

Signatures

Lecture 24

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- Definitions sometimes have subtleties (not all of them have ideal functionality specifications)

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- Security requirement: Unforgeability (chosen message security)

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- Extended to a multi-signature scheme [BN'06]

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- General aggregation: signatures can be created independently and then aggregated in arbitrary order

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- Extended to a sequential aggregate scheme [LOSSW'06]

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 - Aggregate signatures saves on bandwidth and verification time, but does not allow un-aggregating the signatures

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- Similarly for pairing equations, but with further optimizations
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 - Can save on number of pairing operations using $\prod_i e(S_i,g)^{w_i} = \prod_i e(S_i^{w_i},g) = e(\prod_i S_i^{w_i},g)$

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 - But a group manager can “trace” the signer
 - However, the group manager or other group members “cannot frame” a member

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- **Full-Traceability**: If a set of group members collude and create a valid signature, the tracing algorithm will trace at least one member of the set. This holds even if the group manager is passively corrupt.
 - **Implies unforgeability** (i.e., with no group members colluding with it, adversary cannot produce a valid signature) and **framing-resistance** (even colluding with the group manager)

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- Tracing algorithm decrypts C to find SK^*_i and hence ID_i

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- Unwitting collaborators: F_i 's could be the verification keys for a standard signature scheme

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 - Security requirements: Unforgeability and Hiding

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- c.f. Mesh signatures: here, instead of multiple parties signing a message, a single party with multiple attributes

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- Note: Still allows multiple (mutually distrusting) verifiers to be convinced if they run a secure MPC protocol to implement a virtual verifier

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 - e.g. a ring signature with a ring of size 2, containing the signer and the designated verifier

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 - Using Blind signatures and \mathcal{P} -signatures