

Cryptography

Lecture 0

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University of Illinois Urbana-Champaign

In the News



- “Properly implemented strong crypto systems are one of the few things that you can rely on.”

In the News



- “Properly implemented strong crypto systems are one of the few things that you can rely on.”
- “... Unfortunately, endpoint security is so terrifically weak that [the adversary] can frequently find ways around it.”

What is Cryptography?

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- It's all about controlling **access** to **information**



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 - A tool for enforcing policies on who can learn and/or influence information



What is Cryptography?

- It's all about controlling **access** to **information**
 - A tool for enforcing policies on who can learn and/or influence information
 - Do we know what we are talking about?



What is information?

What is information?

- Or rather the lack of it?

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 - Uncertainty

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Rudolf Clausius

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Ludwig Boltzmann

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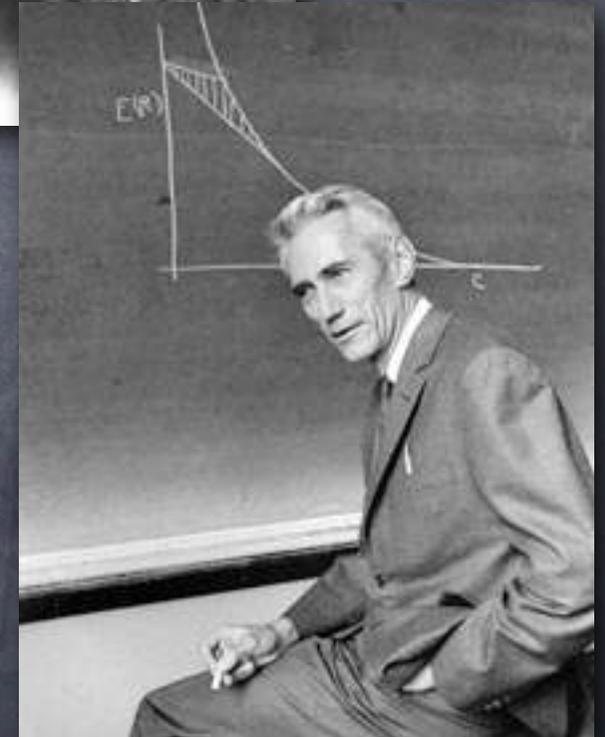
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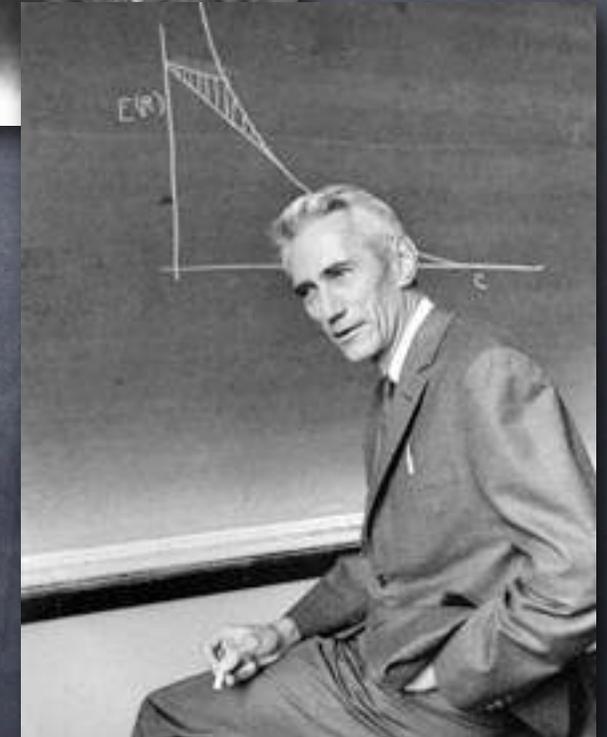
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 - Uncertainty
 - The word is **Entropy**
 - Borrowed from thermodynamics
 - An inherently “probabilistic” notion



Rudolf Clausius



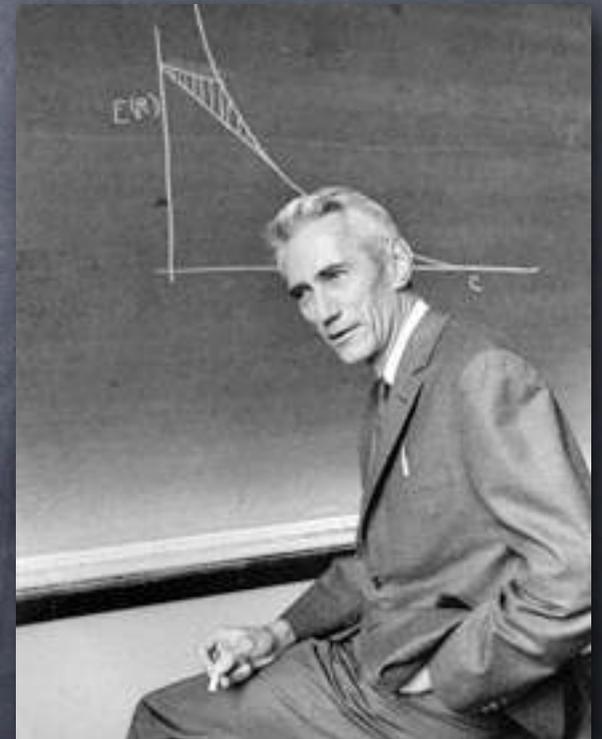
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Claude Shannon

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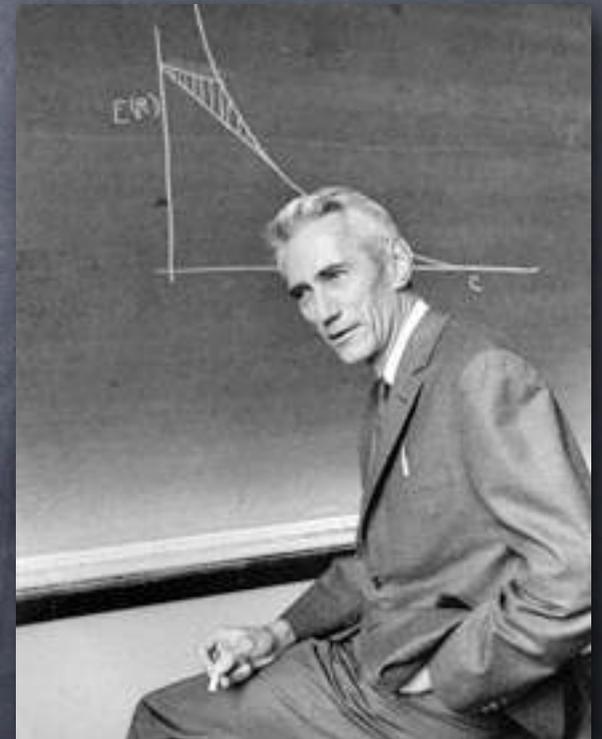
Claude Shannon



What is information?

- Information Theory: ways to quantify information

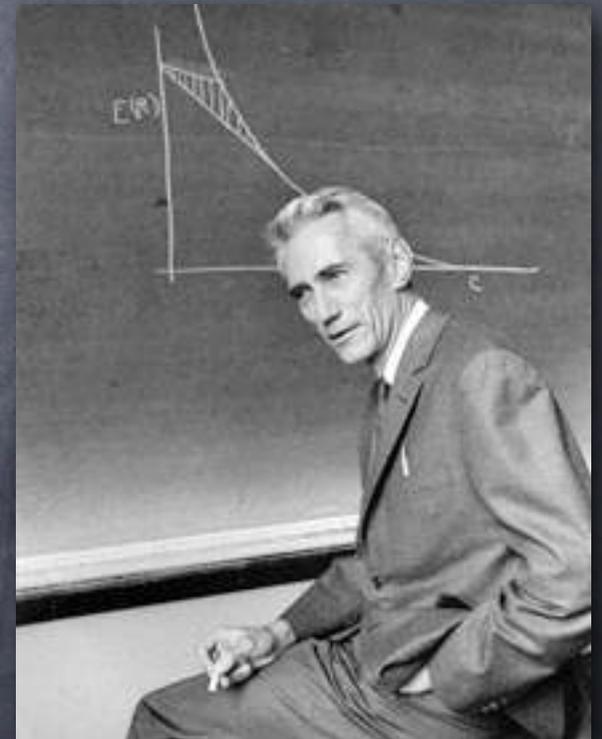
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 - Application 1: to study efficiency of communication (compression, error-correction)

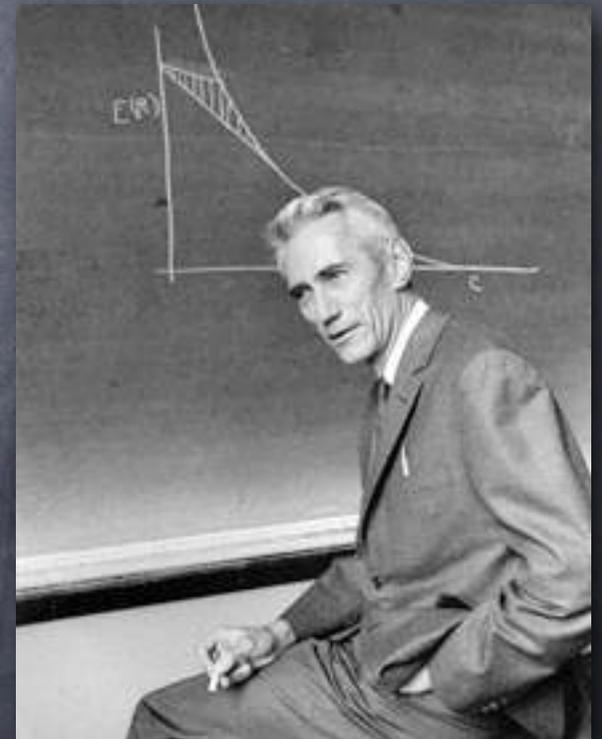
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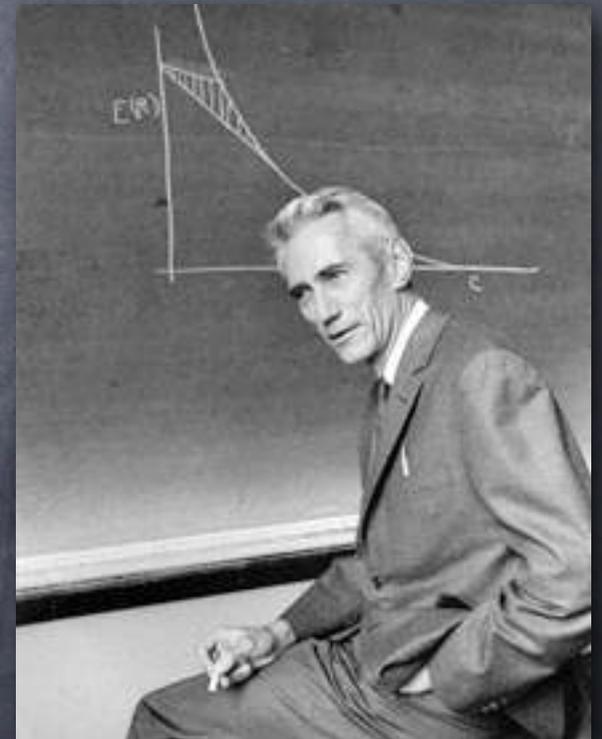
Claude Shannon



What is information?

- Information Theory: ways to quantify information
 - Application 1: to study efficiency of communication (compression, error-correction)
 - **Application 2: to study the possibility of secret communication**
 - The latter turned out to be a relatively easy question! Secret communication possible only if (an equally long) secret key is shared ahead of time

Claude Shannon



Access to Information

Access to Information

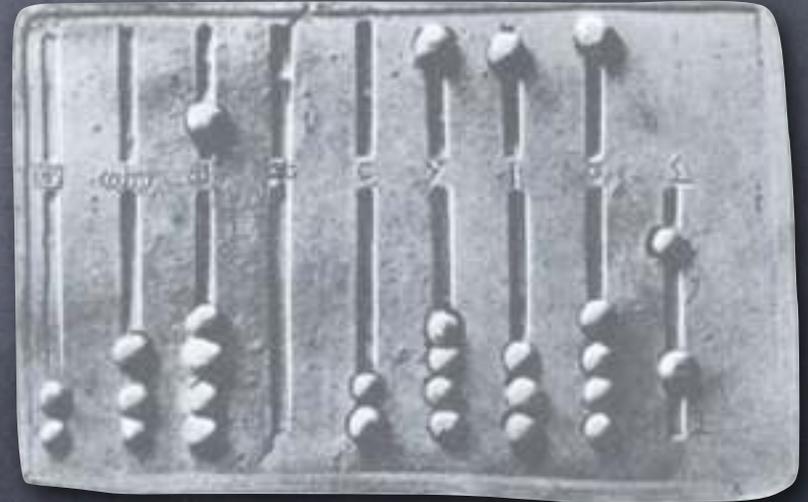
- A second look

Access to Information

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- Information at hand may still not be “accessible” if it is hard to work with it

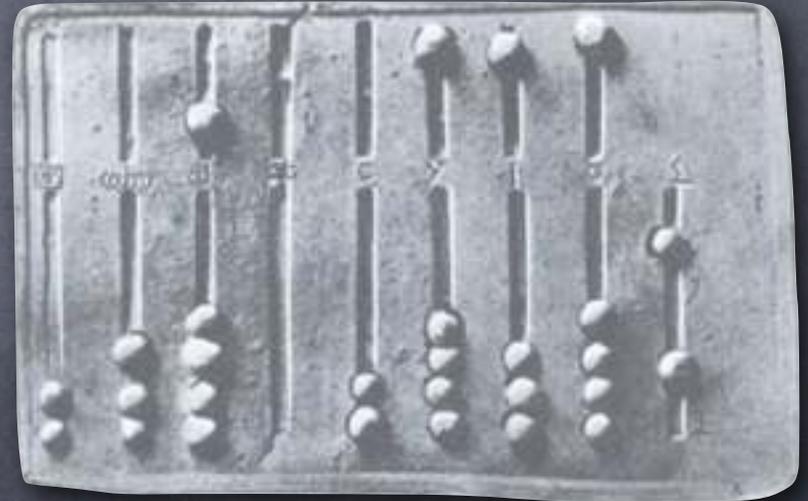
Access to Information

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 - Computation!



Access to Information

- A second look
- Information at hand may still not be “accessible” if it is hard to work with it
 - Computation!
- Shannon’s information may reduce uncertainty only for computationally all-powerful parties



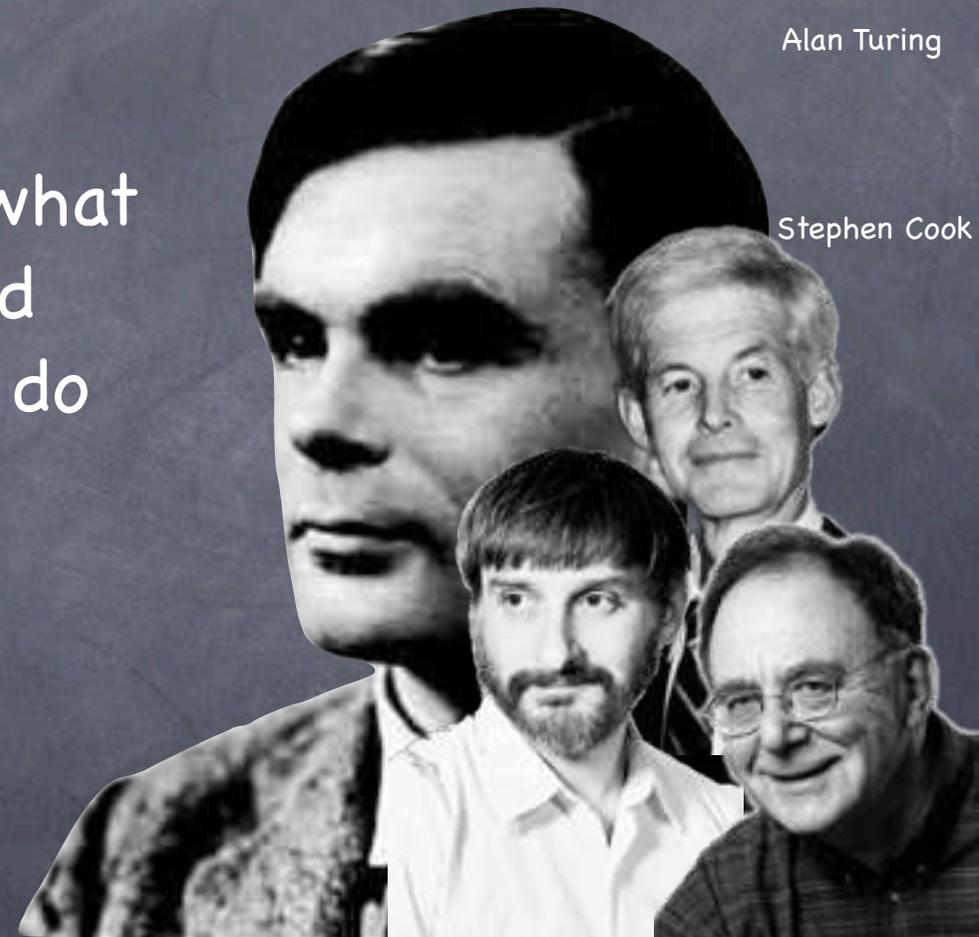
Computational Complexity

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Alan Turing

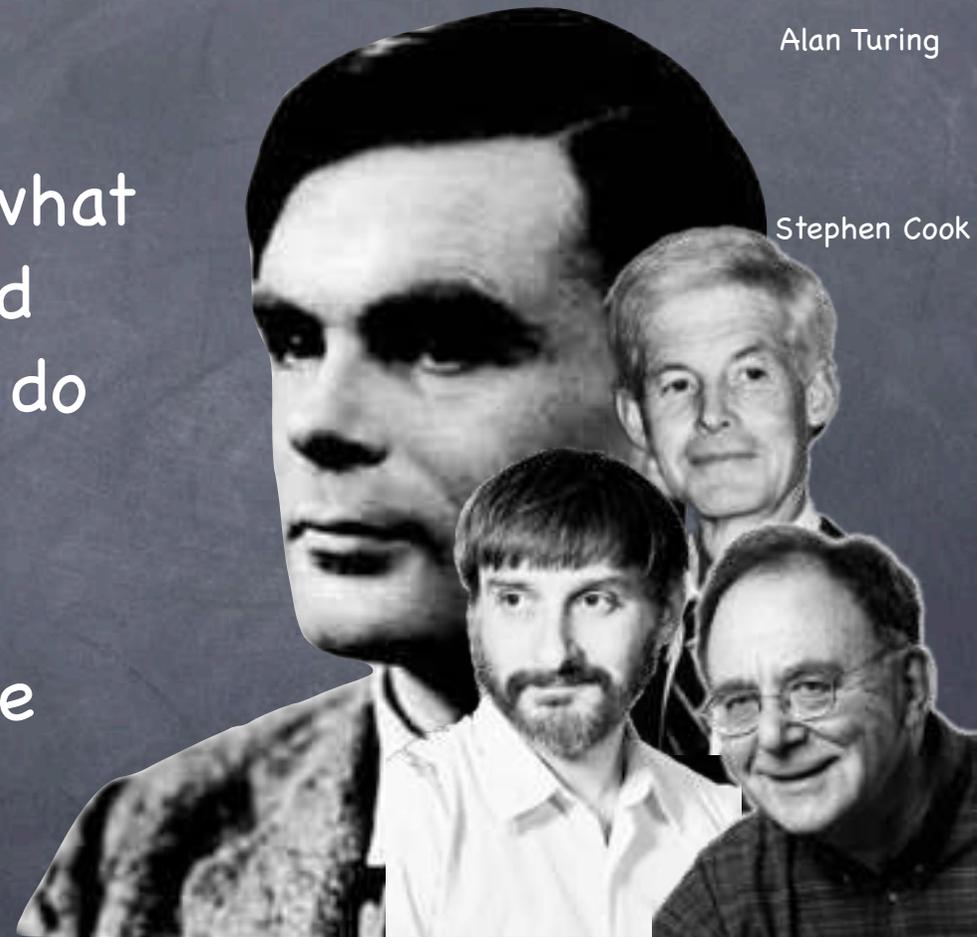
Stephen Cook

Leonid Levin

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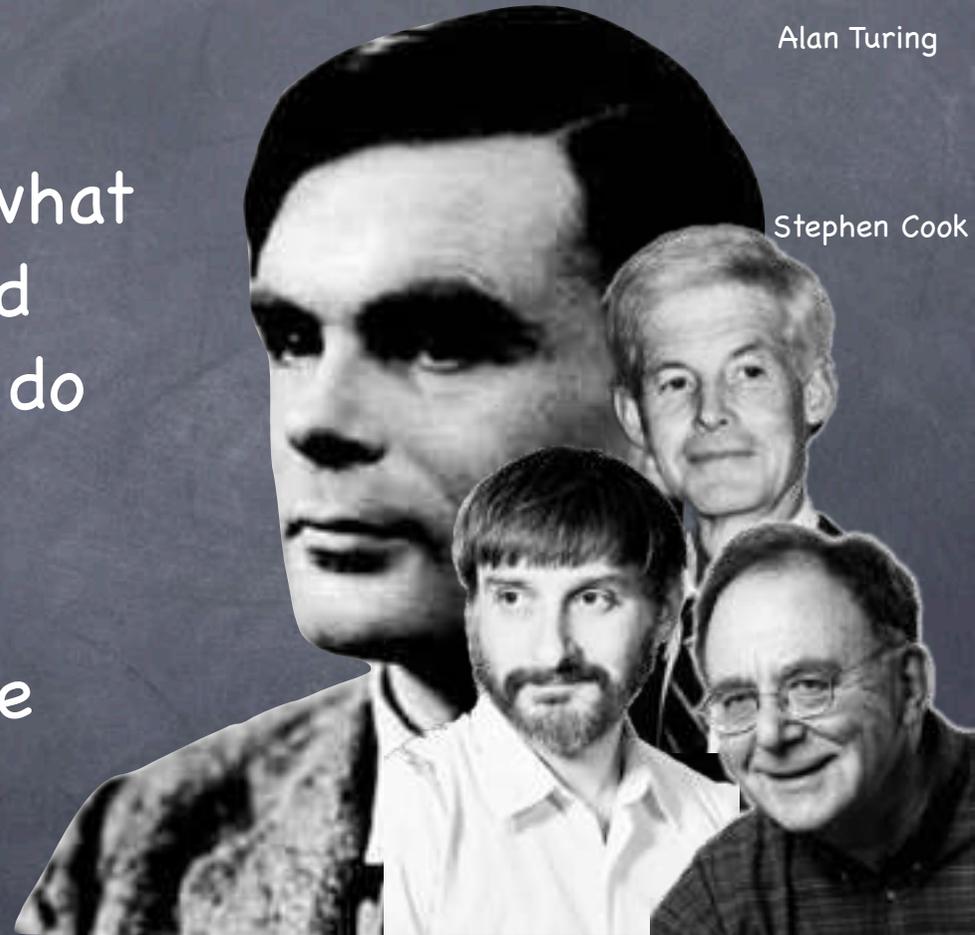
Richard Karp

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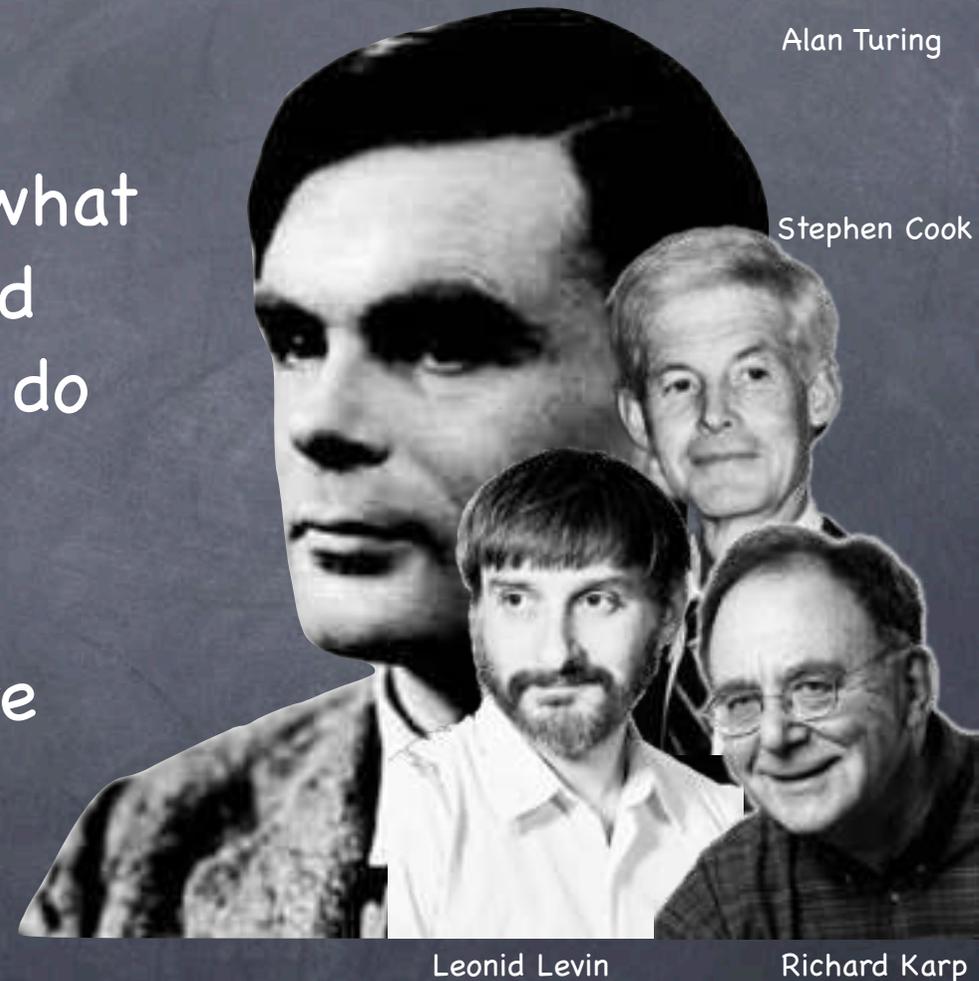


Leonid Levin

Richard Karp

Computational Complexity

- A systematic study of what computationally bounded parties can and cannot do
- A young and rich field
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 - Much “believed”
- Basis of the Modern Theory of Cryptography



Alan Turing

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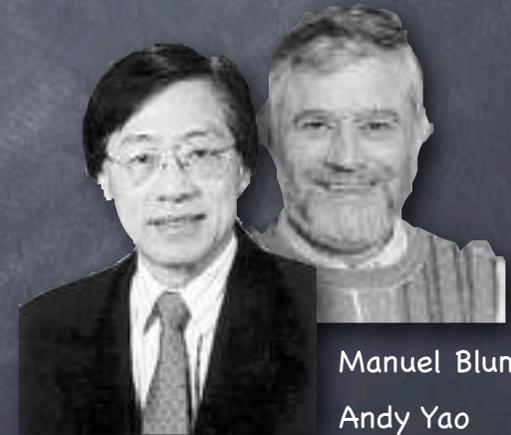
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 - But possible against computationally bounded players:
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- Pseudo-random number generator
 - a.k.a Stream Cipher
 - Generate a long string of random-looking bits from a short random seed



Manuel Blum

Andy Yao

The Public-Key Revolution

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Clifford Cocks

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Malcolm Williamson

Clifford Cocks



Merkle, Hellman, Diffie

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Clifford Cocks



Merkle, Hellman, Diffie



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The Public-Key Revolution

- “Non-Secret Encryption”
 - No a priori shared secrets
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- Publicly verifiable digital signatures
- Forms the backbone of today’s secure communication



Malcolm Williamson

James Ellis

Clifford Cocks



Merkle, Hellman, Diffie



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Crypto-Mania

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Crypto-Mania

- Public-Key cryptography and beyond!
- Secret computation: collaboration among mutually distrusting parties
 - Compute on distributed data, without revealing their private information to each other
 - Compute on encrypted data
- And other fancy things... with sophisticated control over more complex "access" to information
- Do it all faster, better, more conveniently and more securely (or find out if one cannot). And also make sure we know what we are trying to do.

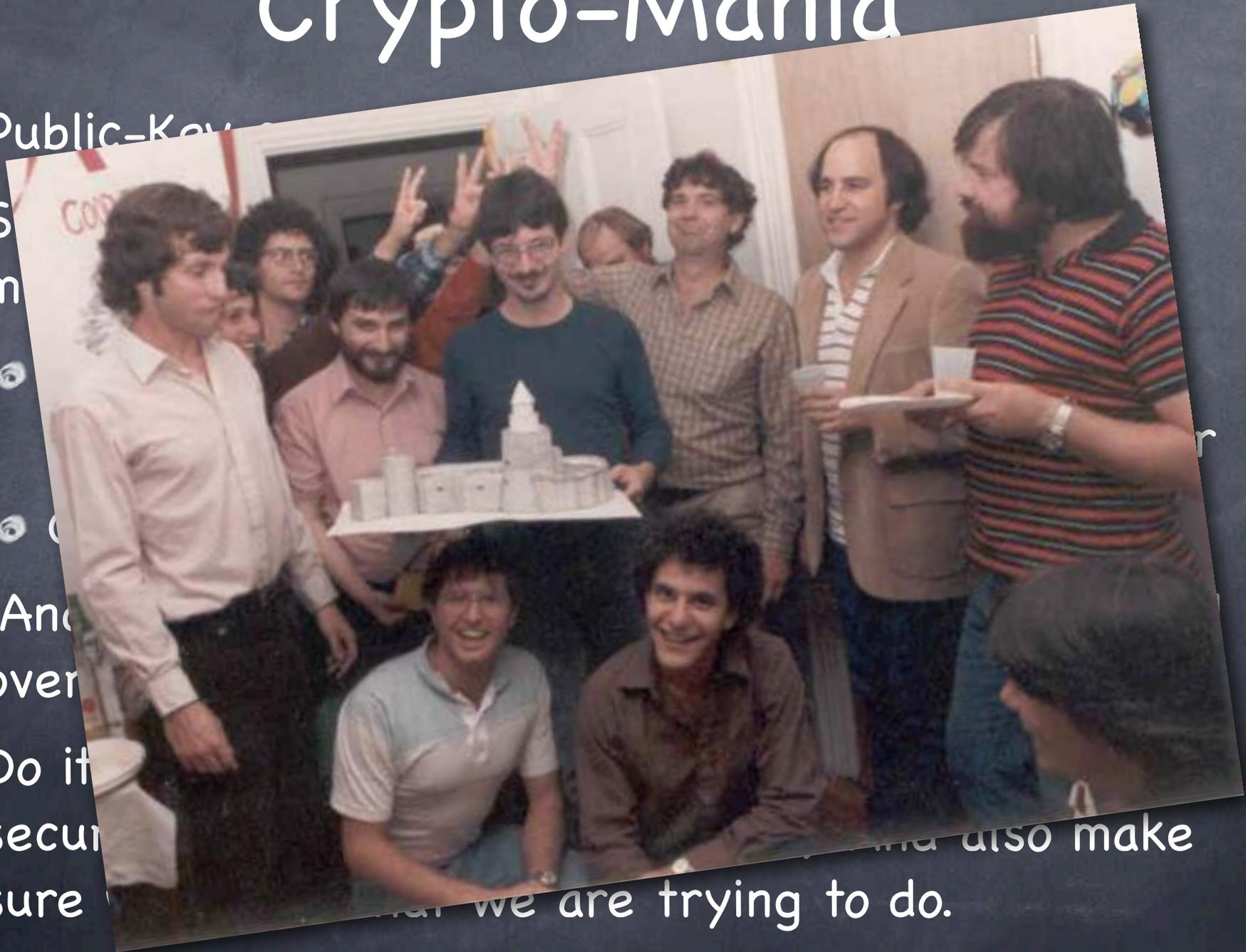
Crypto-Mania

- Public-Key

- Some

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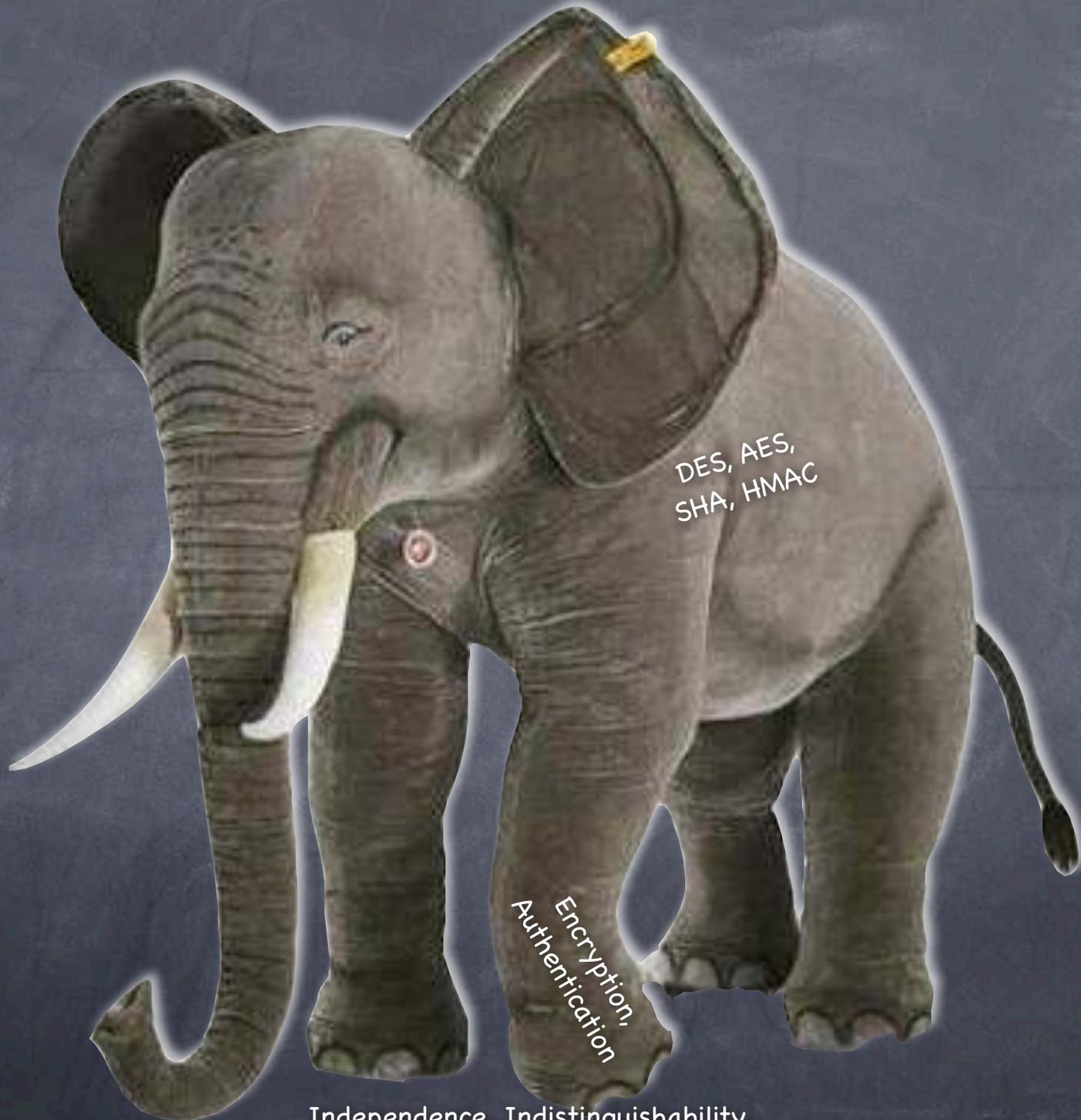




Independence, Indistinguishability,
Infeasibility, Zero-Knowledge, ...



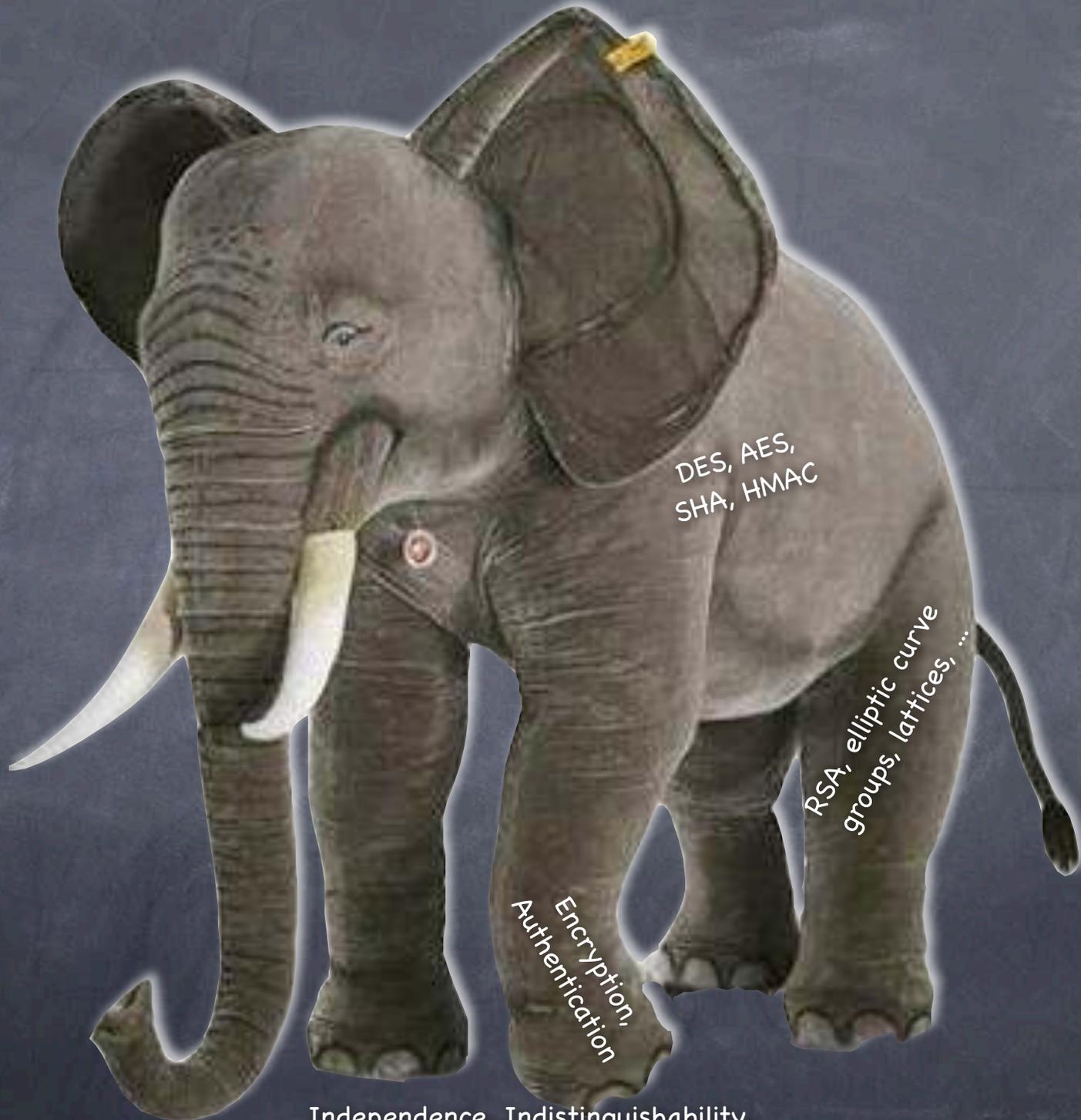
Independence, Indistinguishability,
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DES, AES,
SHA, HMAC

Encryption,
Authentication

Independence, Indistinguishability,
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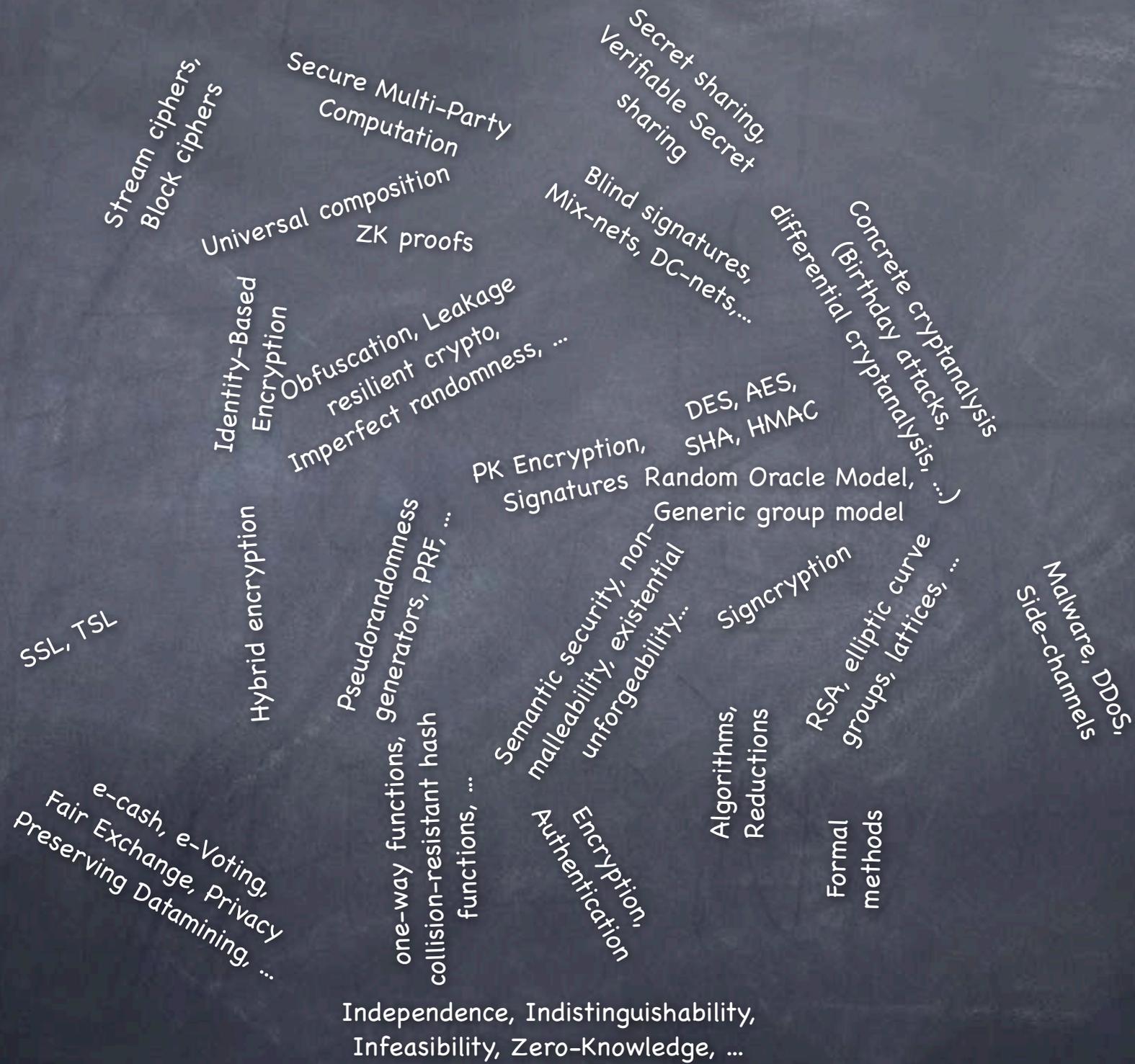


DES, AES,
SHA, HMAC

RSA, elliptic curve
groups, lattices, ...

Encryption,
Authentication

Independence, Indistinguishability,
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In This Course

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(how to tame the elephant...)



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- Fundamental notions: **secrecy, infeasibility**

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- A few assignments

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- <http://courses.engr.illinois.edu/cs598man/fa2014/>

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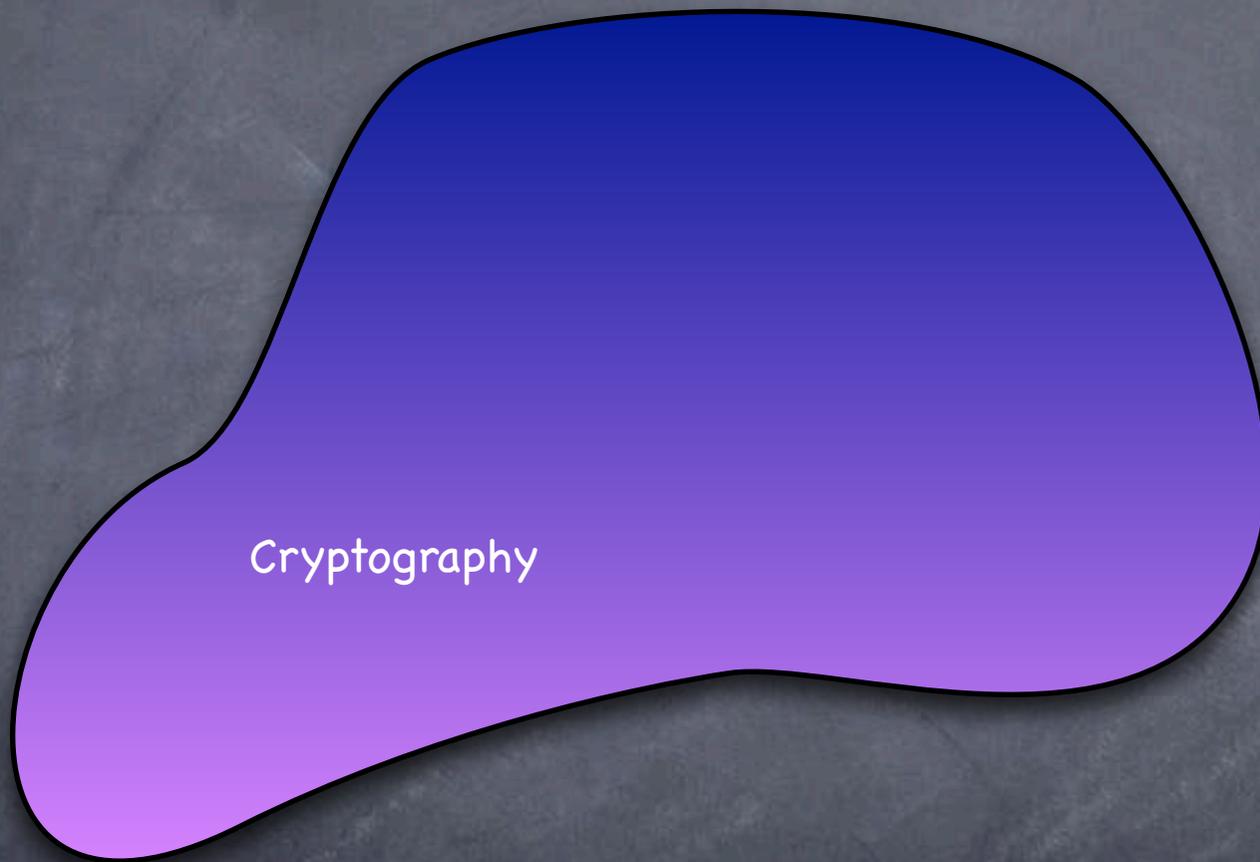
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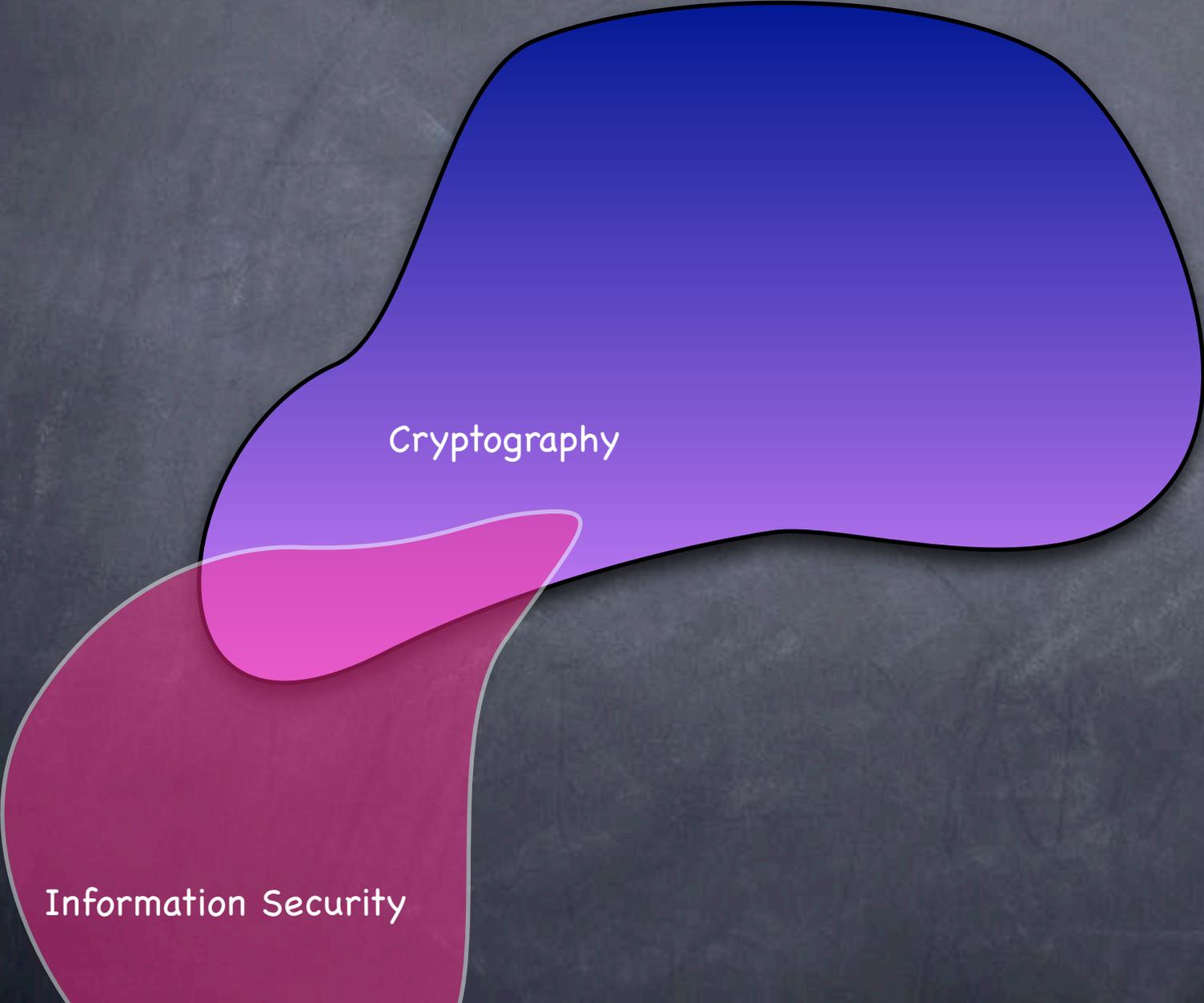


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The Big Picture



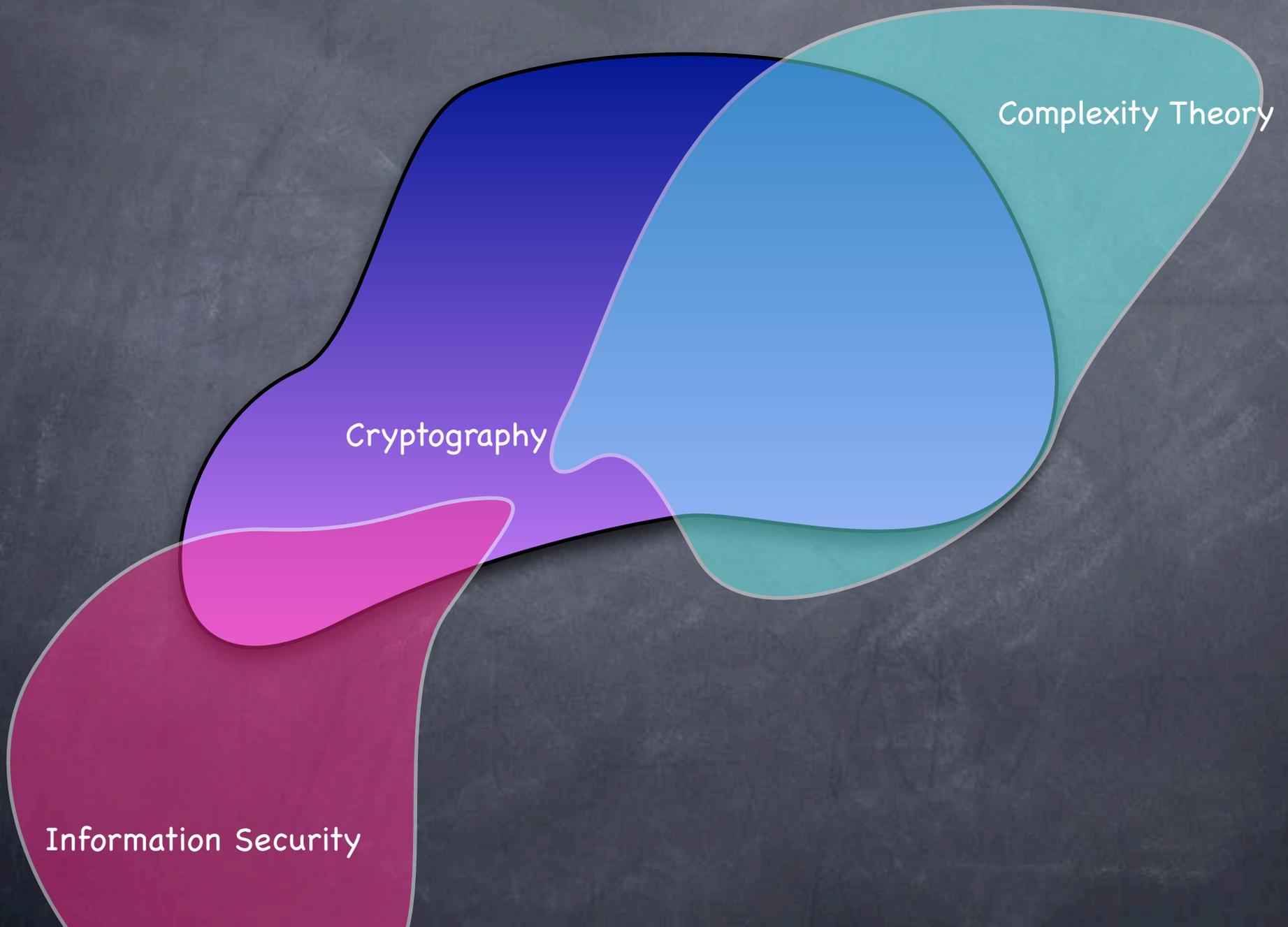
The Big Picture



Cryptography

Information Security

The Big Picture



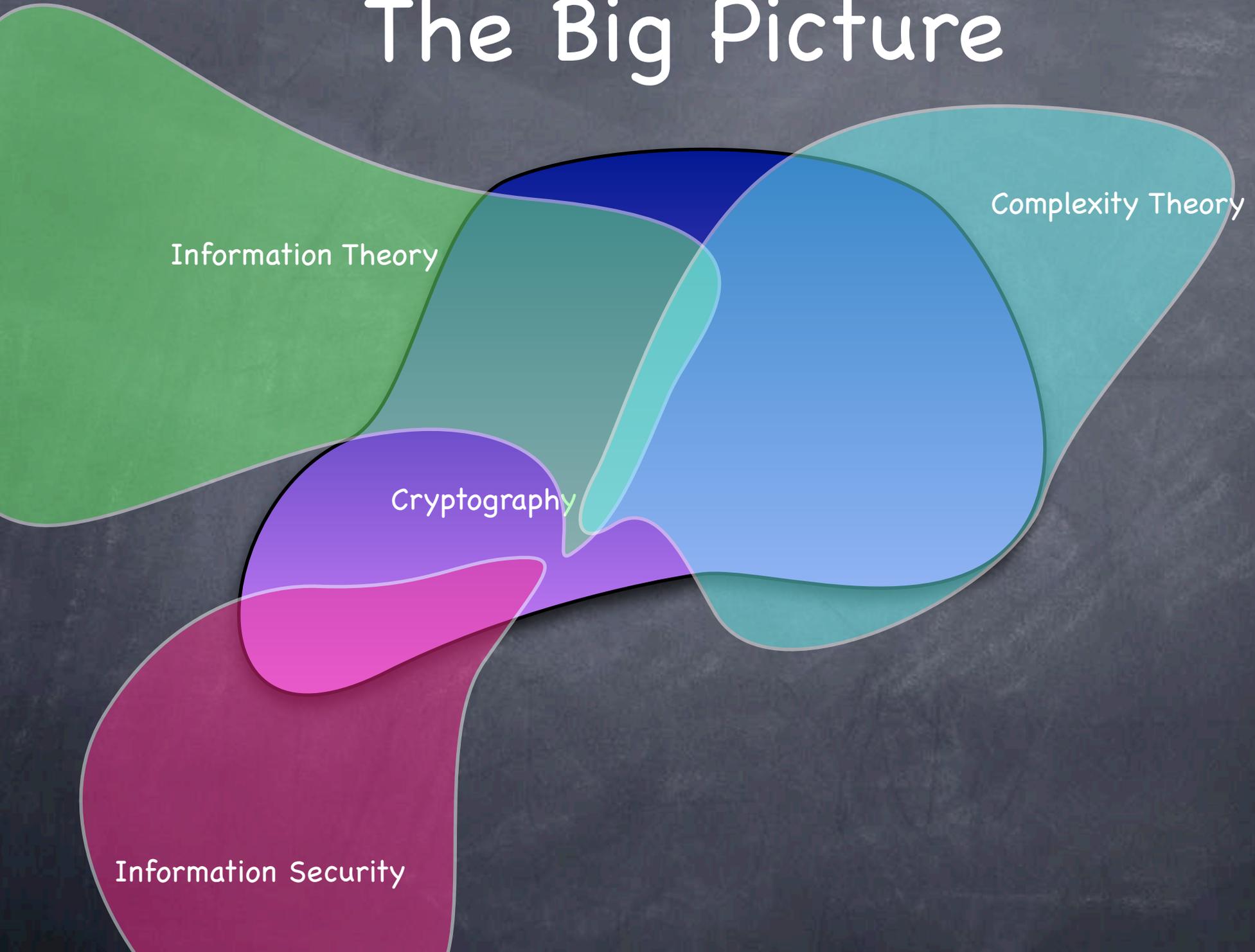
The Big Picture

Information Theory

Complexity Theory

Cryptography

Information Security



The Big Picture

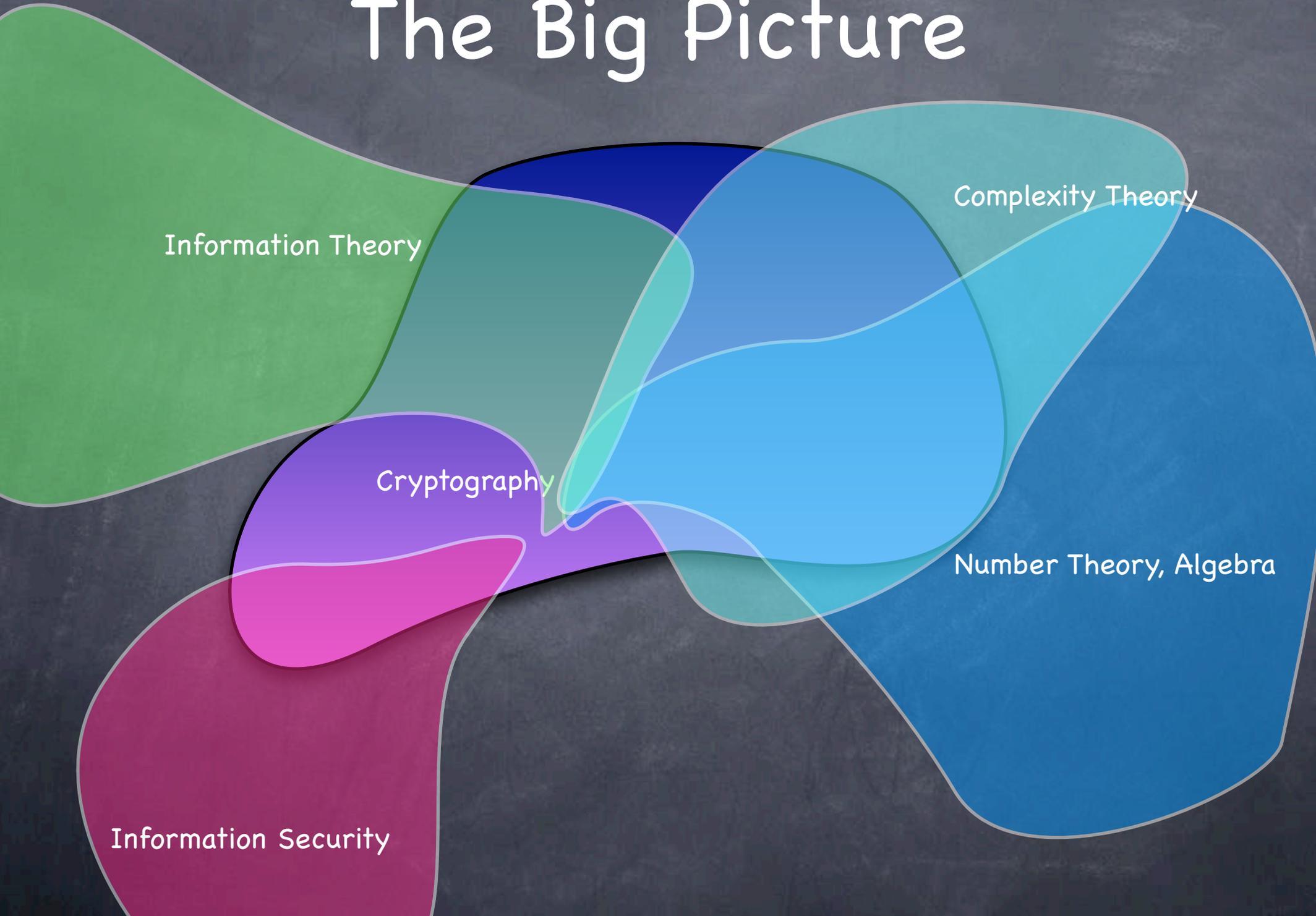
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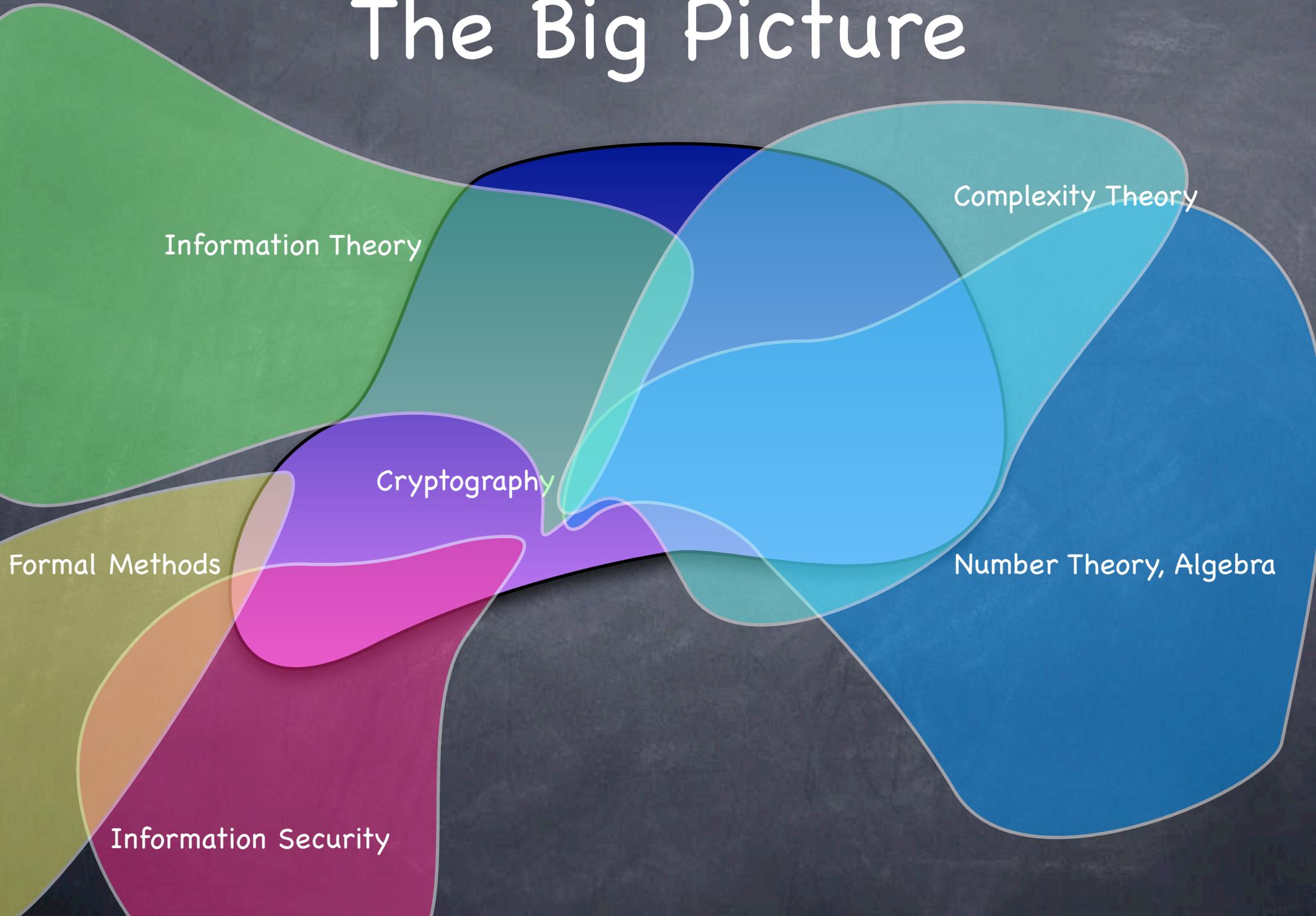
Cryptography

Number Theory, Algebra

Information Security



The Big Picture



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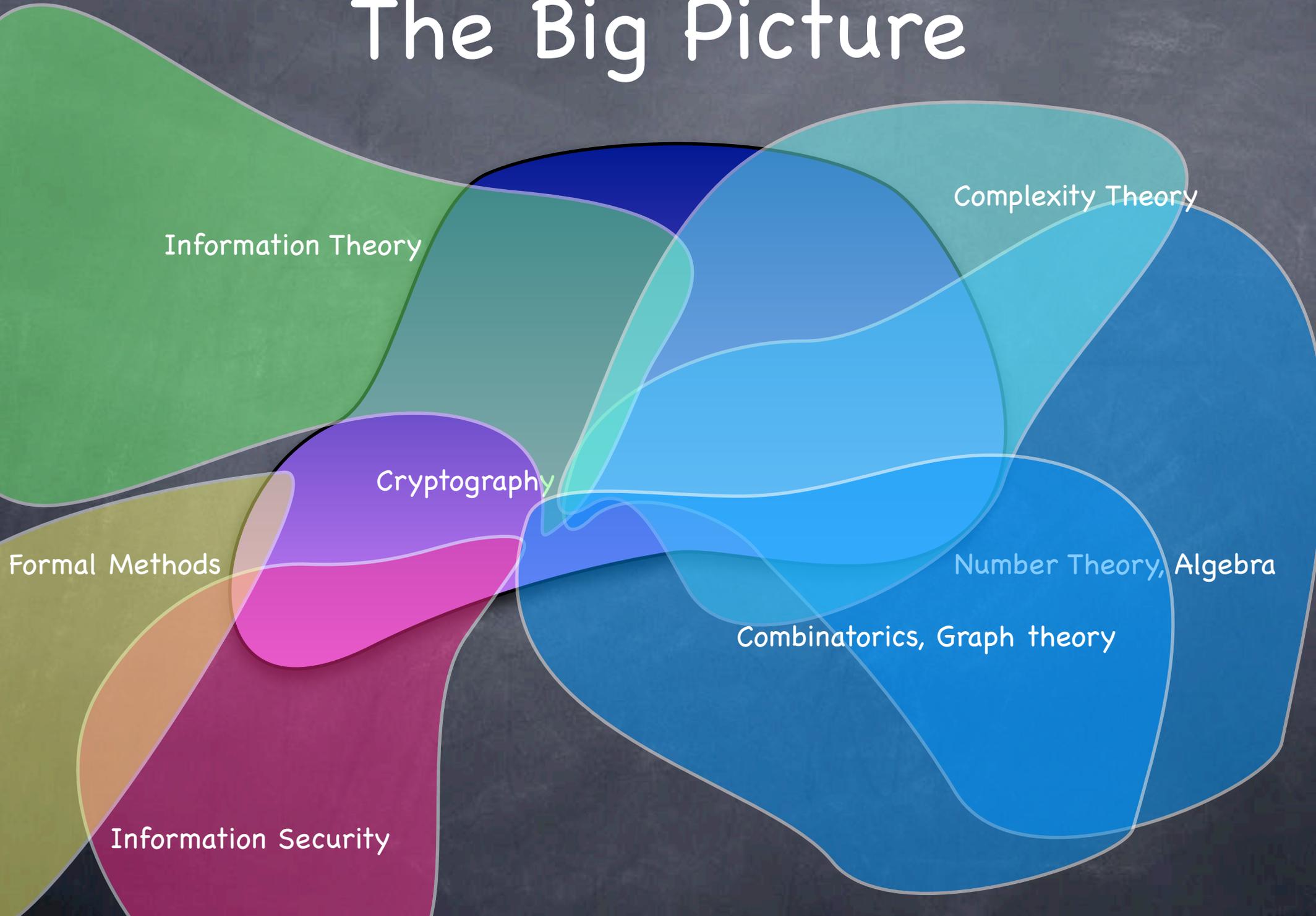
Cryptography

Formal Methods

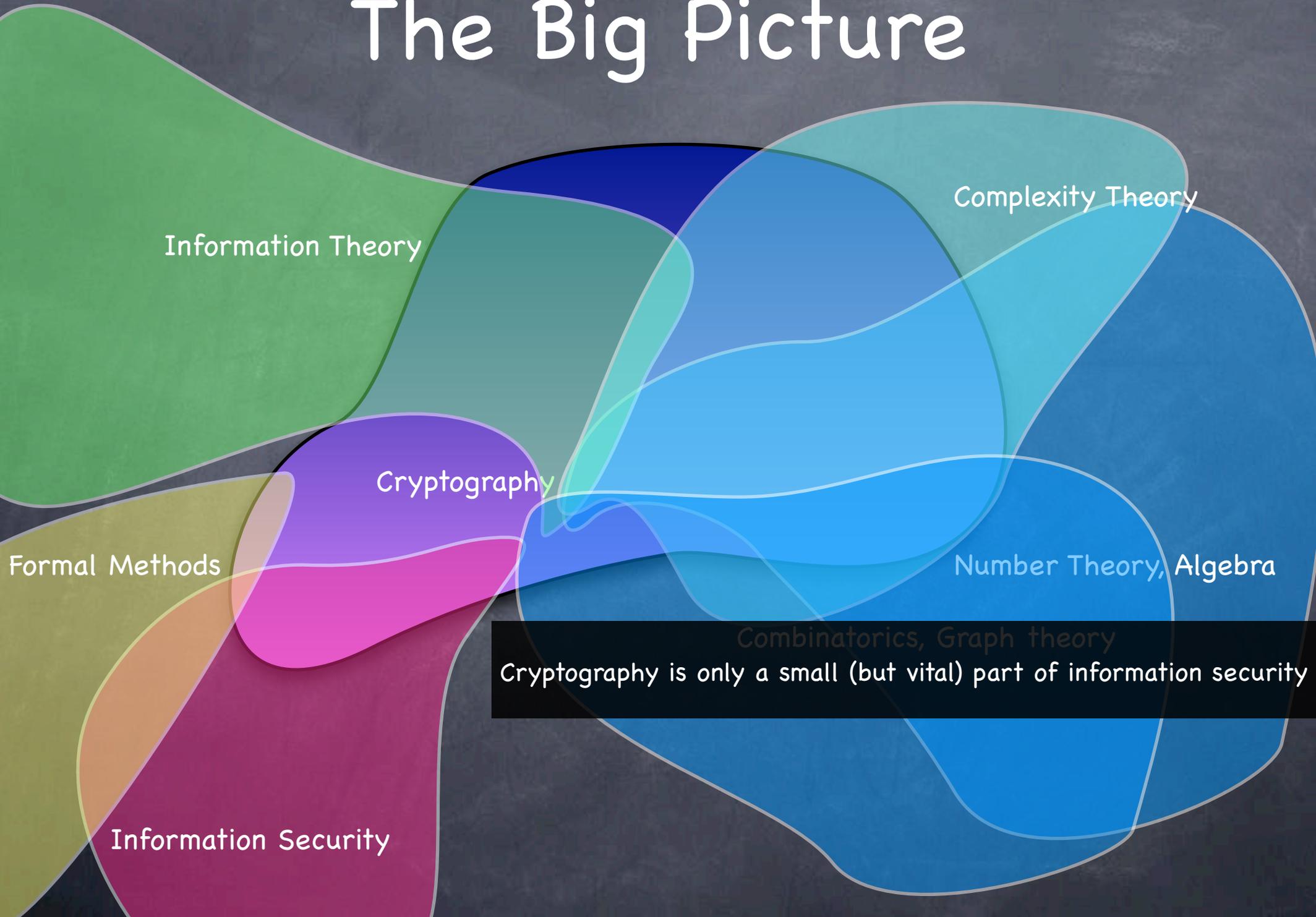
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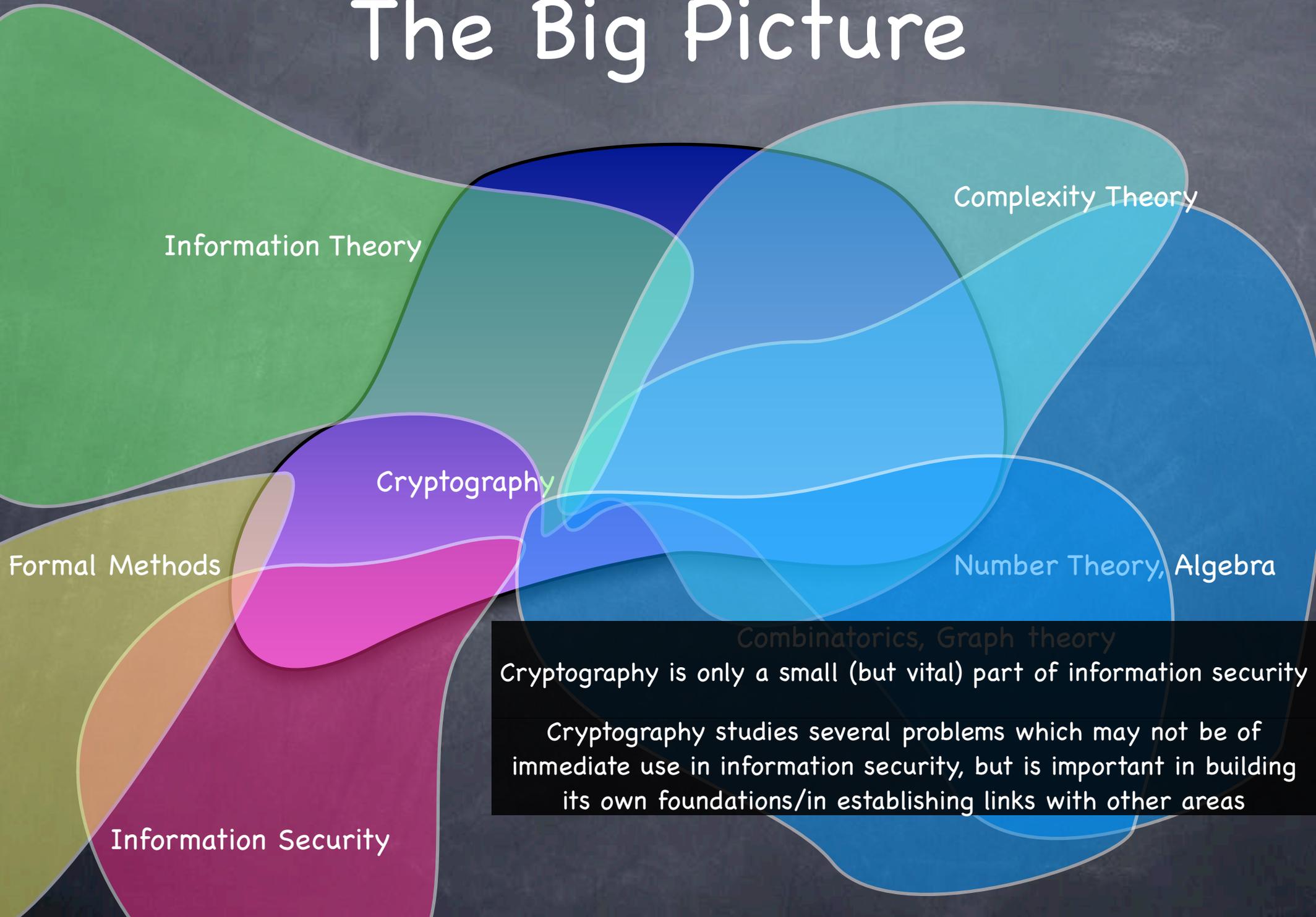


The Big Picture



Cryptography is only a small (but vital) part of information security

The Big Picture



Information Theory

Complexity Theory

Cryptography

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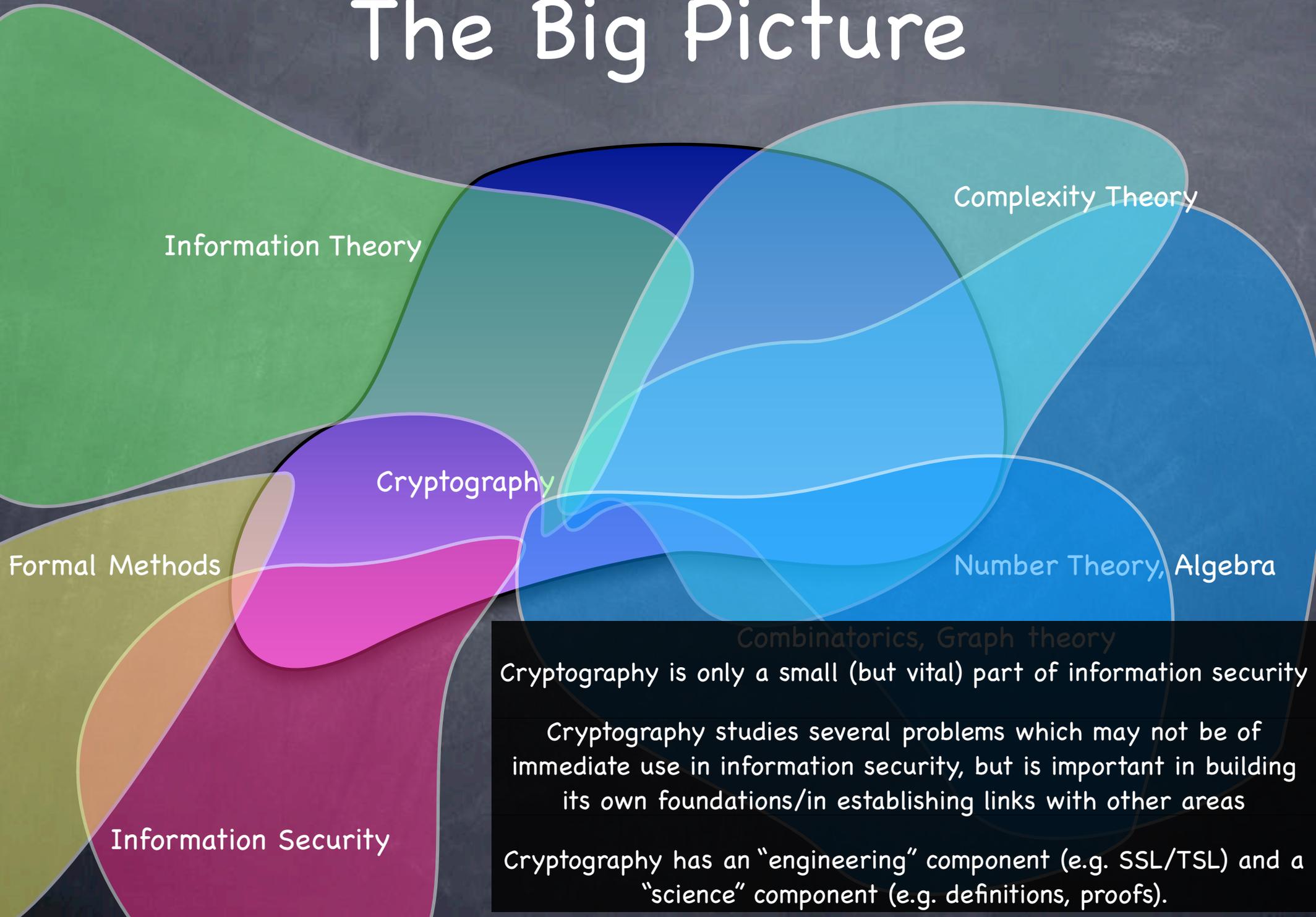
Combinatorics, Graph theory

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Cryptography studies several problems which may not be of immediate use in information security, but is important in building its own foundations/in establishing links with other areas

Information Security

The Big Picture



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Complexity Theory

Cryptography

Formal Methods

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Combinatorics, Graph theory

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Information Security

Cryptography has an "engineering" component (e.g. SSL/TSL) and a "science" component (e.g. definitions, proofs).

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 - i.e., Alice should learn nothing about y , nor Bob about x . Carol shouldn't learn anything else about x, y "other than" $x+y$

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- Can they do it, just by talking to each other (using private channels between every pair of parties)?

Puzzle #2

- Alice and Bob hold **secret bits x and y**
- Carol wants to learn **$x \wedge y$** . Alice and Bob are OK with that.
- But they don't want Carol/each other to learn anything else!
 - i.e., Alice should learn nothing about y , nor Bob about x . Carol shouldn't learn anything else about x, y "other than" $x \wedge y$
- Can they do it, just by talking to each other (using private channels between every pair of parties)?