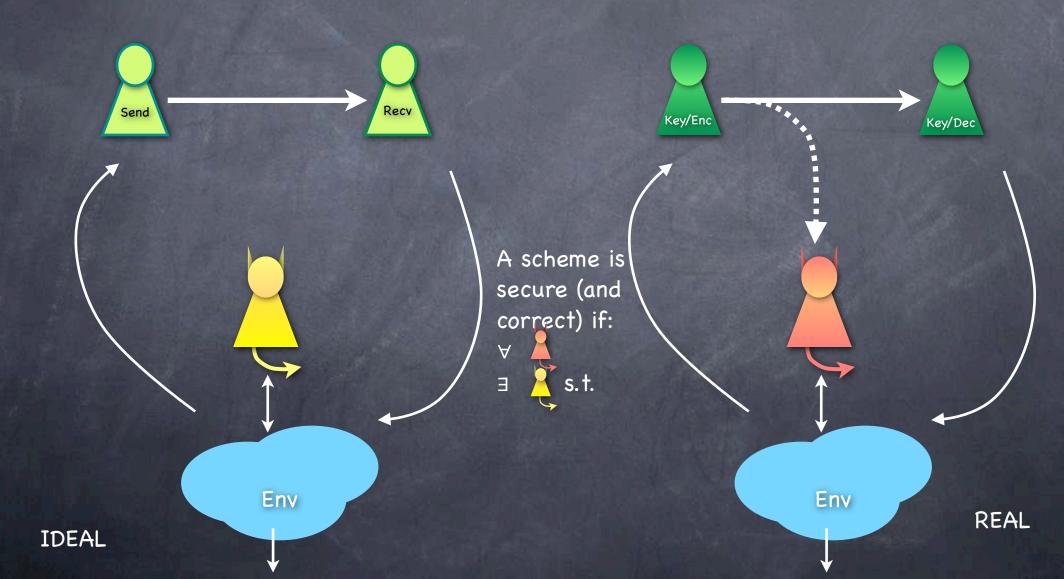


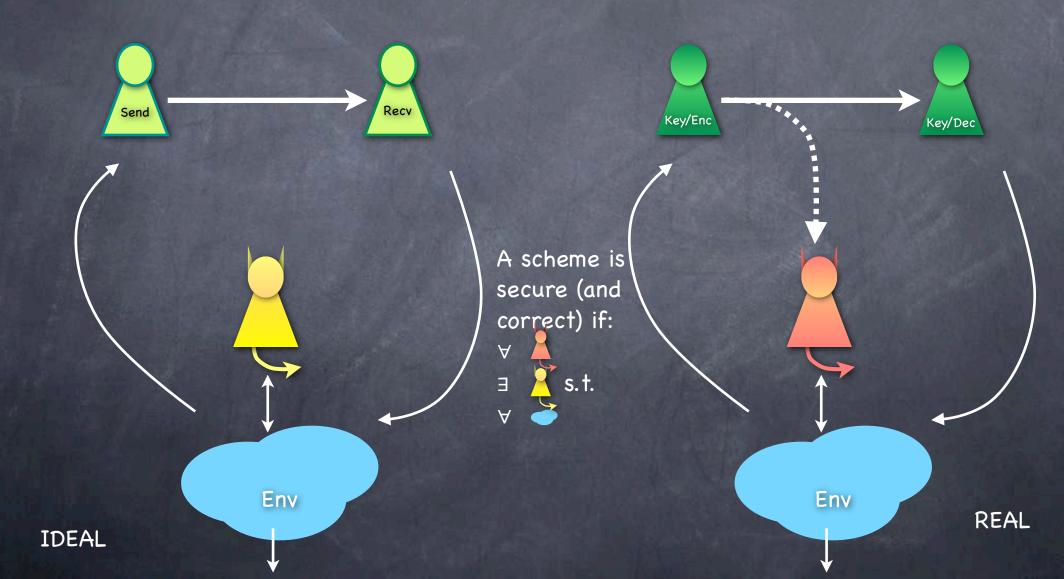


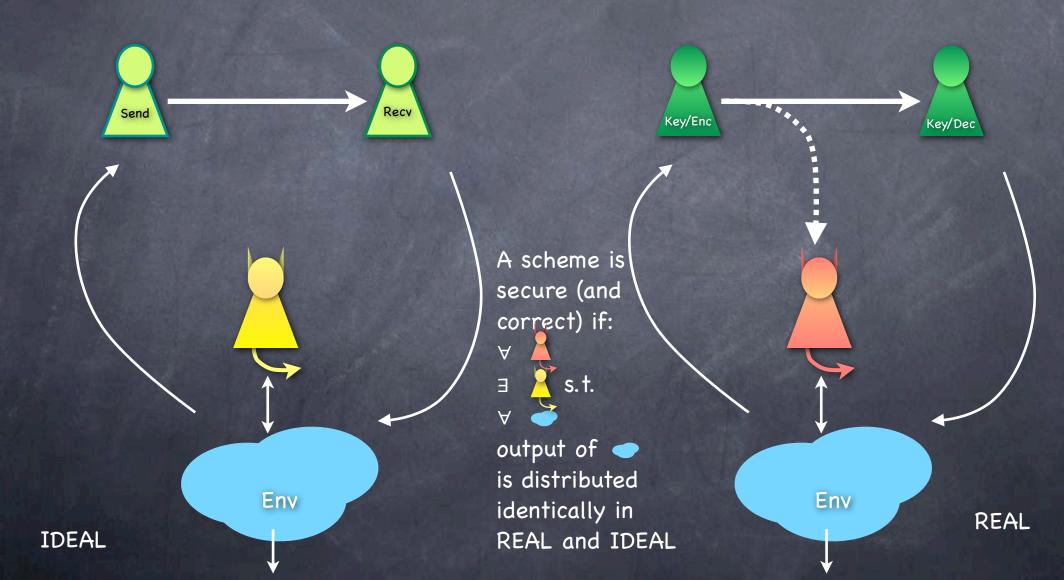












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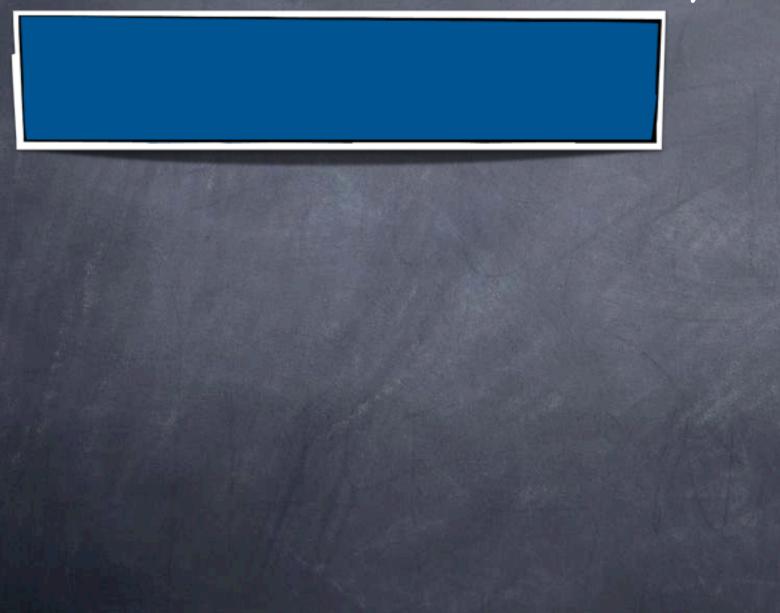
- Security of "one-time encryption"
- Security of (muti-message) encryption
- Security against "active attacks"

- REAL/IDEAL (a.k.a simulation-based) security forms the basic template for a large variety of security definitions
- We will see three definitions of symmetric-key encryption
 - Security of "one-time encryption"
 - Security of (muti-message) encryption
 - Security against "active attacks"
- Will also see alternate (but essentially equivalent) security definitions

Onetime Encryption

Onetime Encryption The Syntax Shared-key (Private-key) Encryption Key Generation: Randomized $K \leftarrow K$, uniformly randomly drawn from the key-space (or according to a key-distribution) Encryption: Deterministic • Enc: $\mathcal{M} \times \mathcal{K} \rightarrow \mathcal{C}$ Decryption: Deterministic

• Dec: $\mathcal{C} \times \mathcal{K} \rightarrow \mathcal{M}$



• Perfect secrecy: \forall m, m' \in \mathscr{M}

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Distribution of the ciphertext

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Distribution of the ciphertext

N M	0	1	2	3
۵	×	У	У	Z
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 Distribution of the ciphertext is defined by the randomness in the key

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Assuming K uniformly drawn from ${\mathscr K}$

Pr[Enc(a,K)=x] = ¼,
Pr[Enc(a,K)=y] = ½,
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Same for Enc(b,K).

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In addition, require correctness

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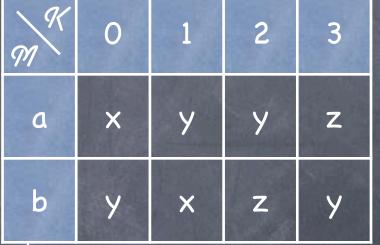
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So E.g. One-time pad: 𝒴 = 𝒴 = 𝒴 = ⟨0,1⟩ⁿ and Enc(m,K) = m⊕K, Dec(c,K) = c⊕K

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Ø ∀ m, K, Dec(Enc(m,K), K) = m

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N
MO123aXYYZbYXZY

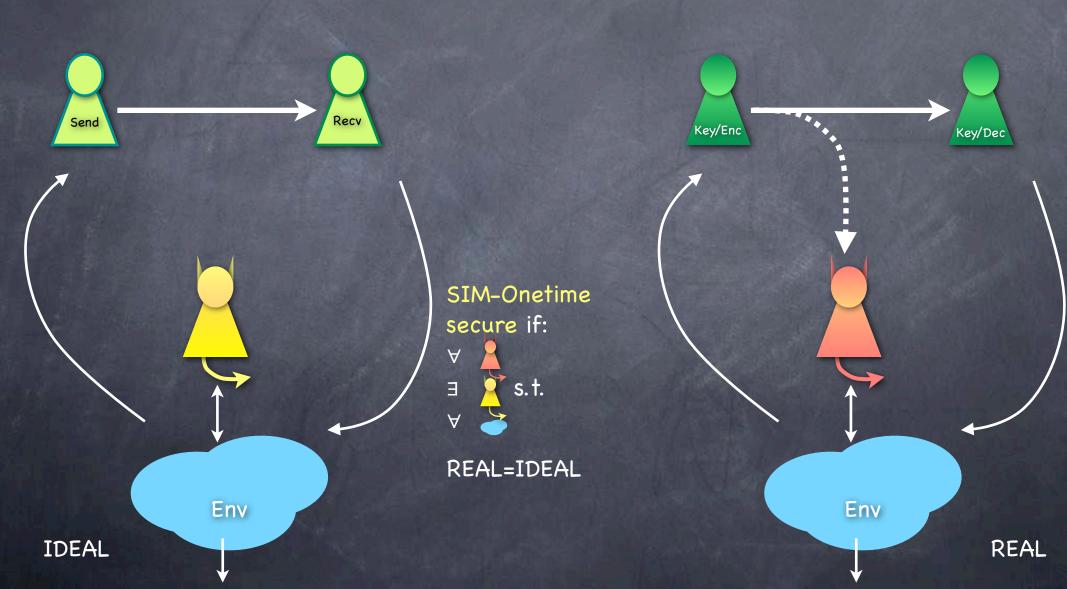
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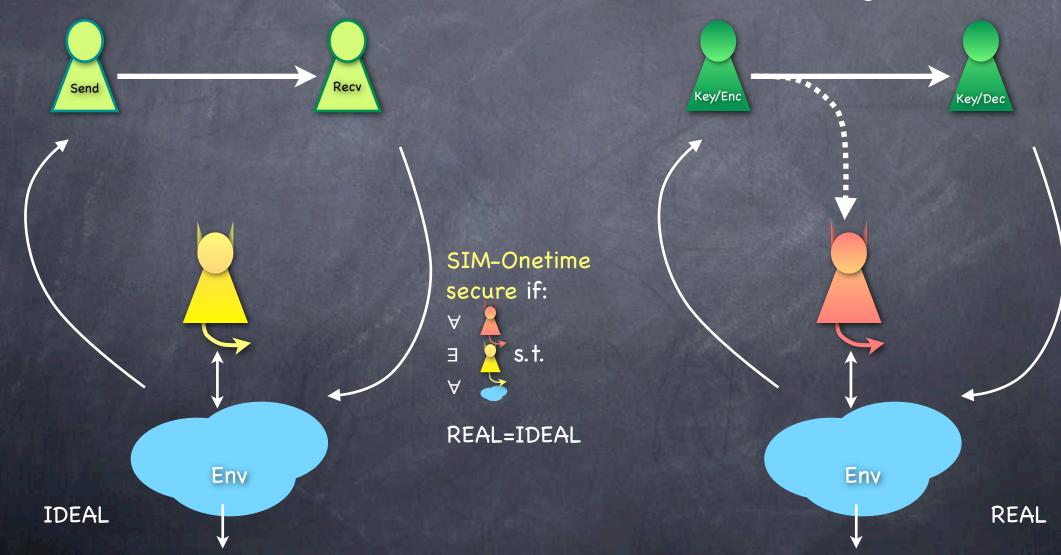
• More generally $\mathcal{M} = \mathcal{K} = \mathcal{C} = \mathcal{C}$ (a finite group) and Enc(m,K) = m+K, Dec(c,K) = c-K

Onetime Encryption SIM-Onetime Security



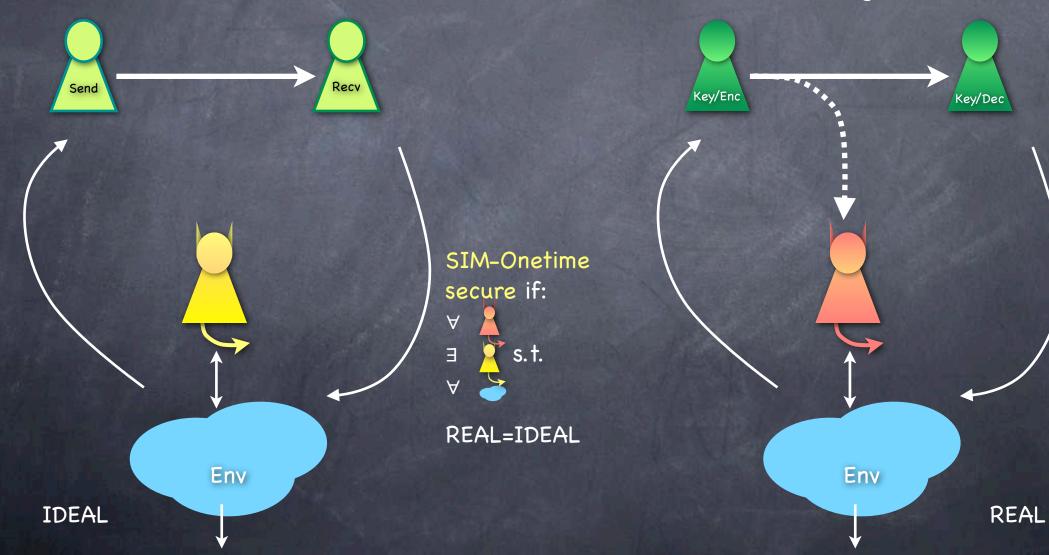
Onetime Encryption SIM-Onetime Security

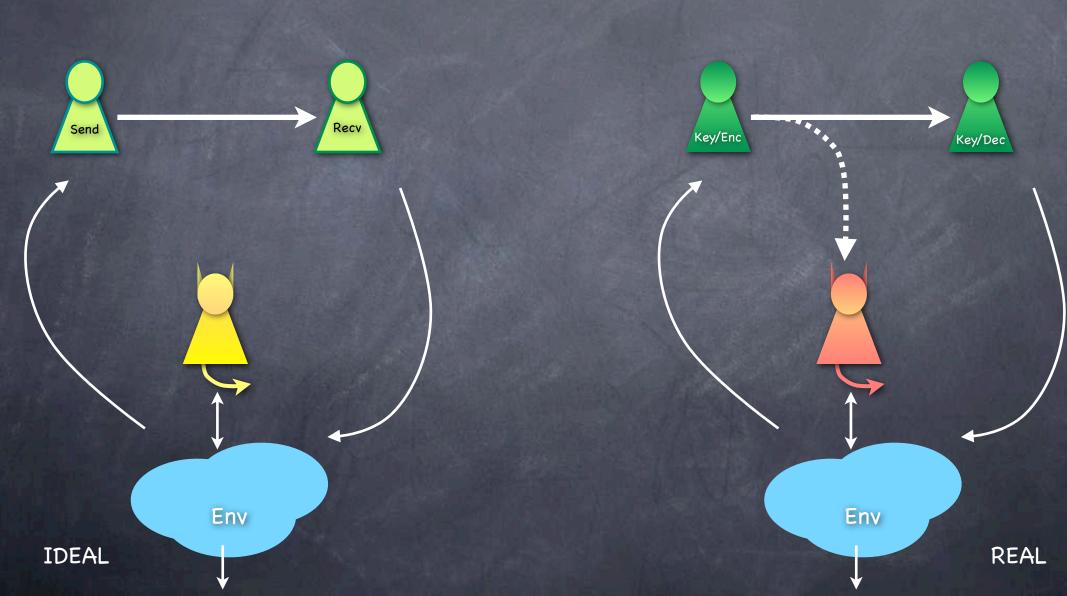
Class of environments which send only one message

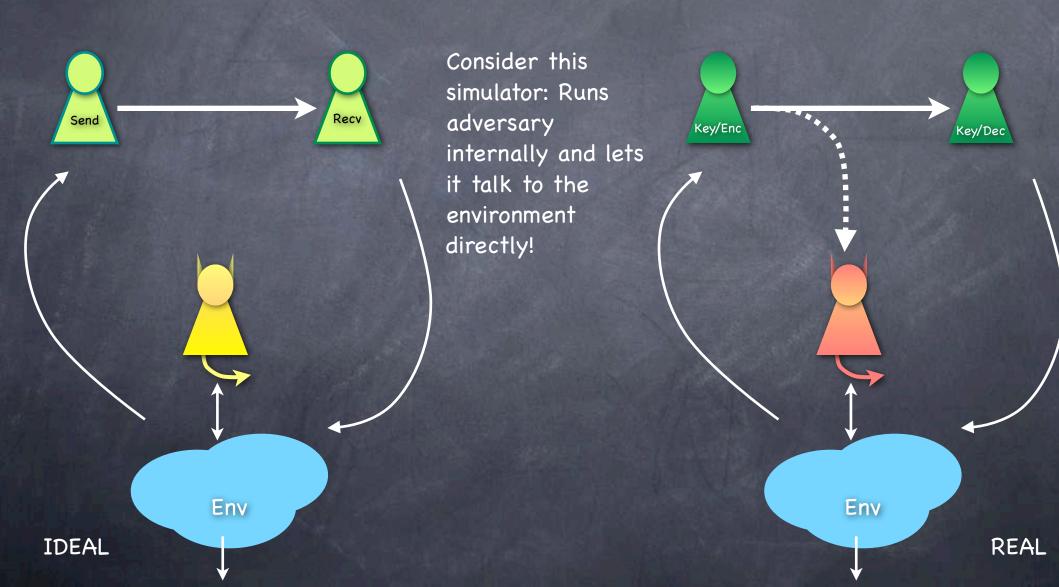


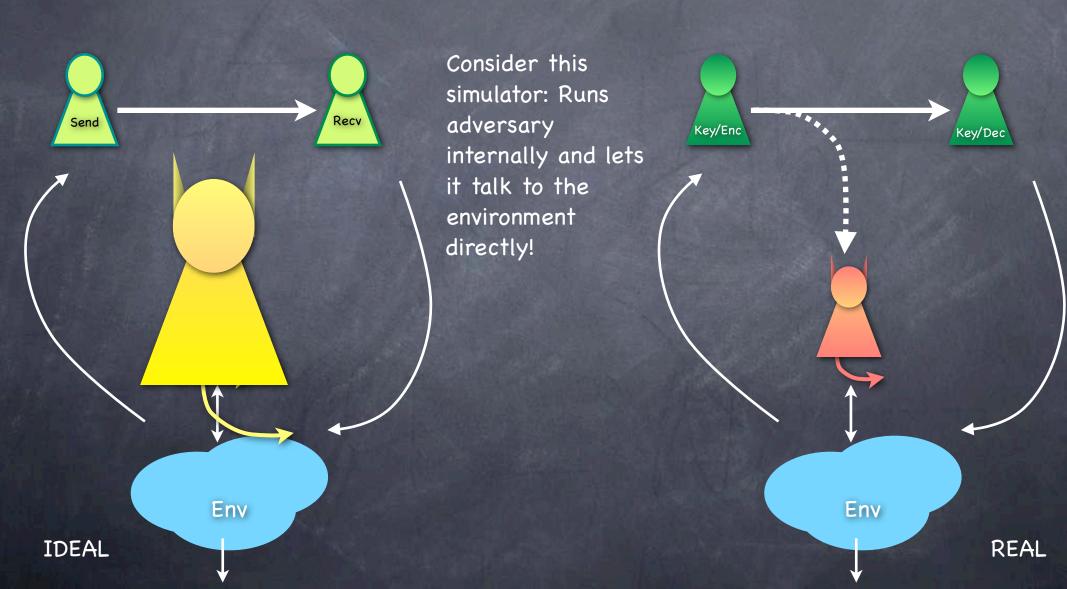
Onetime Encryption Equivalent to SIM-Onetime Security + correctness

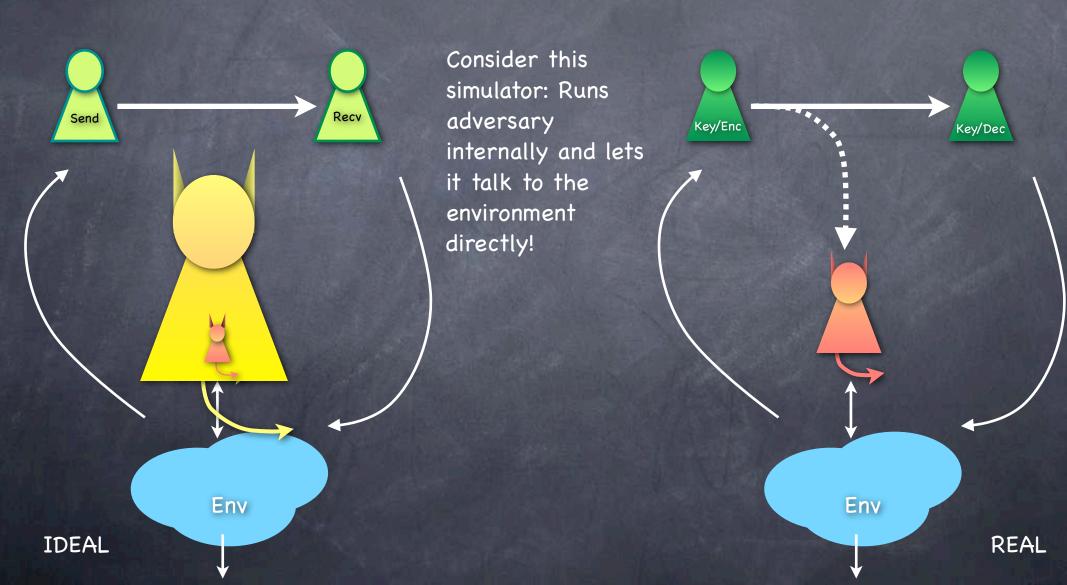
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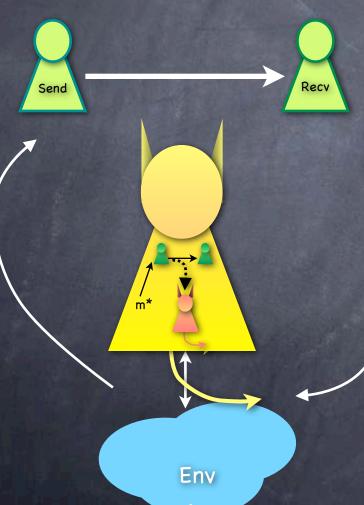












Consider this simulator: Runs adversary internally and lets it talk to the environment directly! Feeds it encryption of a dummy message

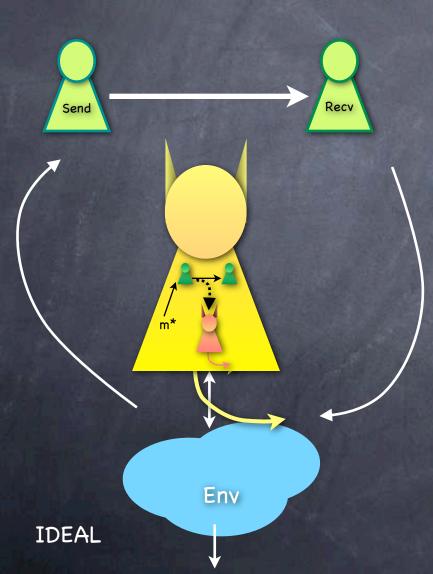
Key/Enc

Env

Key/Dec

REAL

IDEAL



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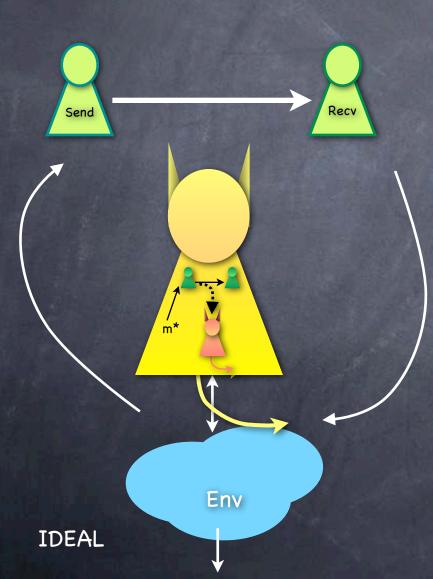
Key/Enc

Can show that REAL=IDEAL



Key/Dec

REAL



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Key/Enc

Key/Dec

REAL

Env

Can show that REAL=IDEAL (Consider view of + for both)

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Also, Eve' allowed to learn the fact that a message is sent

IND-Onetime Experiment

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IND-Onetime Experiment

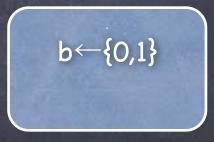




IND-Onetime Experiment

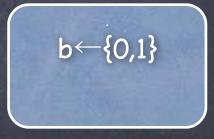
Experiment picks a random bit b. It also runs KeyGen to get a key K



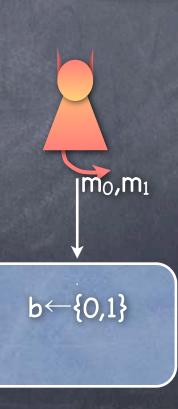


IND-Onetime Experiment

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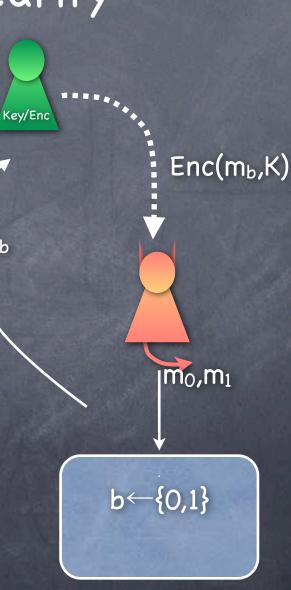
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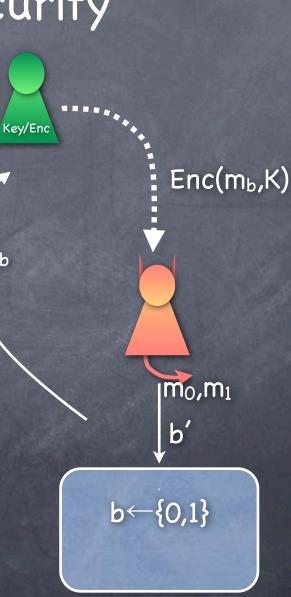
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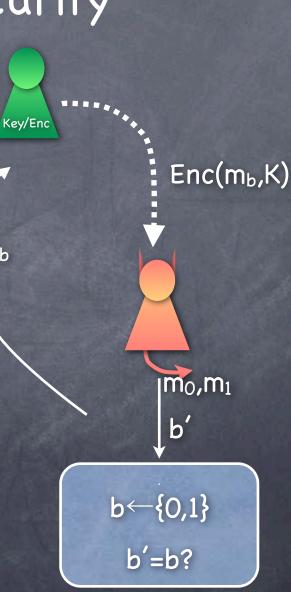
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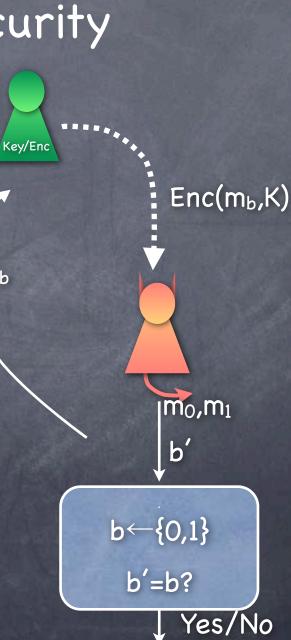
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Adversary returns a guess b'

Experiments outputs 1 iff b'=b



IND-Onetime Experiment

Experiment picks a random bit b. It also runs KeyGen to get a key K

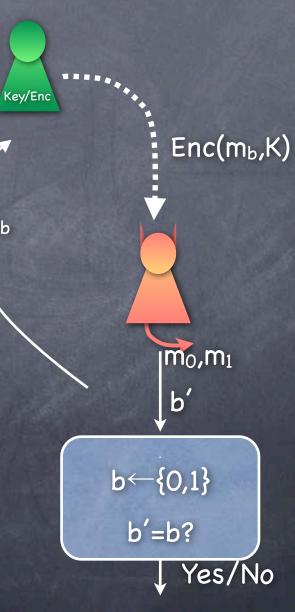
Adversary sends two messages m₀, m_b
 m₁ to the experiment

• Experiment replies with $Enc(m_b,K)$

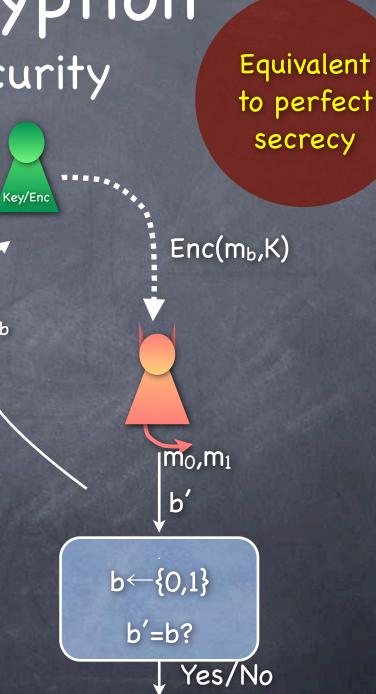
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- Best of both worlds when they are equivalent: use IND- definition while say, proving security of a construction; use SIM- definition when low-level details are not important