

# Signatures

Lecture 23

# Signatures

# Signatures

- Signatures with various functionality/properties

# Signatures

- Signatures with various functionality/properties
- Last time: Blind signatures, P-signatures

# Signatures

- Signatures with various functionality/properties
- Last time: Blind signatures, P-signatures
- Constructions come in different flavors:

# Signatures

- Signatures with various functionality/properties
- Last time: Blind signatures, P-signatures
- Constructions come in different flavors:
  - Using minimal/general assumptions, often simple, but not very efficient (e.g., involving NIZK for general NP statements)

# Signatures

- Signatures with various functionality/properties
- Last time: Blind signatures, P-signatures
- Constructions come in different flavors:
  - Using minimal/general assumptions, often simple, but not very efficient (e.g., involving NIZK for general NP statements)
  - Simple and efficient ones in the Random Oracle Model

# Signatures

- Signatures with various functionality/properties
- Last time: Blind signatures, P-signatures
- Constructions come in different flavors:
  - Using minimal/general assumptions, often simple, but not very efficient (e.g., involving NIZK for general NP statements)
  - Simple and efficient ones in the Random Oracle Model
  - Relatively efficient ones under specific assumptions (often relatively strong/new assumptions)

# Signatures

- Signatures with various functionality/properties
- Last time: Blind signatures, P-signatures
- Constructions come in different flavors:
  - Using minimal/general assumptions, often simple, but not very efficient (e.g., involving NIZK for general NP statements)
  - Simple and efficient ones in the Random Oracle Model
  - Relatively efficient ones under specific assumptions (often relatively strong/new assumptions)
- Definitions sometimes have subtleties (not all of them have ideal functionality specifications)

# Multi-Signatures

# Multi-Signatures

- Multiple signers signing the same message

# Multi-Signatures

- Multiple signers signing the same message
  - Each signer has an (SK,VK) pair

# Multi-Signatures

- Multiple signers signing the same message
  - Each signer has an (SK,VK) pair
- Resulting signature must be “compact”: size independent of the number of signers

# Multi-Signatures

- Multiple signers signing the same message
  - Each signer has an (SK,VK) pair
- Resulting signature must be “compact”: size independent of the number of signers
- Security requirement: Unforgeability (chosen message security)

# A Multi-Signature Scheme

# A Multi-Signature Scheme

- Following Schnorr Signature: a digital signature scheme secure in the Random Oracle model under the discrete log assumption

# A Multi-Signature Scheme

- Following Schnorr Signature: a digital signature scheme secure in the Random Oracle model under the discrete log assumption
  - Signing key is  $x$  and Verification key is  $X = g^x$

# A Multi-Signature Scheme

- Following Schnorr Signature: a digital signature scheme secure in the Random Oracle model under the discrete log assumption
  - Signing key is  $x$  and Verification key is  $X = g^x$
  - $\text{Sign}(m;x) = (h,s)$  where  $h=H(m,R)$ ,  $R=g^r$ ,  $s = r + hx$

# A Multi-Signature Scheme

- Following Schnorr Signature: a digital signature scheme secure in the Random Oracle model under the discrete log assumption
  - Signing key is  $x$  and Verification key is  $X = g^x$
  - $\text{Sign}(m;x) = (h,s)$  where  $h=H(m,R)$ ,  $R=g^r$ ,  $s = r + hx$
  - $\text{Verify}(m,(h,s);X)$ : check if  $h = H(m,g^sX^{-h})$

# A Multi-Signature Scheme

- Following Schnorr Signature: a digital signature scheme secure in the Random Oracle model under the discrete log assumption
  - Signing key is  $x$  and Verification key is  $X = g^x$
  - $\text{Sign}(m;x) = (h,s)$  where  $h=H(m,R)$ ,  $R=g^r$ ,  $s = r + hx$
  - $\text{Verify}(m,(h,s);X)$ : check if  $h = H(m,g^s X^{-h})$
- Alternately  $\text{Sign}(m;x) = (R,s)$  where  $R=g^r$ ,  $s = r + hx$  for  $h=H(m,R)$ .  
 $\text{Verify}(m,(R,s);X)$  checks if  $g^s = R X^h$  for  $h = H(m,R)$

# A Multi-Signature Scheme

- Following Schnorr Signature: a digital signature scheme secure in the Random Oracle model under the discrete log assumption
  - Signing key is  $x$  and Verification key is  $X = g^x$
  - $\text{Sign}(m;x) = (h,s)$  where  $h=H(m,R)$ ,  $R=g^r$ ,  $s = r + hx$
  - $\text{Verify}(m,(h,s);X)$ : check if  $h = H(m,g^s X^{-h})$
- Alternately  $\text{Sign}(m;x) = (R,s)$  where  $R=g^r$ ,  $s = r + hx$  for  $h=H(m,R)$ .  
 $\text{Verify}(m,(R,s);X)$  checks if  $g^s = R X^h$  for  $h = H(m,R)$
- Security by showing that a forger can be used to get distinct signatures  $(h_1,s_1)$ ,  $(h_2,s_2)$  with same  $(m,R)$  (but different  $h$ , by programming the RO) that lets us solve for  $x$

# A Multi-Signature Scheme

- Following Schnorr Signature: a digital signature scheme secure in the Random Oracle model under the discrete log assumption
  - Signing key is  $x$  and Verification key is  $X = g^x$
  - $\text{Sign}(m;x) = (h,s)$  where  $h=H(m,R)$ ,  $R=g^r$ ,  $s = r + hx$
  - $\text{Verify}(m,(h,s);X)$ : check if  $h = H(m,g^s X^{-h})$
- Alternately  $\text{Sign}(m;x) = (R,s)$  where  $R=g^r$ ,  $s = r + hx$  for  $h=H(m,R)$ .  $\text{Verify}(m,(R,s);X)$  checks if  $g^s = R X^h$  for  $h = H(m,R)$
- Security by showing that a forger can be used to get distinct signatures  $(h_1,s_1)$ ,  $(h_2,s_2)$  with same  $(m,R)$  (but different  $h$ , by programming the RO) that lets us solve for  $x$
- Extended to a multi-signature scheme [BN'06]

# A Multi-Signature Scheme

# A Multi-Signature Scheme

- Schnorr:  $\text{Sign}(m;x) = (R,s)$  where  $R=g^r$ ,  $s = r + hx$  for  $h=H(m,R)$ .  
 $\text{Verify}(m,(R,s);X)$  checks if  $g^s = RX^h$  for  $h = H(m,R)$

# A Multi-Signature Scheme

- Schnorr:  $\text{Sign}(m;x) = (R,s)$  where  $R=g^r$ ,  $s = r + hx$  for  $h=H(m,R)$ .  
 $\text{Verify}(m,(R,s);X)$  checks if  $g^s = RX^h$  for  $h = H(m,R)$
- For multiple signers with keys  $X_1, \dots, X_n$  can create an "aggregated" signature  $(R,s)$  such that  $g^s = R \cdot X_1^{h_1} \dots X_n^{h_n}$ , where  $h_i = H(m,R,X_i)$

# A Multi-Signature Scheme

- Schnorr:  $\text{Sign}(m;x) = (R,s)$  where  $R=g^r$ ,  $s = r + hx$  for  $h=H(m,R)$ .  
 $\text{Verify}(m,(R,s);X)$  checks if  $g^s = RX^h$  for  $h = H(m,R)$
- For multiple signers with keys  $X_1, \dots, X_n$  can create an “aggregated” signature  $(R,s)$  such that  $g^s = R \cdot X_1^{h_1} \dots X_n^{h_n}$ , where  $h_i = H(m,R,X_i)$ 
  - Signing done sequentially by individual signers (user  $i$  has  $x_i$ ).  
Initially set  $R=1$  (identity in the group) and  $s=0$ . Then:

# A Multi-Signature Scheme

- Schnorr:  $\text{Sign}(m;x) = (R,s)$  where  $R=g^r$ ,  $s = r + hx$  for  $h=H(m,R)$ .  
 $\text{Verify}(m,(R,s);X)$  checks if  $g^s = RX^h$  for  $h = H(m,R)$
- For multiple signers with keys  $X_1, \dots, X_n$  can create an "aggregated" signature  $(R,s)$  such that  $g^s = R \cdot X_1^{h_1} \dots X_n^{h_n}$ , where  $h_i = H(m,R,X_i)$ 
  - Signing done sequentially by individual signers (user  $i$  has  $x_i$ ).  
Initially set  $R=1$  (identity in the group) and  $s=0$ . Then:
  - $\text{AddSign}(m;(R',s');x_i) = (R,s)$  where  $R=R' \cdot g^{r_i}$  and  $s = s' + r_i + h_i x_i$

# A Multi-Signature Scheme

- Schnorr:  $\text{Sign}(m;x) = (R,s)$  where  $R=g^r$ ,  $s = r + hx$  for  $h=H(m,R)$ .  
 $\text{Verify}(m,(R,s);X)$  checks if  $g^s = RX^h$  for  $h = H(m,R)$
- For multiple signers with keys  $X_1, \dots, X_n$  can create an "aggregated" signature  $(R,s)$  such that  $g^s = R \cdot X_1^{h_1} \dots X_n^{h_n}$ , where  $h_i = H(m,R,X_i)$ 
  - Signing done sequentially by individual signers (user  $i$  has  $x_i$ ).  
Initially set  $R=1$  (identity in the group) and  $s=0$ . Then:
  - $\text{AddSign}(m;(R',s');x_i) = (R,s)$  where  $R=R' \cdot g^{r_i}$  and  $s = s' + r_i + h_i x_i$ 
    - So that finally  $R = g^r = g^{r_1 + \dots + r_n}$  and  $s = r + h_1 x_1 + \dots + h_n x_n$

# A Multi-Signature Scheme

- Schnorr:  $\text{Sign}(m;x) = (R,s)$  where  $R=g^r$ ,  $s = r + hx$  for  $h=H(m,R)$ .  
 $\text{Verify}(m,(R,s);X)$  checks if  $g^s = RX^h$  for  $h = H(m,R)$
- For multiple signers with keys  $X_1, \dots, X_n$  can create an “aggregated” signature  $(R,s)$  such that  $g^s = R.X_1^{h_1} \dots X_n^{h_n}$ , where  $h_i = H(m,R,X_i)$ 
  - Signing done sequentially by individual signers (user  $i$  has  $x_i$ ).  
Initially set  $R=1$  (identity in the group) and  $s=0$ . Then:
  - $\text{AddSign}(m;(R',s');x_i) = (R,s)$  where  $R=R'.g^{r_i}$  and  $s = s' + r_i + h_i x_i$ 
    - So that finally  $R = g^r = g^{r_1 + \dots + r_n}$  and  $s = r + h_1 x_1 + \dots + h_n x_n$
- Security by showing that a forger succeeds with same  $(m,R)$  when given two distinct answers for  $h_1$  (by programming the RO)

# Aggregate Signatures

# Aggregate Signatures

- Generalization of multi-signatures where multiple signers may have different messages

# Aggregate Signatures

- Generalization of multi-signatures where multiple signers may have different messages
- Sequential aggregation: each signer gets the aggregated signature so far and adds her signature into it

# Aggregate Signatures

- Generalization of multi-signatures where multiple signers may have different messages
- Sequential aggregation: each signer gets the aggregated signature so far and adds her signature into it
- General aggregation: signatures can be created independently and then aggregated in arbitrary order

# A Sequential Aggregate Signature Scheme

# A Sequential Aggregate Signature Scheme

- Water's Signature: Secure if the Computational Diffie-Hellman assumption holds in a group with bilinear pairings (no RO)

# A Sequential Aggregate Signature Scheme

- Water's Signature: Secure if the Computational Diffie-Hellman assumption holds in a group with bilinear pairings (no RO)
  - Signing key is  $x$  and verification key is  $X := e(g,g)^x$ , and generators  $u_0, u_1, \dots, u_k$  (for  $k$  bit long messages)

# A Sequential Aggregate Signature Scheme

- Water's Signature: Secure if the Computational Diffie-Hellman assumption holds in a group with bilinear pairings (no RO)
  - Signing key is  $x$  and verification key is  $X := e(g,g)^x$ , and generators  $u_0, u_1, \dots, u_k$  (for  $k$  bit long messages)
  - $\text{Sign}(m;x) = (R,S)$  where  $R=g^r$  and  $S = g^x h^r$ , where  $h = u_0 \cdot u_1^{m_1} \dots u_k^{m_k}$

# A Sequential Aggregate Signature Scheme

- Water's Signature: Secure if the Computational Diffie-Hellman assumption holds in a group with bilinear pairings (no RO)
  - Signing key is  $x$  and verification key is  $X := e(g,g)^x$ , and generators  $u_0, u_1, \dots, u_k$  (for  $k$  bit long messages)
  - $\text{Sign}(m;x) = (R,S)$  where  $R=g^r$  and  $S = g^x h^r$ , where  $h = u_0 \cdot u_1^{m_1} \dots u_k^{m_k}$
  - $\text{Verify}(m,(R,S);X,u_0,u_1,\dots,u_k)$ : check  $e(S,g) = e(R,h) \cdot X$

# A Sequential Aggregate Signature Scheme

- Water's Signature: Secure if the Computational Diffie-Hellman assumption holds in a group with bilinear pairings (no RO)
  - Signing key is  $x$  and verification key is  $X := e(g,g)^x$ , and generators  $u_0, u_1, \dots, u_k$  (for  $k$  bit long messages)
  - $\text{Sign}(m;x) = (R,S)$  where  $R=g^r$  and  $S = g^x h^r$ , where  $h = u_0 \cdot u_1^{m_1} \dots u_k^{m_k}$
  - $\text{Verify}(m,(R,S);X,u_0,u_1,\dots,u_k)$ : check  $e(S,g) = e(R,h) \cdot X$
- Extended to a sequential aggregate scheme [LOSSW'06]

# A Sequential Aggregate Signature Scheme

# A Sequential Aggregate Signature Scheme

- Water's Signature:  $\text{Sign}(m;x) = (R,S)$  where  $R=g^r$  and  $S = g^x h^r$ , where  $h = u_0.u_1^{m_1} \dots u_k^{m_k}$ .  $\text{Verify}(m,(R,S);X)$ : check  $e(S,g) = e(R,h).X$

# A Sequential Aggregate Signature Scheme

- Water's Signature:  $\text{Sign}(m;x) = (R,S)$  where  $R=g^r$  and  $S = g^x h^r$ , where  $h = u_0.u_1^{m_1} \dots u_k^{m_k}$ .  $\text{Verify}(m,(R,S);X)$ : check  $e(S,g) = e(R,h).X$
- For user  $i$  verification key is  $X_i := e(g,g)^{x_i}$ , and  $u^i_0, u^i_1, \dots, u^i_k$ . Signing key is  $x_i$  and  $y^i_0, y^i_1, \dots, y^i_k$  where  $u^i_j = g^{y^i_j}$

# A Sequential Aggregate Signature Scheme

- Water's Signature:  $\text{Sign}(m;x) = (R,S)$  where  $R=g^r$  and  $S = g^x h^r$ , where  $h = u_0.u_1^{m_1} \dots u_k^{m_k}$ .  $\text{Verify}(m,(R,S);X)$ : check  $e(S,g) = e(R,h).X$
- For user  $i$  verification key is  $X_i := e(g,g)^{x_i}$ , and  $u^i_0, u^i_1, \dots, u^i_k$ . Signing key is  $x_i$  and  $y^i_0, y^i_1, \dots, y^i_k$  where  $u^i_j = g^{y^i_j}$
- Verification of signature  $(R,S)$  for messages  $(m^1, \dots, m^n)$ : check if  $e(S,g) = e(R,h_1)X_1 \dots e(R,h_n)X_n$  where  $h_i = u^i_0.(u^i_1)^{m^1} \dots (u^i_k)^{m^k}$

# A Sequential Aggregate Signature Scheme

- Water's Signature:  $\text{Sign}(m;x) = (R,S)$  where  $R=g^r$  and  $S = g^x h^r$ , where  $h = u_0.u_1^{m_1} \dots u_k^{m_k}$ .  $\text{Verify}(m,(R,S);X)$ : check  $e(S,g) = e(R,h).X$
- For user  $i$  verification key is  $X_i := e(g,g)^{x_i}$ , and  $u^i_0, u^i_1, \dots, u^i_k$ . Signing key is  $x_i$  and  $y^i_0, y^i_1, \dots, y^i_k$  where  $u^i_j = g^{y^i_j}$
- Verification of signature  $(R,S)$  for messages  $(m^1, \dots, m^n)$ : check if  $e(S,g) = e(R,h_1)X_1 \dots e(R,h_n)X_n$  where  $h_i = u^i_0.(u^i_1)^{m^1} \dots (u^i_k)^{m^k}$
- Signing done sequentially by individual signers. Initially set  $R=1$  and  $S = 1$  (identity in the group). Then:

# A Sequential Aggregate Signature Scheme

- Water's Signature:  $\text{Sign}(m;x) = (R,S)$  where  $R=g^r$  and  $S = g^x h^r$ , where  $h = u_0.u_1^{m^1}...u_k^{m^k}$ .  $\text{Verify}(m,(R,S);X)$ : check  $e(S,g) = e(R,h).X$
- For user  $i$  verification key is  $X_i := e(g,g)^{x_i}$ , and  $u^i_0, u^i_1, \dots, u^i_k$ . Signing key is  $x_i$  and  $y^i_0, y^i_1, \dots, y^i_k$  where  $u^i_j = g^{y^i_j}$
- Verification of signature  $(R,S)$  for messages  $(m^1, \dots, m^n)$ : check if  $e(S,g) = e(R,h_1)X_1 \dots e(R,h_n)X_n$  where  $h_i = u^i_0.(u^i_1)^{m^1}... (u^i_k)^{m^k}$
- Signing done sequentially by individual signers. Initially set  $R=1$  and  $S = 1$  (identity in the group). Then:
  - $\text{AddSign}(m^i,(R',S'); x_i, y^i_0, y^i_1, \dots, y^i_k) = \text{ReRand}(R,S)$ , where  $R=R'$  and  $S = S'.g^{x_i}.(R')^{f_i}$  where  $f_i$  s.t.  $g^{f_i} = h_i$

# A Sequential Aggregate Signature Scheme

- Water's Signature:  $\text{Sign}(m;x) = (R,S)$  where  $R=g^r$  and  $S = g^x h^r$ , where  $h = u_0.u_1^{m_1} \dots u_k^{m_k}$ .  $\text{Verify}(m,(R,S);X)$ : check  $e(S,g) = e(R,h).X$
- For user  $i$  verification key is  $X_i := e(g,g)^{x_i}$ , and  $u^i_0, u^i_1, \dots, u^i_k$ . Signing key is  $x_i$  and  $y^i_0, y^i_1, \dots, y^i_k$  where  $u^i_j = g^{y^i_j}$
- Verification of signature  $(R,S)$  for messages  $(m^1, \dots, m^n)$ : check if  $e(S,g) = e(R,h_1)X_1 \dots e(R,h_n)X_n$  where  $h_i = u^i_0.(u^i_1)^{m^1} \dots (u^i_k)^{m^k}$
- Signing done sequentially by individual signers. Initially set  $R=1$  and  $S = 1$  (identity in the group). Then:
  - $\text{AddSign}(m^i,(R',S'); x_i, y^i_0, y^i_1, \dots, y^i_k) = \text{ReRand}(R,S)$ , where  $R=R'$  and  $S = S'.g^{x_i}.(R')^{f_i}$  where  $f_i$  s.t.  $g^{f_i} = h_i$
  - $\text{ReRand}(R,S) = (R^*,S^*)$ , where  $R^* = Rg^\dagger$  and  $S^* = S.h_1^\dagger \dots h_i^\dagger$

# Batch Verification

# Batch Verification

- To speed up verification of a collection of signatures

# Batch Verification

- To speed up verification of a collection of signatures
  - Batching done by the verifier

# Batch Verification

- To speed up verification of a collection of signatures
  - Batching done by the verifier
  - Incomparable to aggregate signatures

# Batch Verification

- To speed up verification of a collection of signatures
  - Batching done by the verifier
  - Incomparable to aggregate signatures
    - Batch verifiable signature scheme reduces verification time, but does not reduce the total size of signatures that verifier gets. Retains individual signatures (that can be forwarded/re-batched).

# Batch Verification

- To speed up verification of a collection of signatures
  - Batching done by the verifier
  - Incomparable to aggregate signatures
    - Batch verifiable signature scheme reduces verification time, but does not reduce the total size of signatures that verifier gets. Retains individual signatures (that can be forwarded/re-batched).
    - Aggregate signatures saves on bandwidth and verification time, but does not allow un-aggregating the signatures

# Batch Verification

# Batch Verification

- Idea: to verify several equations of the form  $Z_i = g^{z_i}$ , pick random weights  $w_i$  and check  $\prod_i Z_i^{w_i} = g^{\sum z_i \cdot w_i}$

# Batch Verification

- Idea: to verify several equations of the form  $Z_i = g^{z_i}$ , pick random weights  $w_i$  and check  $\prod_i Z_i^{w_i} = g^{\sum z_i \cdot w_i}$
- If one (or more) equation is wrong, probability of verifying is  $1/q$ , where  $q$  is the domain of  $w_i$  (at most order of  $g$ , but may be smaller for efficiency)

# Batch Verification

- Idea: to verify several equations of the form  $Z_i = g^{z_i}$ , pick random weights  $w_i$  and check  $\prod_i Z_i^{w_i} = g^{\sum z_i \cdot w_i}$ 
  - If one (or more) equation is wrong, probability of verifying is  $1/q$ , where  $q$  is the domain of  $w_i$  (at most order of  $g$ , but may be smaller for efficiency)
- Similarly for pairing equations, but with further optimizations

# Batch Verification

- Idea: to verify several equations of the form  $Z_i = g^{z_i}$ , pick random weights  $w_i$  and check  $\prod_i Z_i^{w_i} = g^{\sum z_i \cdot w_i}$ 
  - If one (or more) equation is wrong, probability of verifying is  $1/q$ , where  $q$  is the domain of  $w_i$  (at most order of  $g$ , but may be smaller for efficiency)
- Similarly for pairing equations, but with further optimizations
  - e.g. Waters' signature:  $e(S, g) = e(R, h) \cdot X$  ( $g$  same for all signers)

# Batch Verification

- Idea: to verify several equations of the form  $Z_i = g^{z_i}$ , pick random weights  $w_i$  and check  $\prod_i Z_i^{w_i} = g^{\sum z_i \cdot w_i}$ 
  - If one (or more) equation is wrong, probability of verifying is  $1/q$ , where  $q$  is the domain of  $w_i$  (at most order of  $g$ , but may be smaller for efficiency)
- Similarly for pairing equations, but with further optimizations
  - e.g. Waters' signature:  $e(S, g) = e(R, h) \cdot X$  ( $g$  same for all signers)
    - Can save using  $\prod_i e(S_i, g)^{w_i} = \prod_i e(S_i^{w_i}, g) = e(\prod_i S_i^{w_i}, g)$

# Group Signatures

# Group Signatures

- To sign a message “anonymously” [CvH'91]

# Group Signatures

- To sign a message “anonymously” [CvH’91]
  - Signature shows that message was signed by some member of a group

# Group Signatures

- To sign a message “anonymously” [CvH’91]
  - Signature shows that message was signed by some member of a group
  - But a group manager can “trace” the signer

# Group Signatures

- To sign a message “anonymously” [CvH’91]
  - Signature shows that message was signed by some member of a group
  - But a group manager can “trace” the signer
  - However, the group manager or other group members “cannot frame” a member

# Group Signatures

# Group Signatures

- Full-Anonymity: Adversary gives  $(m, ID_0, ID_1)$  and gets back  $\text{Sign}(m; ID_b)$  for a random bit  $b$ . Advantage of the adversary in finding  $b$  should be negligible. Adversary knows secret keys of all group-members, and has oracle access to the tracing algorithm (but not allowed to query it on the challenge)

# Group Signatures

- Full-Anonymity: Adversary gives  $(m, ID_0, ID_1)$  and gets back  $\text{Sign}(m; ID_b)$  for a random bit  $b$ . Advantage of the adversary in finding  $b$  should be negligible. Adversary knows secret keys of all group-members, and has oracle access to the tracing algorithm (but not allowed to query it on the challenge)
  - Implies unlinkability (can't link signatures from same user)

# Group Signatures

- Full-Anonymity: Adversary gives  $(m, ID_0, ID_1)$  and gets back  $\text{Sign}(m; ID_b)$  for a random bit  $b$ . Advantage of the adversary in finding  $b$  should be negligible. Adversary knows secret keys of all group-members, and has oracle access to the tracing algorithm (but not allowed to query it on the challenge)
  - Implies unlinkability (can't link signatures from same user)
- Full-Traceability: If a set of group members collude and create a valid signature, the tracing algorithm will trace at least one member of the set. This holds even if the group manager is passively corrupt.

# Group Signatures

- Full-Anonymity: Adversary gives  $(m, ID_0, ID_1)$  and gets back  $\text{Sign}(m; ID_b)$  for a random bit  $b$ . Advantage of the adversary in finding  $b$  should be negligible. Adversary knows secret keys of all group-members, and has oracle access to the tracing algorithm (but not allowed to query it on the challenge)
  - Implies unlinkability (can't link signatures from same user)
- Full-Traceability: If a set of group members collude and create a valid signature, the tracing algorithm will trace at least one member of the set. This holds even if the group manager is passively corrupt.
  - Implies unforgeability (i.e., with no group members colluding with it, adversary cannot produce a valid signature) and framing-resistance (even colluding with the group manager)

# Group Signatures

# Group Signatures

- A general construction: using a digital signature scheme, a CCA secure encryption scheme, and a “simulation-sound” NIZK [BMW'03]

# Group Signatures

- A general construction: using a digital signature scheme, a CCA secure encryption scheme, and a “simulation-sound” NIZK [BMW'03]
- Each member's signing key  $\text{Key}_i$  consists of a key-pair  $(\text{VK}_i, \text{SK}_i)$  and a certificate from the group-manager for  $\text{VK}_i$  (optionally binding it to  $\text{ID}_i$ )

# Group Signatures

- A general construction: using a digital signature scheme, a CCA secure encryption scheme, and a “simulation-sound” NIZK [BMW'03]
- Each member's signing key  $\text{Key}_i$  consists of a key-pair  $(\text{VK}_i, \text{SK}_i)$  and a certificate from the group-manager for  $\text{VK}_i$  (optionally binding it to  $\text{ID}_i$ )
- Signature is  $(C, \pi)$ ,  $C$  being an encryption of  $(s, \text{Key}_i)$  where  $s$  is a signature on the message using  $\text{SK}_i$  and  $\pi$  being a proof (w.r.t a CRS in the group's public-key) that  $C$  is correct (and in particular has a correct  $s$  and  $\text{Key}$ )

# Group Signatures

- A general construction: using a digital signature scheme, a CCA secure encryption scheme, and a “simulation-sound” NIZK [BMW’03]
- Each member’s signing key  $\text{Key}_i$  consists of a key-pair  $(\text{VK}_i, \text{SK}_i)$  and a certificate from the group-manager for  $\text{VK}_i$  (optionally binding it to  $\text{ID}_i$ )
- Signature is  $(C, \pi)$ ,  $C$  being an encryption of  $(s, \text{Key}_i)$  where  $s$  is a signature on the message using  $\text{SK}_i$  and  $\pi$  being a proof (w.r.t a CRS in the group’s public-key) that  $C$  is correct (and in particular has a correct  $s$  and  $\text{Key}$ )
- Tracing algorithm decrypts  $C$  to find  $\text{Key}_i$  and hence  $\text{ID}_i$

# Ring Signatures

# Ring Signatures

- For “leaking secrets”

# Ring Signatures

- For “leaking secrets”
  - Similar to group signatures, but with unwitting collaborators

# Ring Signatures

- For “leaking secrets”
  - Similar to group signatures, but with unwitting collaborators
    - i.e. the “ring” is not a priori fixed

# Ring Signatures

- For “leaking secrets”
  - Similar to group signatures, but with unwitting collaborators
    - i.e. the “ring” is not a priori fixed
  - And no manager who can trace the signer

# Ring Signatures

# Ring Signatures

- Recall T-OWP/RO based signature

# Ring Signatures

- Recall T-OWP/RO based signature
  - $(SK, VK) = (F^{-1}, F)$

# Ring Signatures

- Recall T-OWP/RO based signature
  - $(SK, VK) = (F^{-1}, F)$
  - $\text{Sign}(m; F^{-1}) = F^{-1}(H(m))$

# Ring Signatures

- Recall T-OWP/RO based signature
  - $(SK, VK) = (F^{-1}, F)$
  - $Sign(m; F^{-1}) = F^{-1}(H(m))$
  - $Verify(S; F)$ : check if  $H(m) = F(S)$

# Ring Signatures

- Recall T-OWP/RO based signature
  - $(SK, VK) = (F^{-1}, F)$
  - $Sign(m; F^{-1}) = F^{-1}(H(m))$
  - $Verify(S; F)$ : check if  $H(m) = F(S)$
- Extended to a ring signature [RST'01]

# Ring Signatures

- Recall T-OWP/RO based signature
  - $(SK, VK) = (F^{-1}, F)$
  - $Sign(m; F^{-1}) = F^{-1}(H(m))$
  - $Verify(S; F)$ : check if  $H(m) = F(S)$
- Extended to a ring signature [RST'01]
- $Verify(m, (S_1, \dots, S_n); (F_1, \dots, F_n))$  : check  $H(m) = F_1(S_1) + \dots + F_n(S_n)$

# Ring Signatures

- Recall T-OWP/RO based signature
  - $(SK, VK) = (F^{-1}, F)$
  - $\text{Sign}(m; F^{-1}) = F^{-1}(H(m))$
  - $\text{Verify}(S; F)$ : check if  $H(m) = F(S)$
- Extended to a ring signature [RST'01]
- $\text{Verify}(m, (S_1, \dots, S_n); (F_1, \dots, F_n))$  : check  $H(m) = F_1(S_1) + \dots + F_n(S_n)$
- $\text{Sign}(m; F_1^{-1}, F_2, \dots, F_n) = (S_1, \dots, S_n)$  where  $S_2, \dots, S_n$  are random and  $S_1 = F_1^{-1}(H(m) - F_2(S_2) - \dots - F_n(S_n))$

# Ring Signatures

- Recall T-OWP/RO based signature
  - $(SK, VK) = (F^{-1}, F)$
  - $\text{Sign}(m; F^{-1}) = F^{-1}(H(m))$
  - $\text{Verify}(S; F)$ : check if  $H(m) = F(S)$
- Extended to a ring signature [RST'01]
- $\text{Verify}(m, (S_1, \dots, S_n); (F_1, \dots, F_n))$  : check  $H(m) = F_1(S_1) + \dots + F_n(S_n)$
- $\text{Sign}(m; F_1^{-1}, F_2, \dots, F_n) = (S_1, \dots, S_n)$  where  $S_2, \dots, S_n$  are random and  $S_1 = F_1^{-1}(H(m) - F_2(S_2) - \dots - F_n(S_n))$
- Unwitting collaborators:  $F_i$ 's could be the verification keys for a standard signature scheme

# Mesh Signatures

# Mesh Signatures

- Ring signature allows statements of the form  
( $P_1$  signed  $m$ ) or ( $P_2$  signed  $m$ ) or ... or ( $P_n$  signed  $m$ )

# Mesh Signatures

- Ring signature allows statements of the form  
( $P_1$  signed  $m$ ) or ( $P_2$  signed  $m$ ) or ... or ( $P_n$  signed  $m$ )
- Mesh signatures extend this to more complex statements

# Mesh Signatures

- Ring signature allows statements of the form  
( $P_1$  signed  $m$ ) or ( $P_2$  signed  $m$ ) or ... or ( $P_n$  signed  $m$ )
- Mesh signatures extend this to more complex statements
  - e.g., ( $P_1$  signed  $m_1$ ) or ( ( $P_2$  signed  $m_2$ ) and ( $P_3$  signed  $m_3$ ) )

# Mesh Signatures

- Ring signature allows statements of the form  $(P_1 \text{ signed } m)$  or  $(P_2 \text{ signed } m)$  or ... or  $(P_n \text{ signed } m)$
- Mesh signatures extend this to more complex statements
  - e.g.,  $(P_1 \text{ signed } m_1)$  or  $( (P_2 \text{ signed } m_2)$  and  $(P_3 \text{ signed } m_3) )$
  - e.g., some two out of the three statements  $(P_1 \text{ signed } m_1)$ ,  $(P_2 \text{ signed } m_2)$ ,  $(P_3 \text{ signed } m_3)$  hold

# Mesh Signatures

- Ring signature allows statements of the form  $(P_1 \text{ signed } m)$  or  $(P_2 \text{ signed } m)$  or ... or  $(P_n \text{ signed } m)$
- Mesh signatures extend this to more complex statements
  - e.g.,  $(P_1 \text{ signed } m_1)$  or  $( (P_2 \text{ signed } m_2)$  and  $(P_3 \text{ signed } m_3) )$
  - e.g., some two out of the three statements  $(P_1 \text{ signed } m_1)$ ,  $(P_2 \text{ signed } m_2)$ ,  $(P_3 \text{ signed } m_3)$  hold
  - Signature is produced by the relevant parties collaborating

# Mesh Signatures

- Ring signature allows statements of the form  $(P_1 \text{ signed } m)$  or  $(P_2 \text{ signed } m)$  or ... or  $(P_n \text{ signed } m)$
- Mesh signatures extend this to more complex statements
  - e.g.,  $(P_1 \text{ signed } m_1)$  or  $( (P_2 \text{ signed } m_2)$  and  $(P_3 \text{ signed } m_3) )$
  - e.g., some two out of the three statements  $(P_1 \text{ signed } m_1)$ ,  $(P_2 \text{ signed } m_2)$ ,  $(P_3 \text{ signed } m_3)$  hold
  - Signature is produced by the relevant parties collaborating
  - Security requirements: Unforgeability and Hiding

# Attribute-Based Signatures

# Attribute-Based Signatures

- “Claim-and-endorse”: Claim to have attributes satisfying a certain policy, and sign a message

# Attribute-Based Signatures

- “Claim-and-endorse”: Claim to have attributes satisfying a certain policy, and sign a message
  - Soundness: can't forge, even by colluding

# Attribute-Based Signatures

- “Claim-and-endorse”: Claim to have attributes satisfying a certain policy, and sign a message
  - Soundness: can't forge, even by colluding
  - Hiding: Verification without learning how the policy was satisfied

# Attribute-Based Signatures

- “Claim-and-endorse”: Claim to have attributes satisfying a certain policy, and sign a message
  - Soundness: can't forge, even by colluding
  - Hiding: Verification without learning how the policy was satisfied
    - Also unlinkable: cannot link multiple signatures as originating from the same signer

# Attribute-Based Signatures

- “Claim-and-endorse”: Claim to have attributes satisfying a certain policy, and sign a message
  - Soundness: can't forge, even by colluding
  - Hiding: Verification without learning how the policy was satisfied
    - Also unlinkable: cannot link multiple signatures as originating from the same signer
- c.f. Mesh signatures: here, instead of multiple parties signing a message, a single party with multiple attributes

# An ABS Construction

# An ABS Construction

- Using “Credential Bundles” and NIZK proofs (in fact, NIWI proofs)

# An ABS Construction

- Using “Credential Bundles” and NIZK proofs (in fact, NIWI proofs)
- Credential Bundle for a set of attributes:

# An ABS Construction

- Using “Credential Bundles” and NIZK proofs (in fact, NIWI proofs)
- Credential Bundle for a set of attributes:
  - Given multiple credential bundles, can't create a credential bundle for a new set, unless it is a subset of attributes in a single given credential bundle

# An ABS Construction

- Using “Credential Bundles” and NIZK proofs (in fact, NIWI proofs)
- Credential Bundle for a set of attributes:
  - Given multiple credential bundles, can't create a credential bundle for a new set, unless it is a subset of attributes in a single given credential bundle
- Map each (claim,message) to a “pseudo-attribute”

# An ABS Construction

- Using “Credential Bundles” and NIZK proofs (in fact, NIWI proofs)
- Credential Bundle for a set of attributes:
  - Given multiple credential bundles, can’t create a credential bundle for a new set, unless it is a subset of attributes in a single given credential bundle
- Map each (claim,message) to a “pseudo-attribute”
- Signing key: credential bundle for (real) attributes possessed

# An ABS Construction

- Using “Credential Bundles” and NIZK proofs (in fact, NIWI proofs)
- Credential Bundle for a set of attributes:
  - Given multiple credential bundles, can’t create a credential bundle for a new set, unless it is a subset of attributes in a single given credential bundle
- Map each (claim,message) to a “pseudo-attribute”
- Signing key: credential bundle for (real) attributes possessed
- Signature: a NIZK proof of knowledge of a credential-bundle for attributes satisfying the claim, or a credential for the pseudo-attribute corresponding to (claim,message)

# An ABS Construction

- Using “Credential Bundles” and NIZK proofs (in fact, NIWI proofs)
- Credential Bundle for a set of attributes:
  - Given multiple credential bundles, can’t create a credential bundle for a new set, unless it is a subset of attributes in a single given credential bundle
- Map each (claim,message) to a “pseudo-attribute”
- Signing key: credential bundle for (real) attributes possessed
- Signature: a NIZK proof of knowledge of a credential-bundle for attributes satisfying the claim, or a credential for the pseudo-attribute corresponding to (claim,message)
- Using conventional tools. More efficiently using bilinear pairings.

# Undeniable Signatures

# Undeniable Signatures

- Suppose Signer wants to control when/how often the signature can be verified, but signature is a commitment to a message

# Undeniable Signatures

- Suppose Signer wants to control when/how often the signature can be verified, but signature is a commitment to a message
  - Verification is via an interactive protocol

# Undeniable Signatures

- Suppose Signer wants to control when/how often the signature can be verified, but signature is a commitment to a message
  - Verification is via an interactive protocol
  - It lets the signer verifiably accept or deny endorsing the message

# Undeniable Signatures

- Suppose Signer wants to control when/how often the signature can be verified, but signature is a commitment to a message
  - Verification is via an interactive protocol
  - It lets the signer verifiably accept or deny endorsing the message
  - Signer refusing to deny can be taken as accepting

# Undeniable Signatures

- Suppose Signer wants to control when/how often the signature can be verified, but signature is a commitment to a message
  - Verification is via an interactive protocol
  - It lets the signer verifiably accept or deny endorsing the message
  - Signer refusing to deny can be taken as accepting
- Zero-knowledge verification: A verifier cannot transfer a signature that it verified

# Undeniable Signatures

- Suppose Signer wants to control when/how often the signature can be verified, but signature is a commitment to a message
  - Verification is via an interactive protocol
  - It lets the signer verifiably accept or deny endorsing the message
  - Signer refusing to deny can be taken as accepting
- Zero-knowledge verification: A verifier cannot transfer a signature that it verified
- Note: Still allows multiple (mutually distrusting) verifiers to be convinced if they run a secure MPC protocol to implement a virtual verifier

# Designated Verifier Signatures

# Designated Verifier Signatures

- Signature addressed to a single designated verifier

# Designated Verifier Signatures

- Signature addressed to a single designated verifier
  - Verifier cannot convince others of the validity of the signature

# Designated Verifier Signatures

- Signature addressed to a single designated verifier
  - Verifier cannot convince others of the validity of the signature
  - e.g. a ring signature with a ring of size 2, containing the signer and the designated verifier

# Today

# Today

- Signatures

# Today

- Signatures
  - Multi-signatures

# Today

- Signatures
  - Multi-signatures
  - Aggregate Signatures

# Today

- Signatures
  - Multi-signatures
  - Aggregate Signatures
  - Signatures with Batch verification

# Today

- Signatures
  - Multi-signatures
  - Aggregate Signatures
  - Signatures with Batch verification
  - Group signatures

# Today

- Signatures
  - Multi-signatures
  - Aggregate Signatures
  - Signatures with Batch verification
  - Group signatures
  - Ring and Mesh signatures

# Today

- Signatures
  - Multi-signatures
  - Aggregate Signatures
  - Signatures with Batch verification
  - Group signatures
  - Ring and Mesh signatures
  - Attribute-Based signatures

# Today

- Signatures
  - Multi-signatures
  - Aggregate Signatures
  - Signatures with Batch verification
  - Group signatures
  - Ring and Mesh signatures
  - Attribute-Based signatures
  - Undeniable signatures

# Today

- Signatures
  - Multi-signatures
  - Aggregate Signatures
  - Signatures with Batch verification
  - Group signatures
  - Ring and Mesh signatures
  - Attribute-Based signatures
  - Undeniable signatures
  - Designated verifier signatures