Universal Composition

Lecture 14

Composition Issues

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Play the GM's against each other

Will not lose against both!

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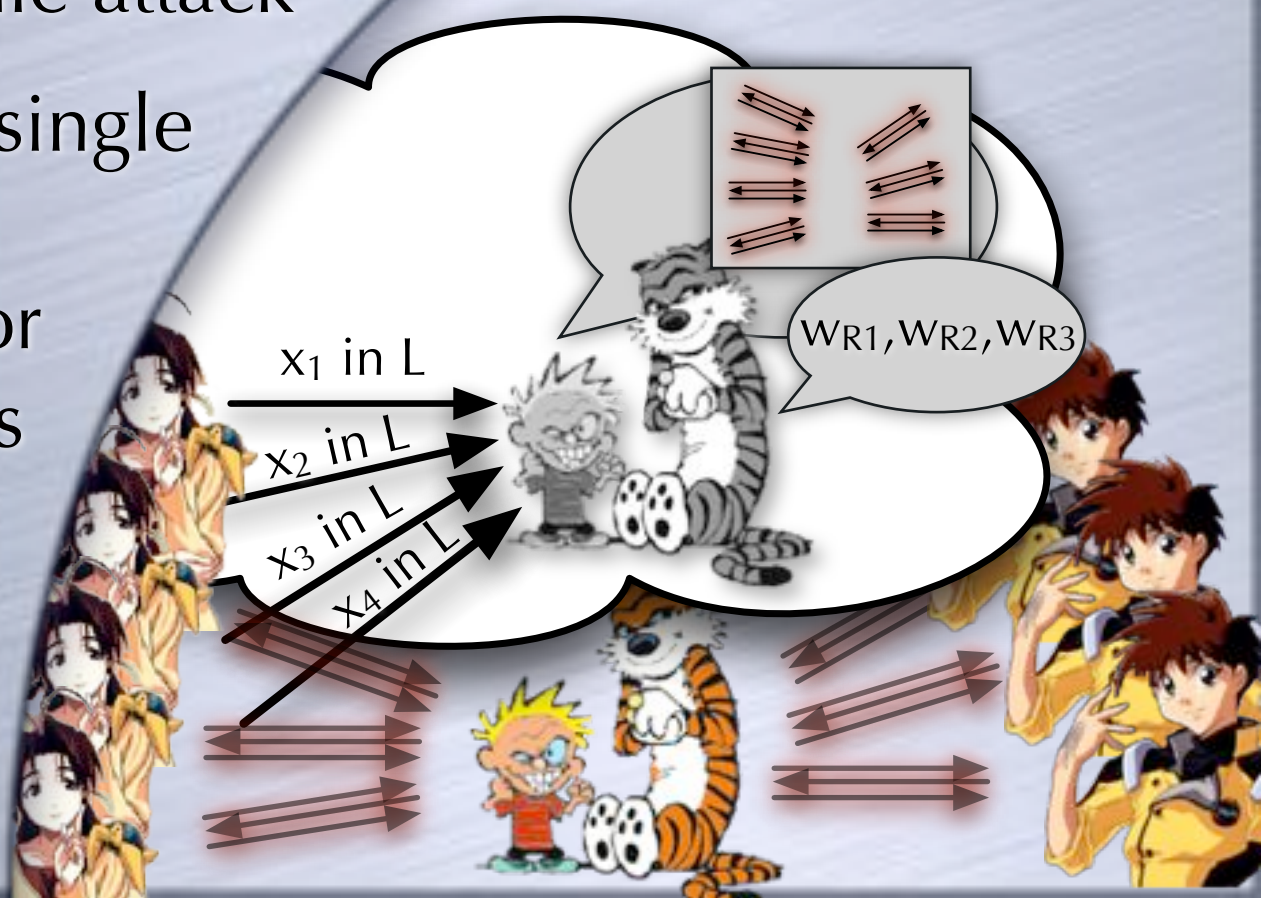
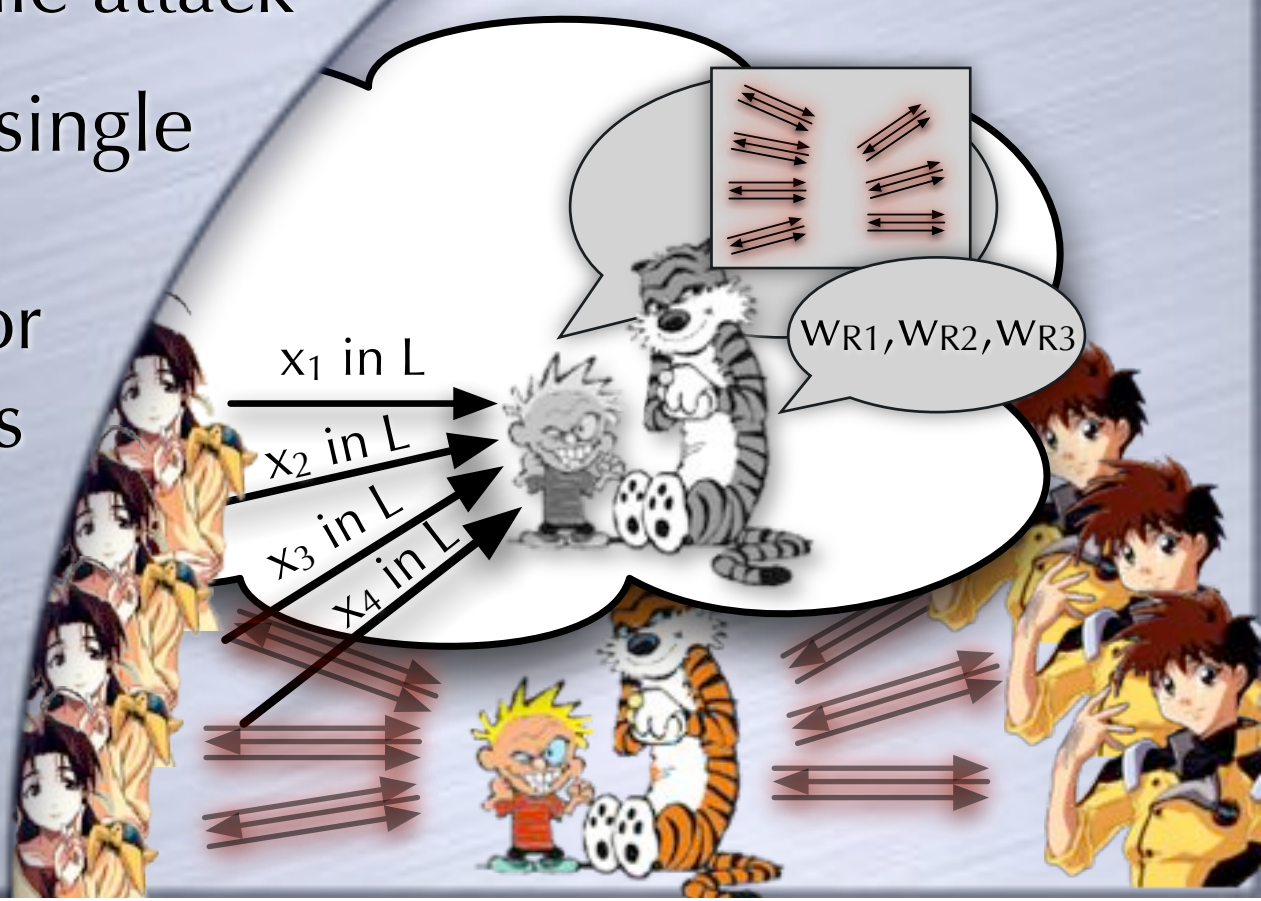


Composition Issues

- Multiple executions provide new opportunities for the hacker
  - Person-in-the-middle attack
  - Simulatability of a single execution doesn't imply simulation for multiple executions
    - Or when run along with other protocols



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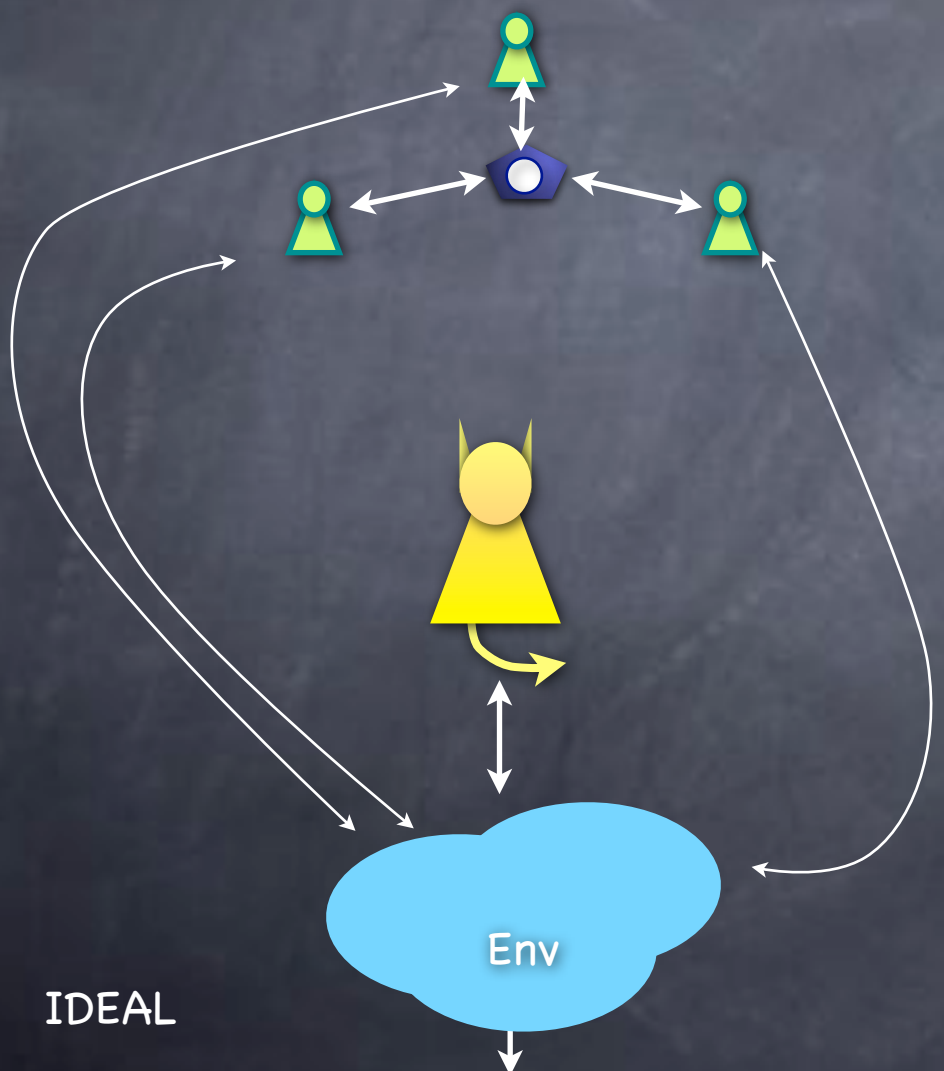
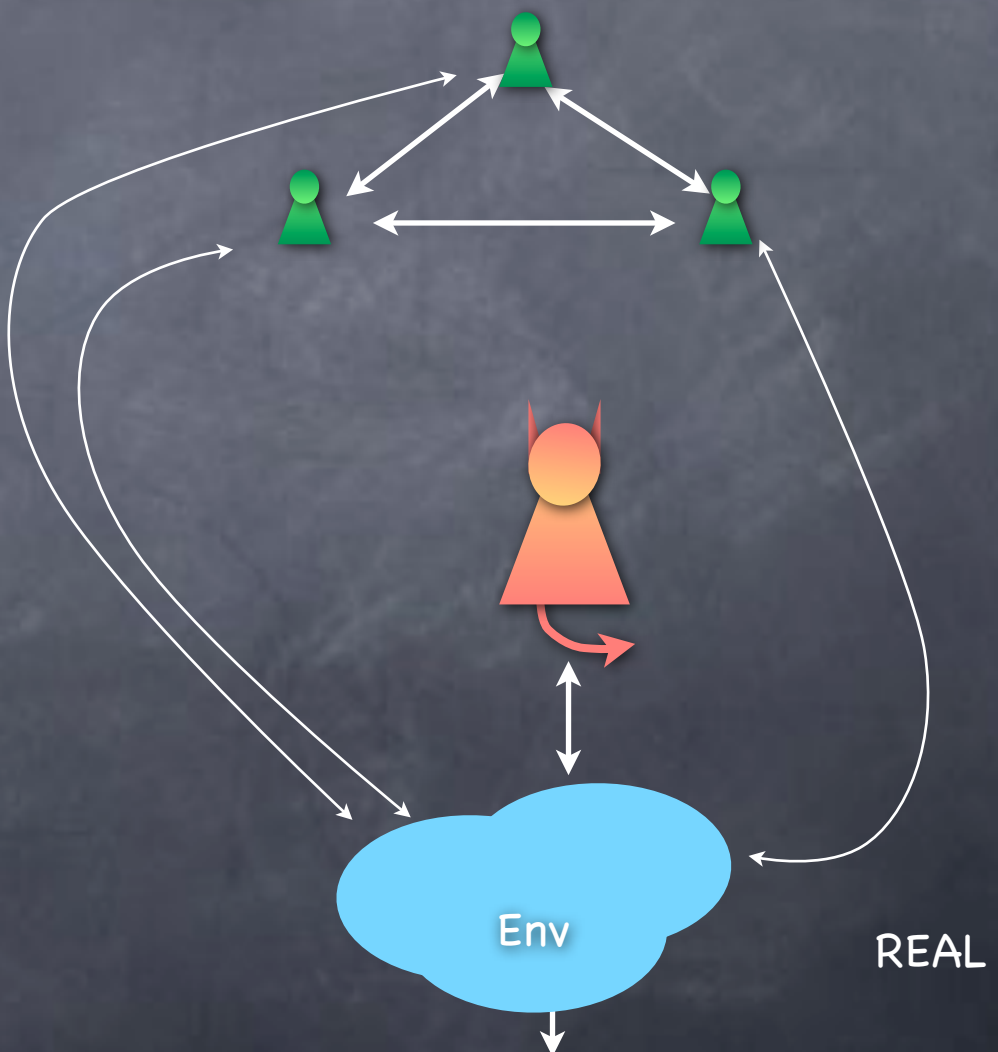
    - Will use SIM security

RECALL

Security

REAL (with protocol) is as secure as IDEAL (with functionality) if:





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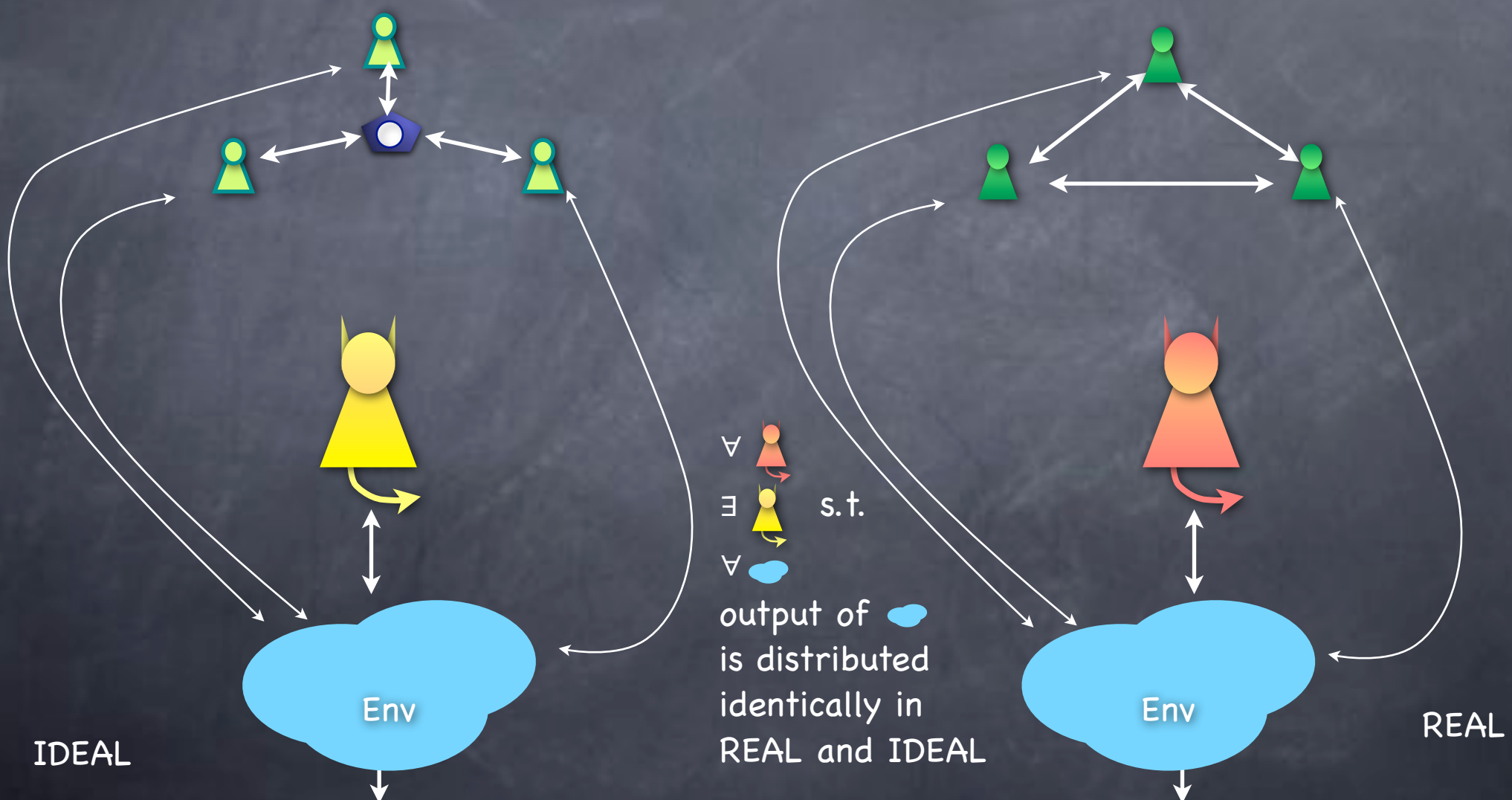
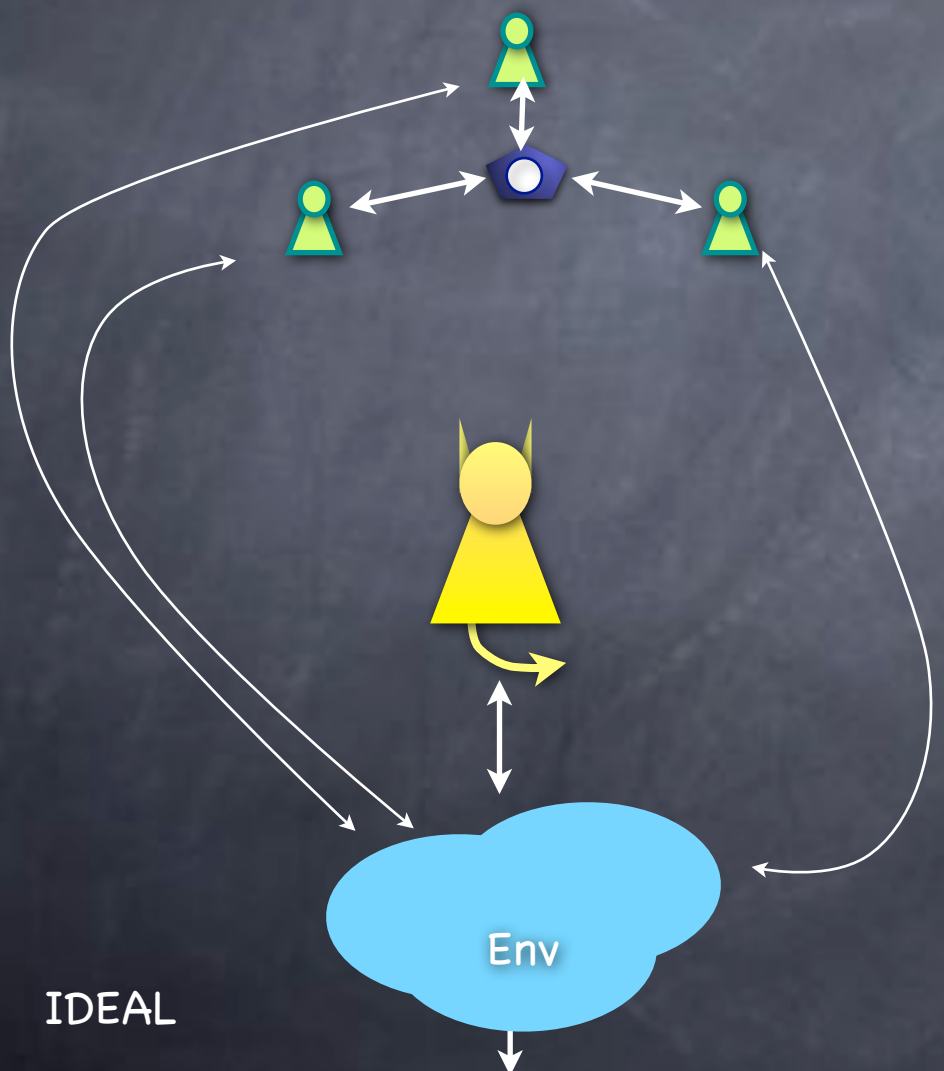
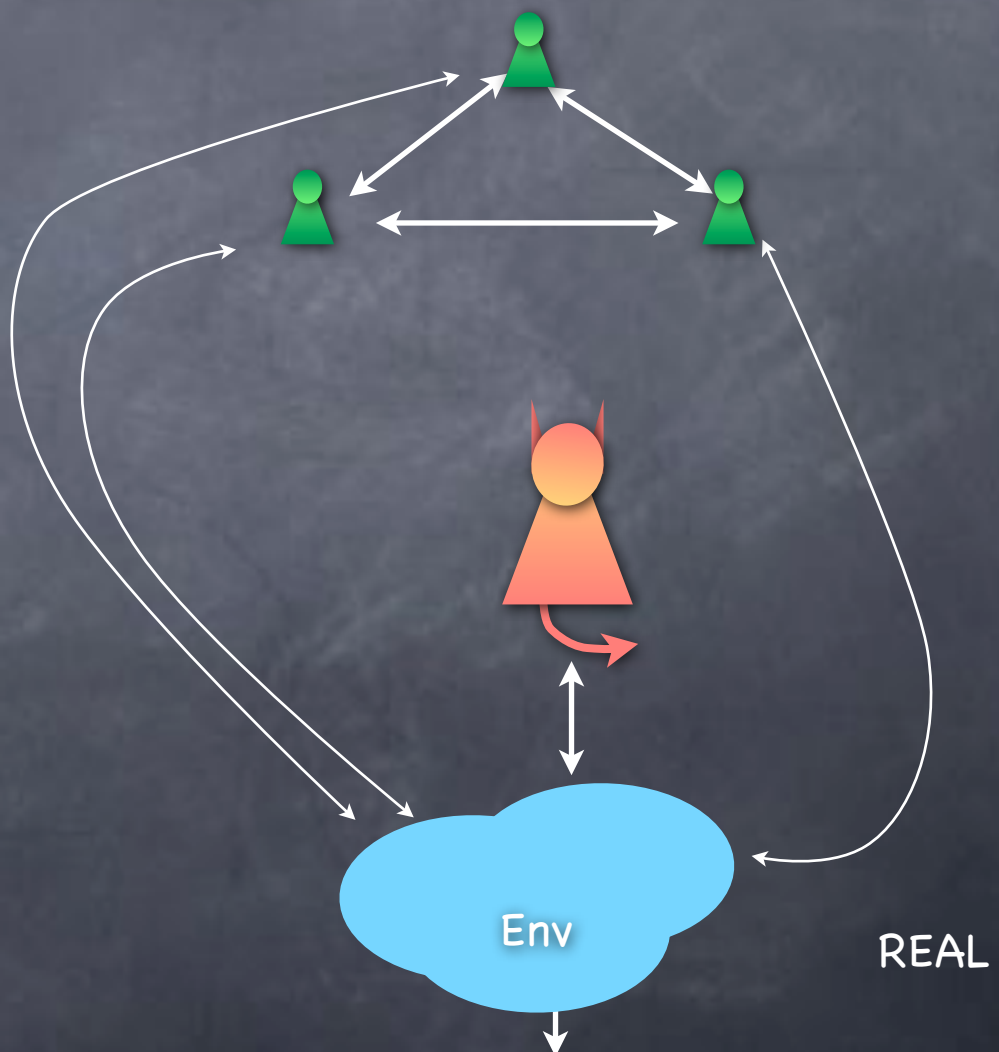


Security of Composed Systems

Extend to allow a "composed system" with multiple functionalities





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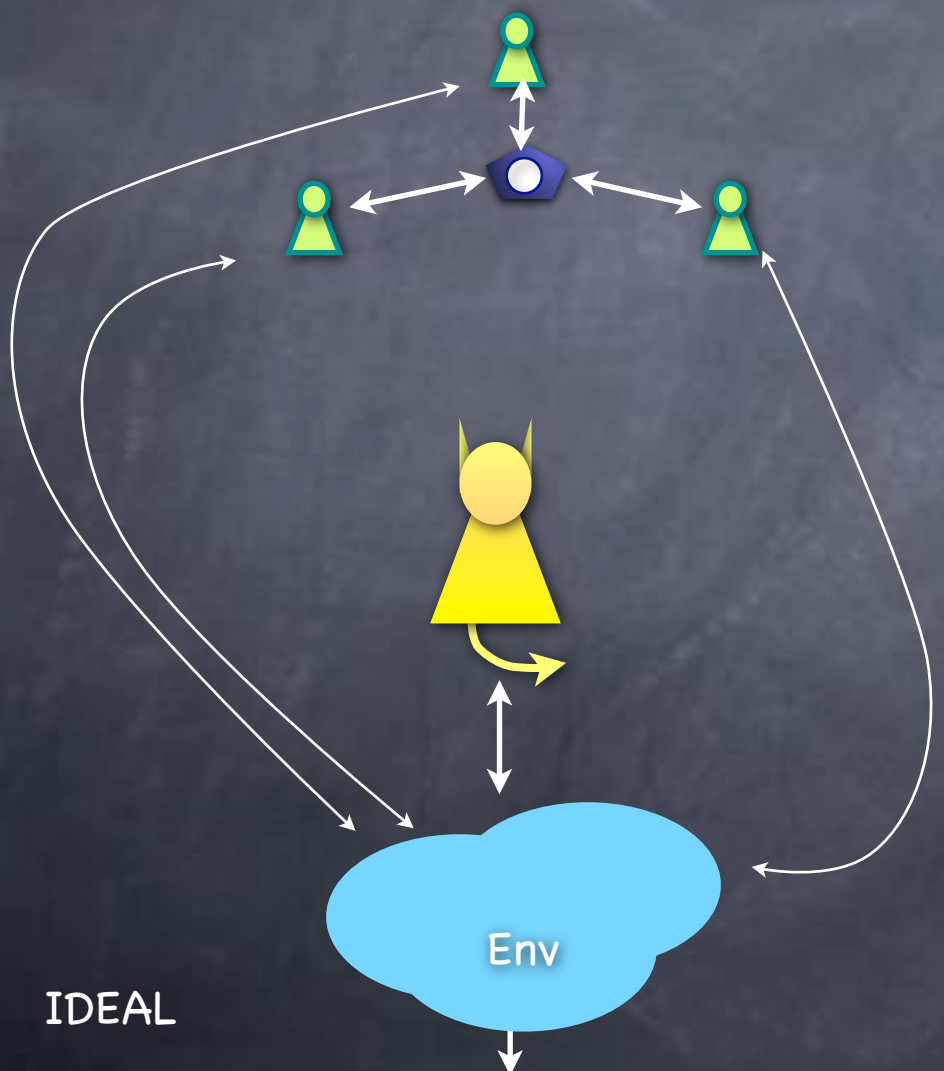

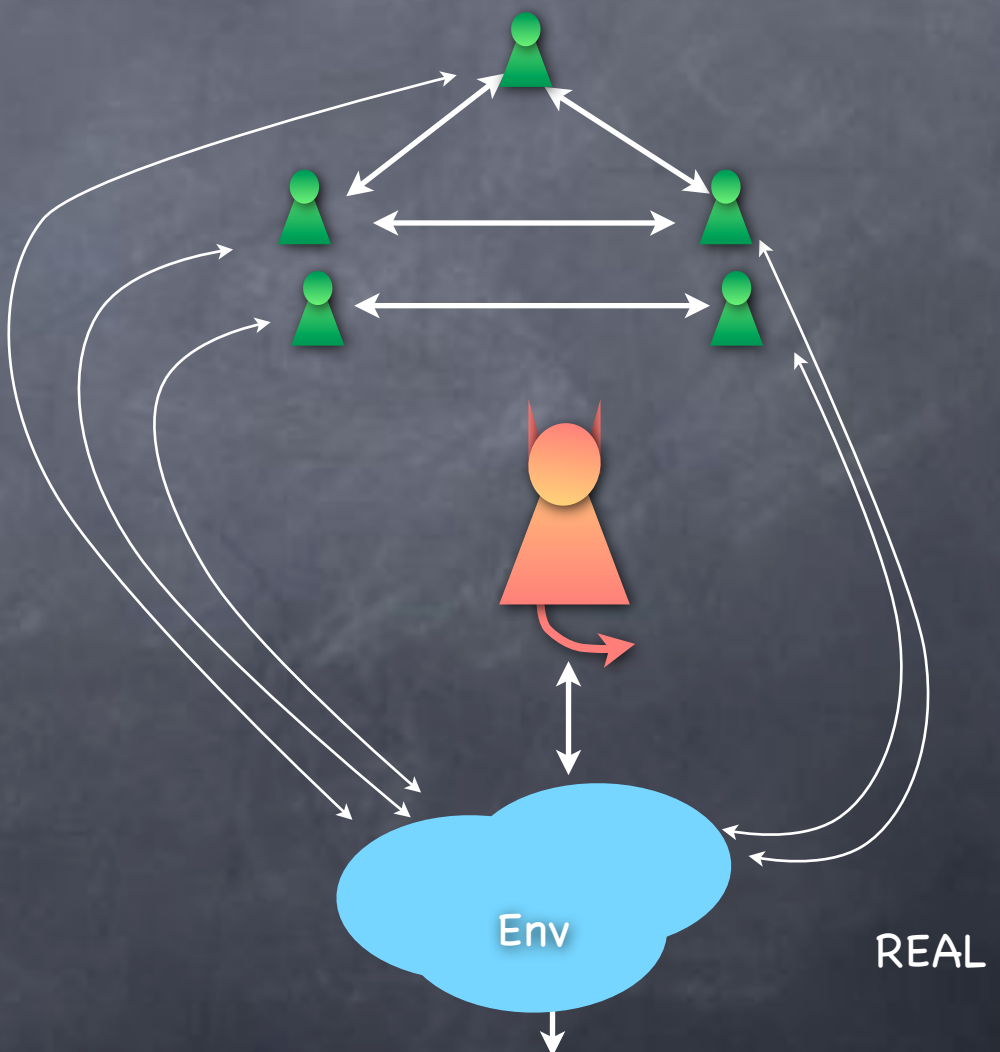

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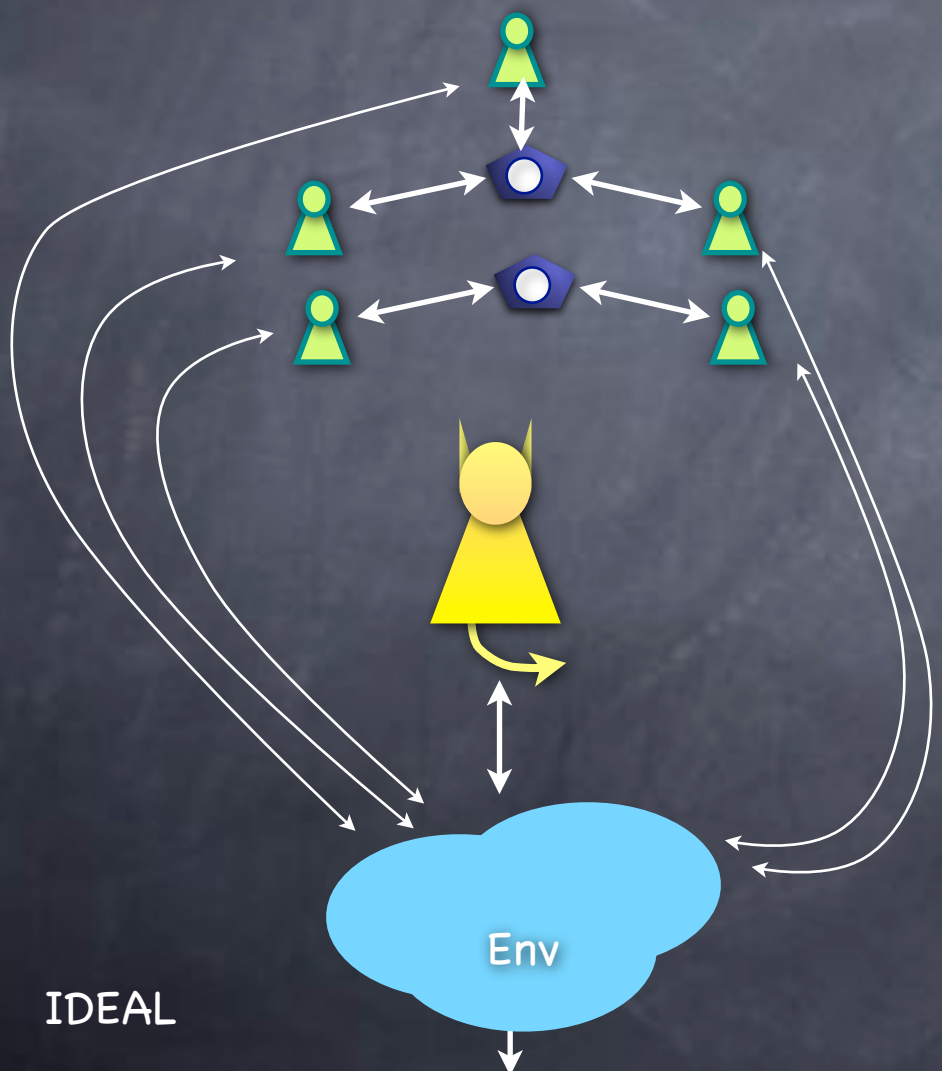

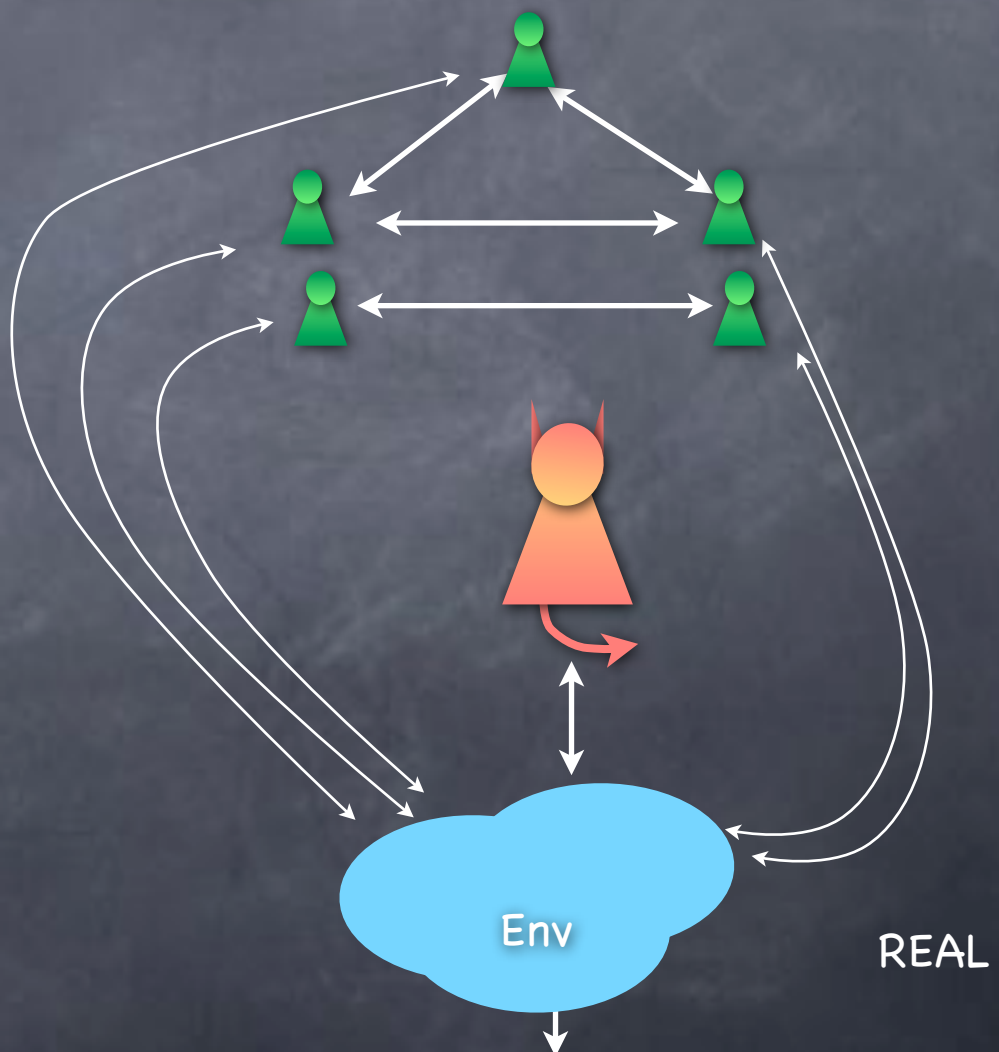

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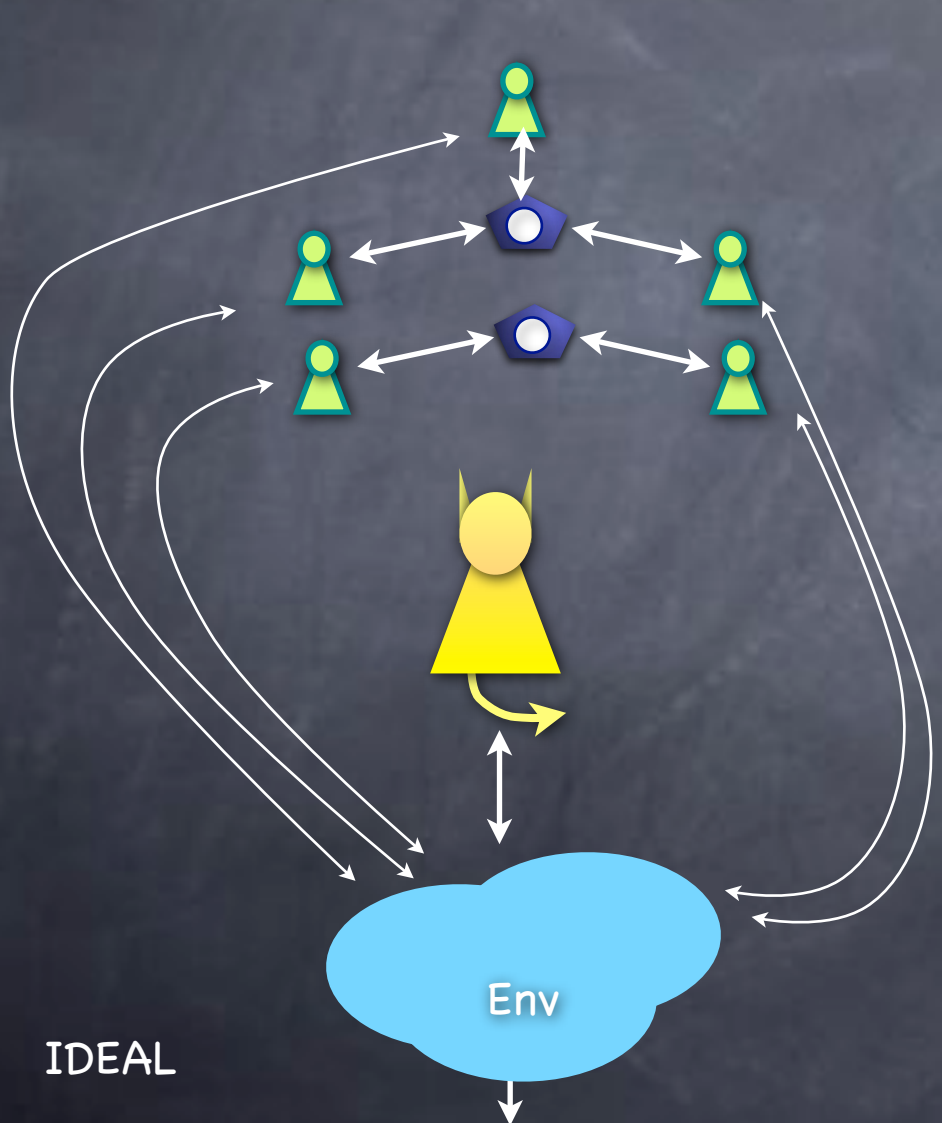

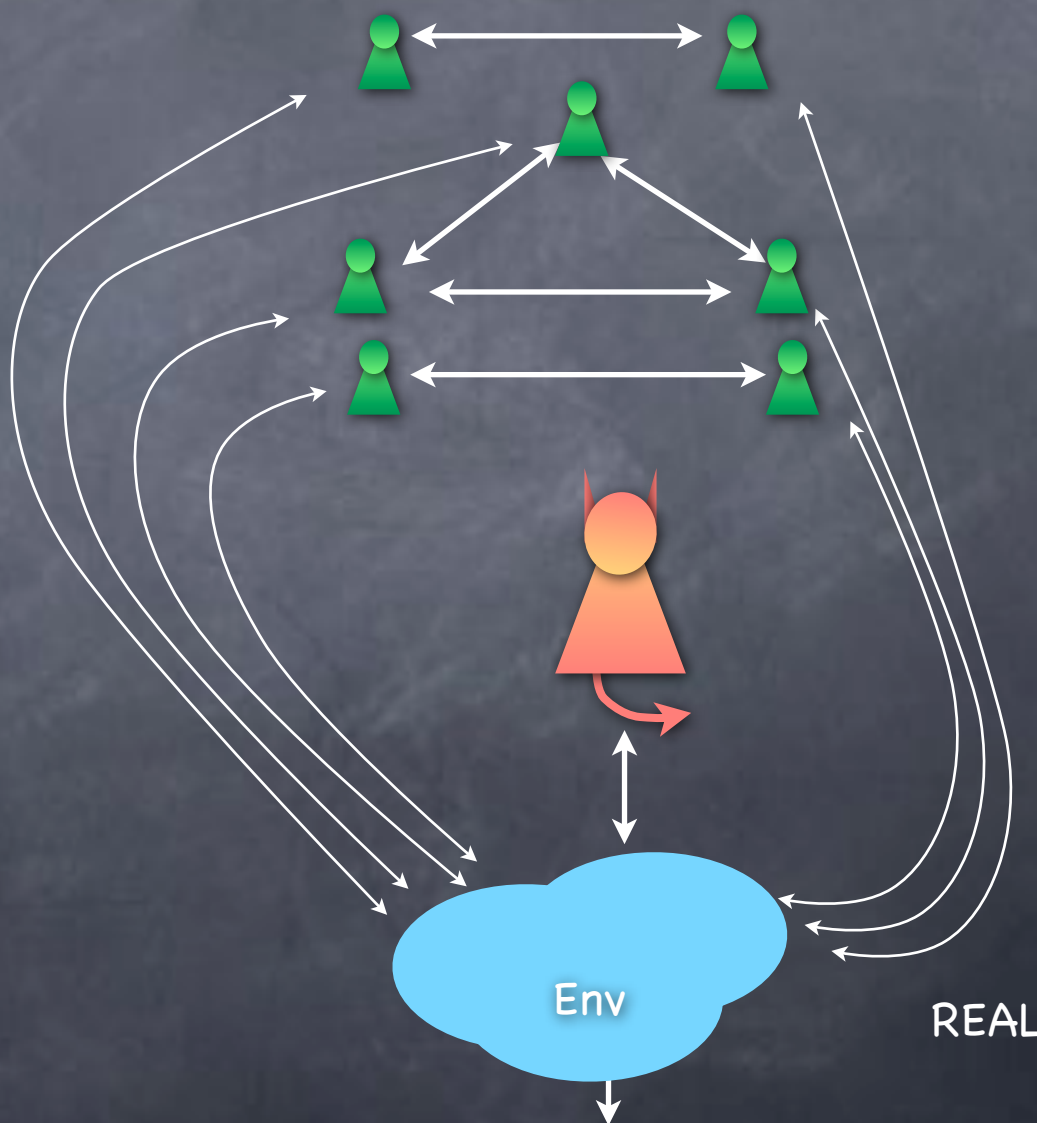

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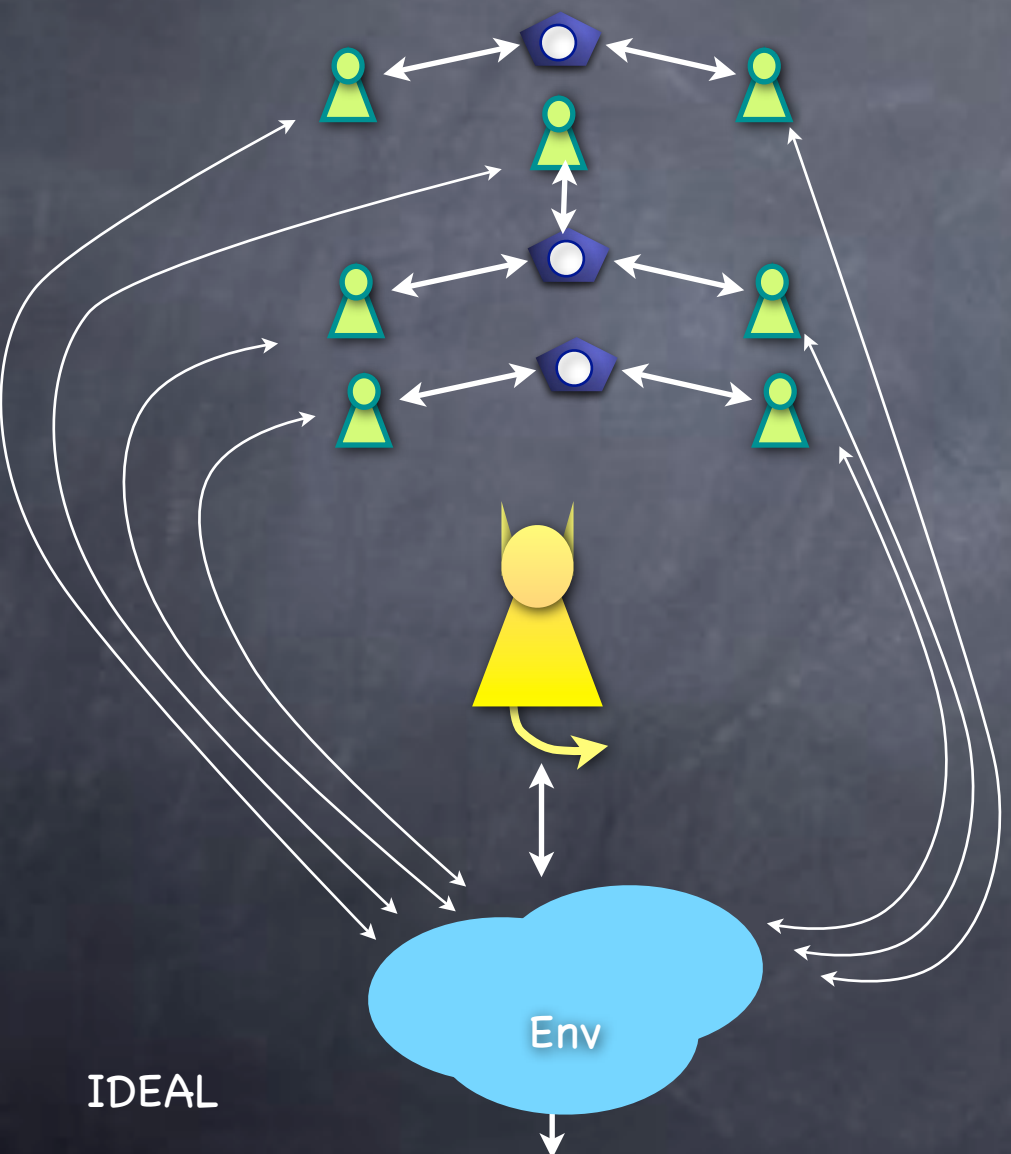

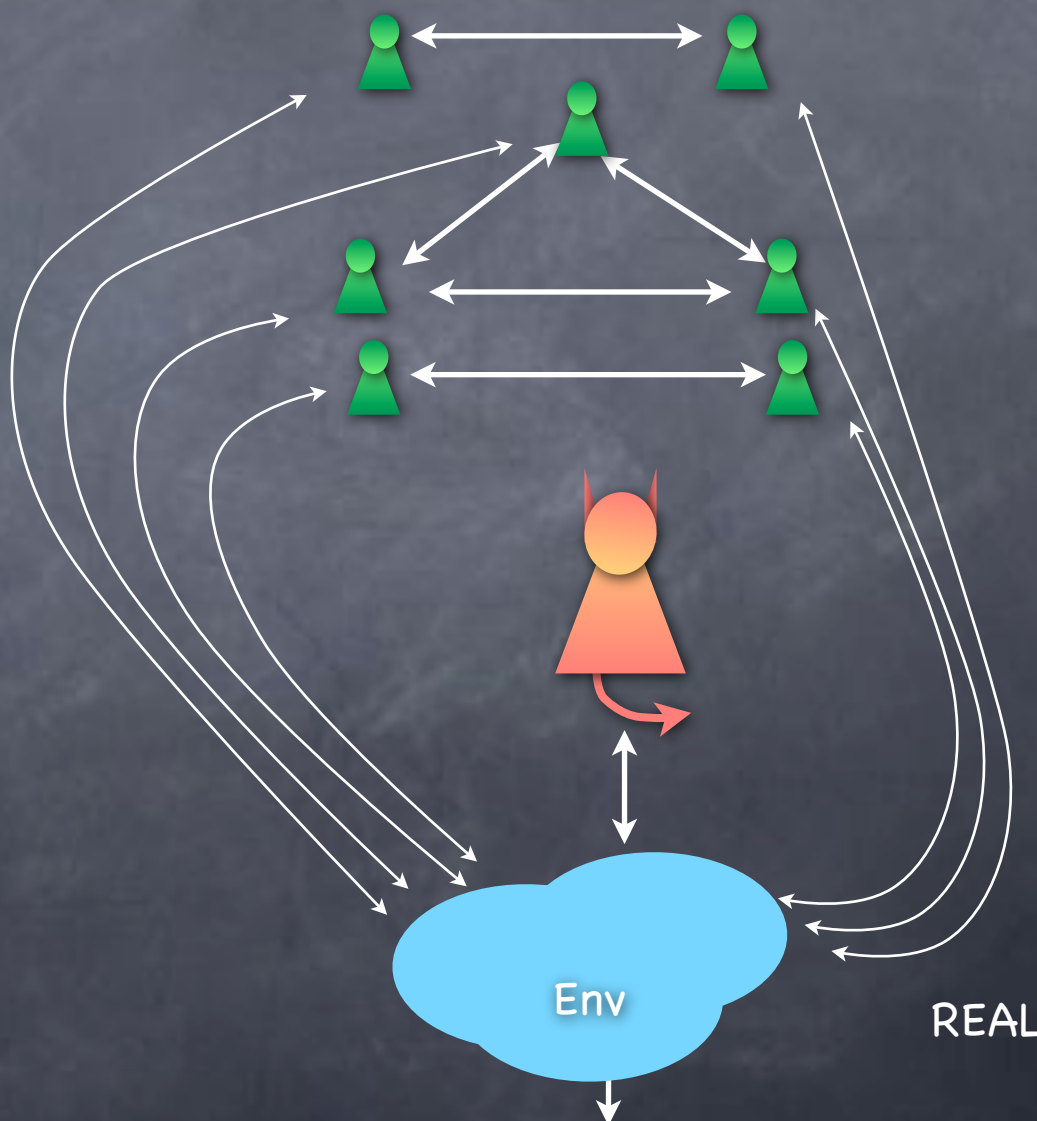

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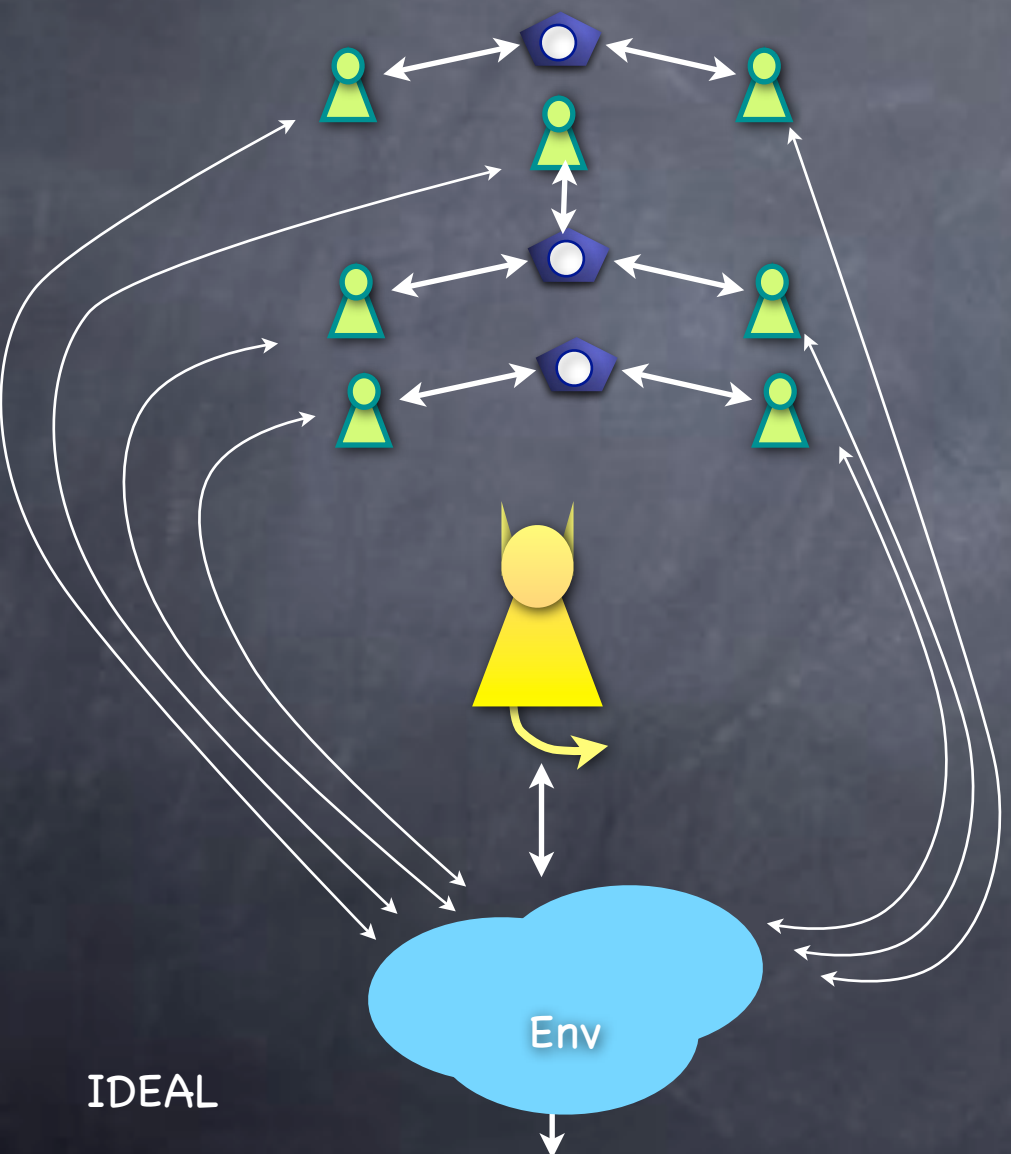

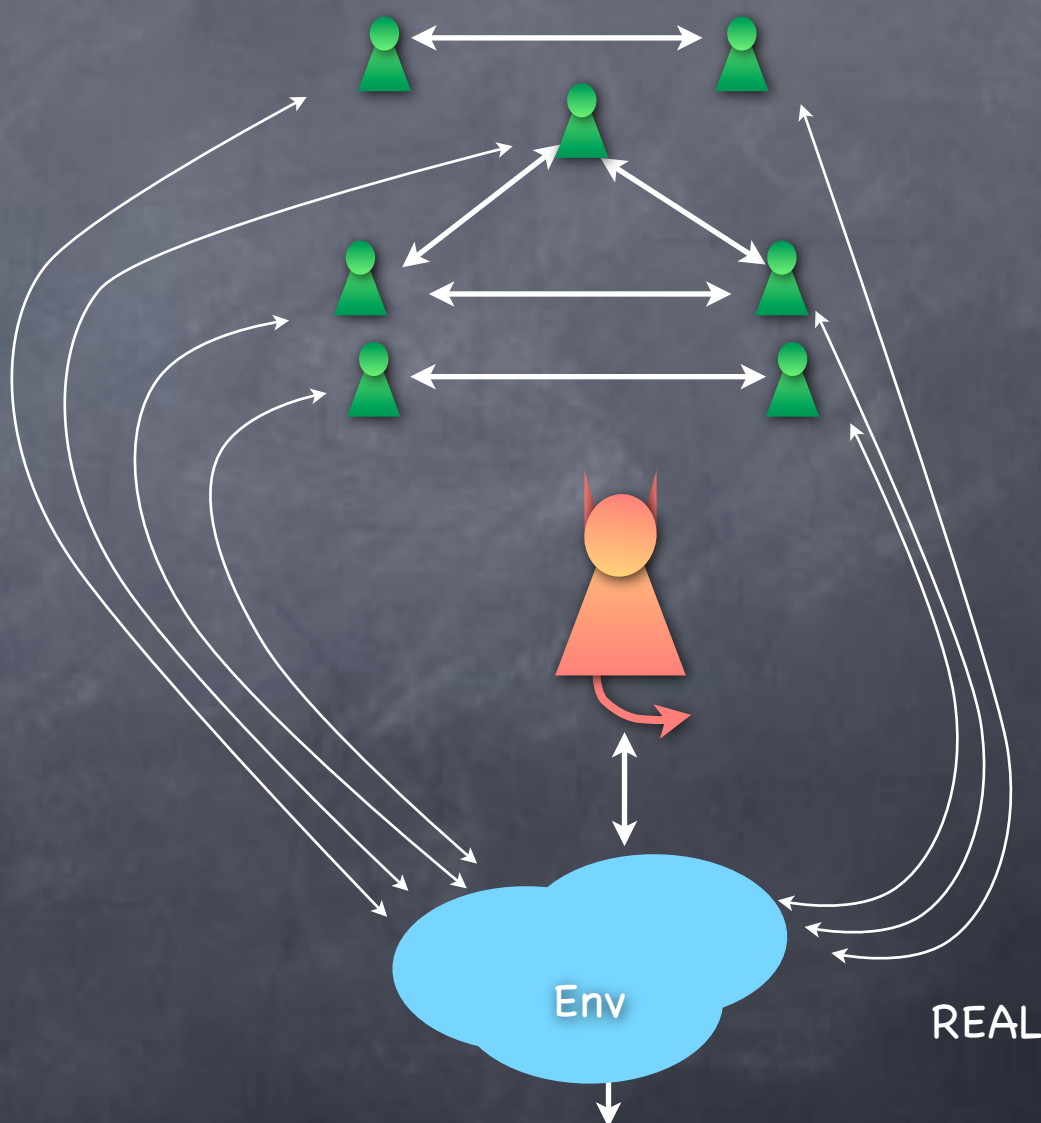

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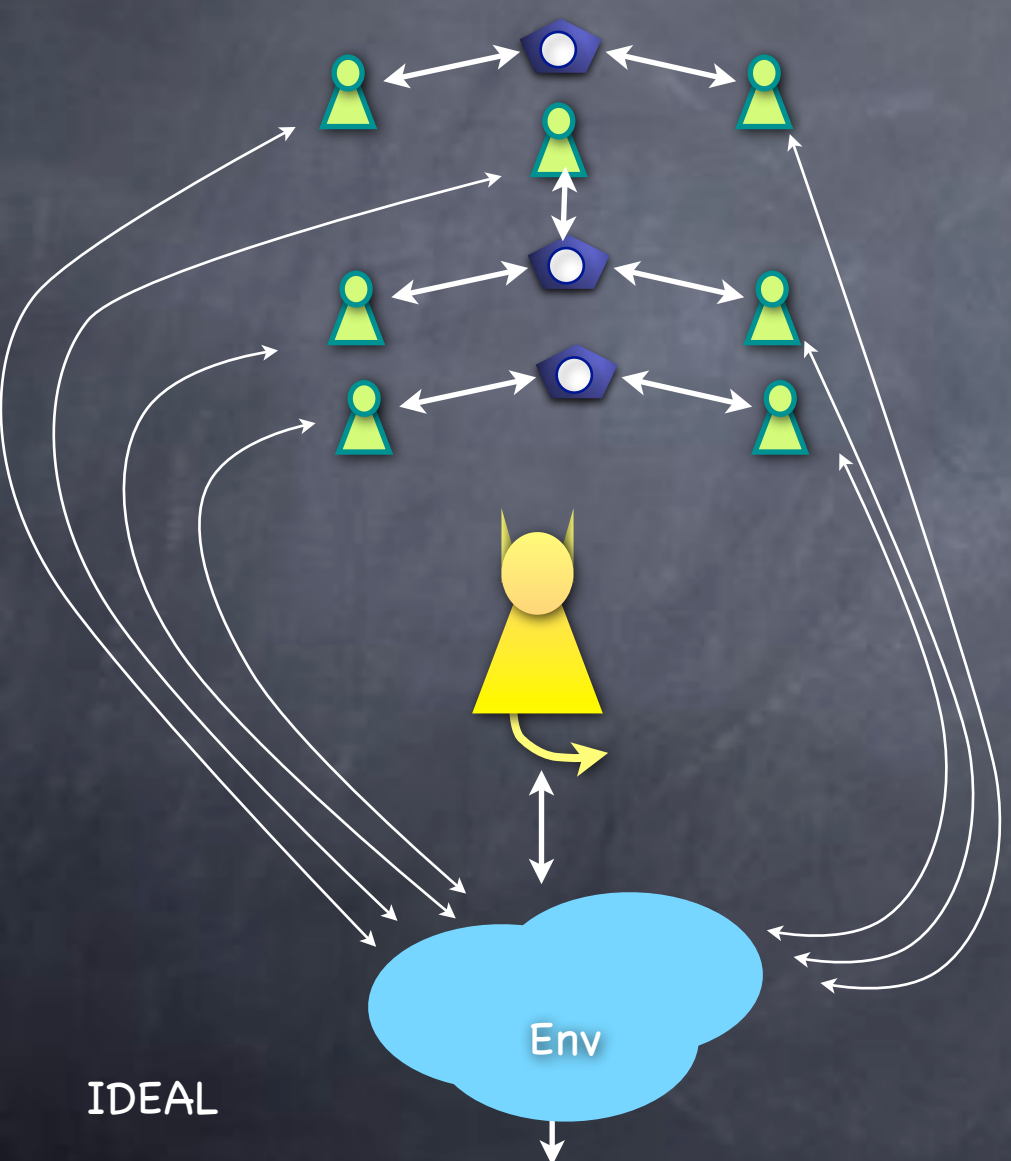

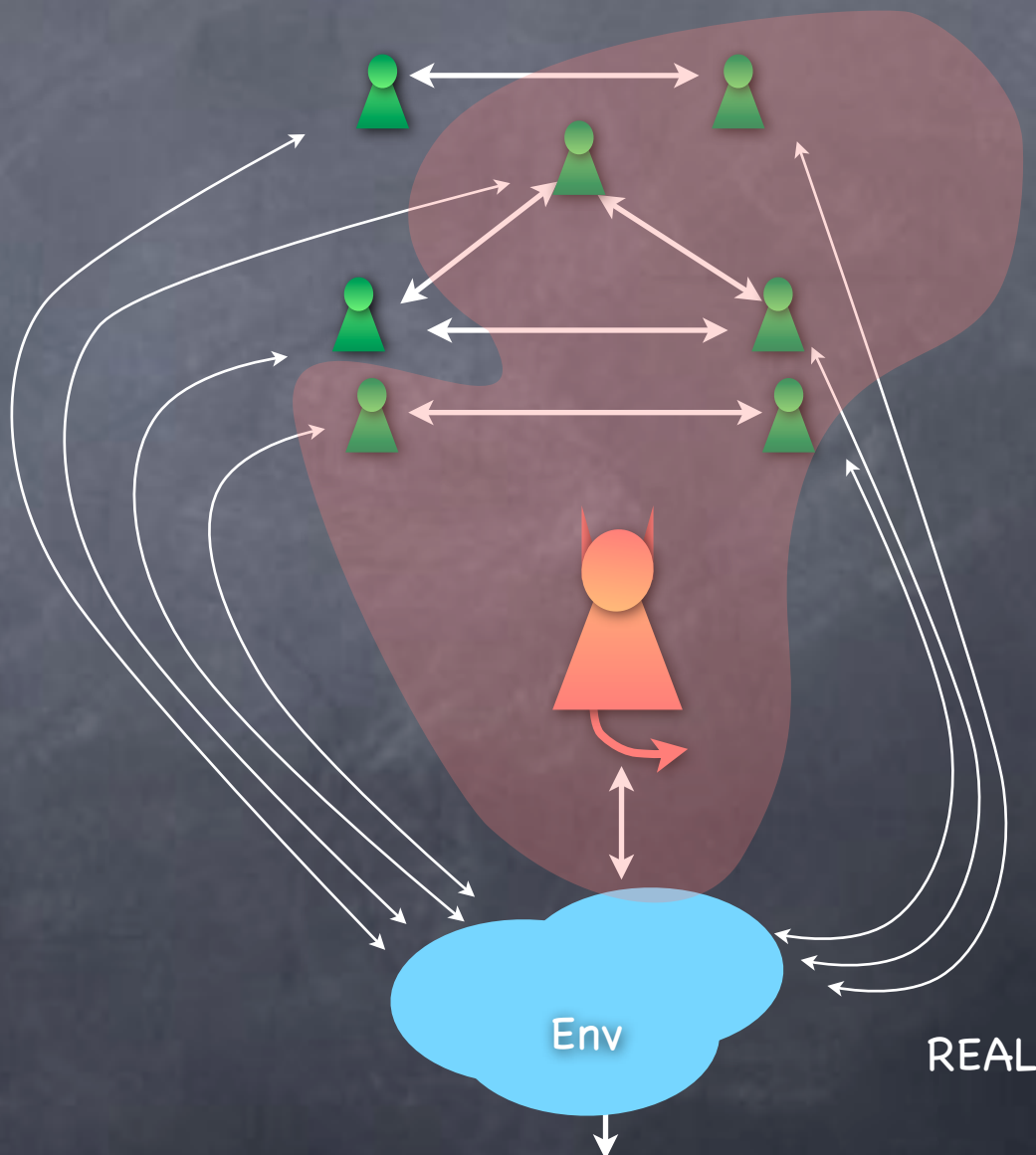

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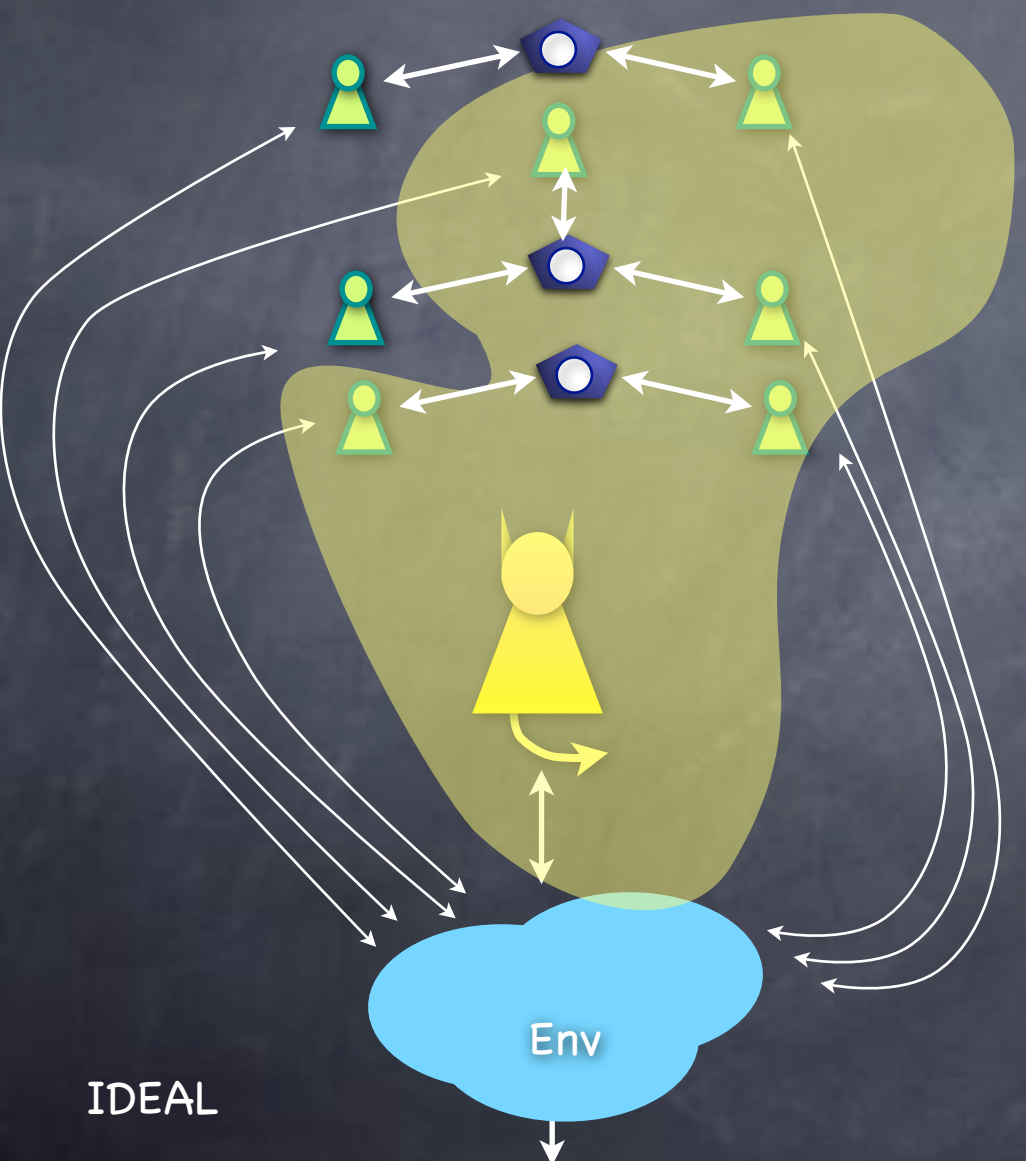

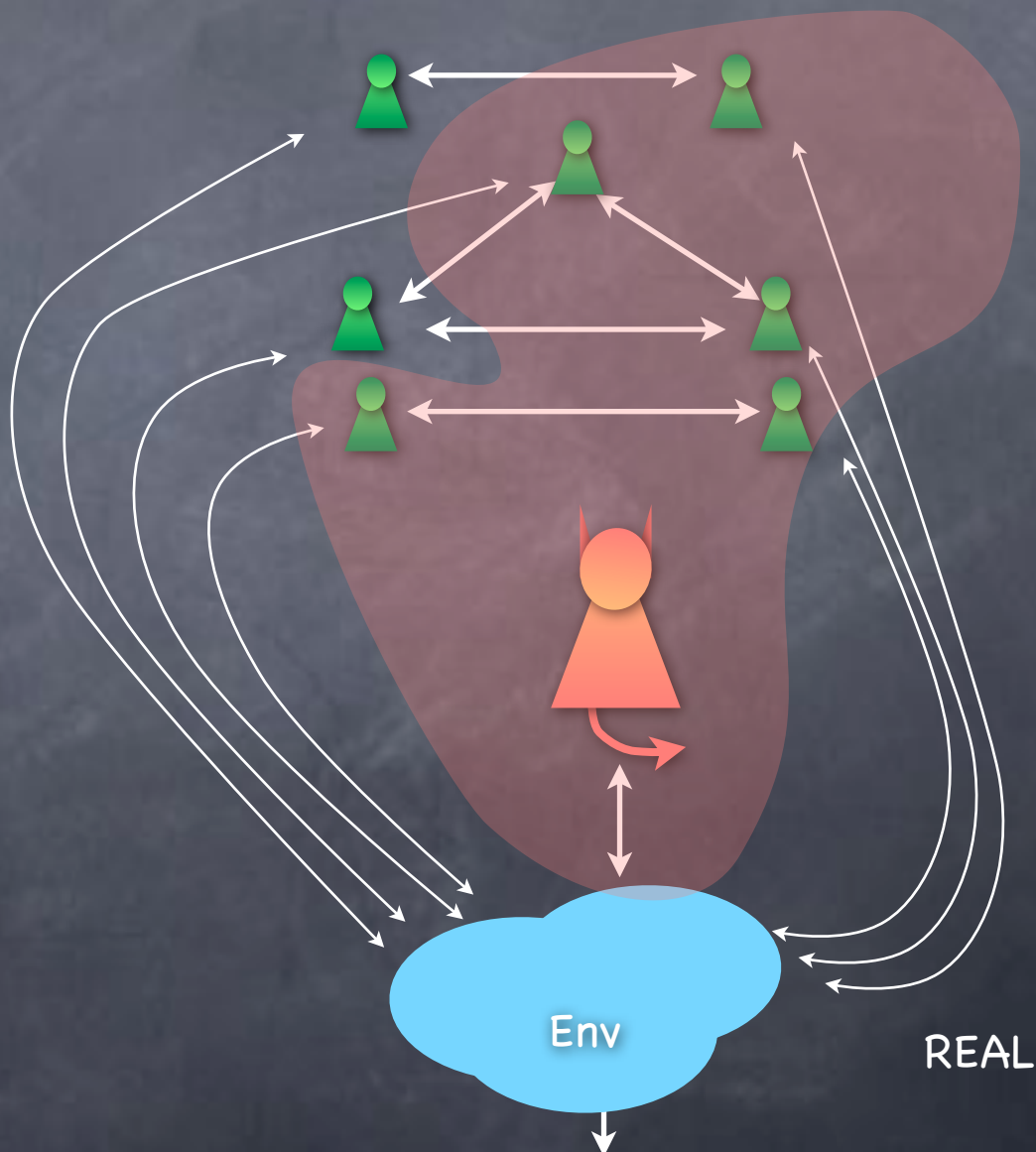

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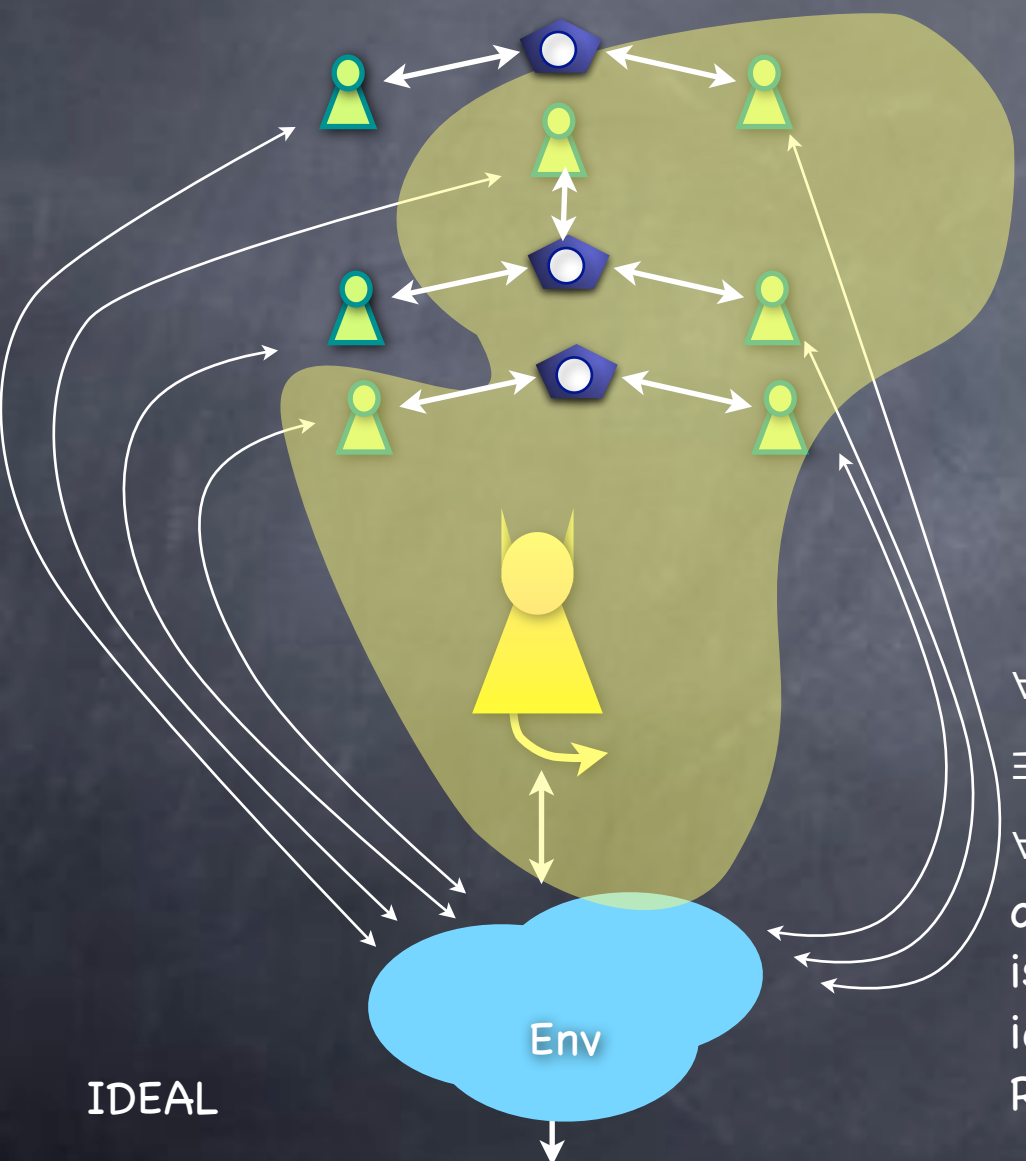

output of ☁ is distributed identically in REAL and IDEAL

Universal Composition - 1

If each protocol secure (i.e., 
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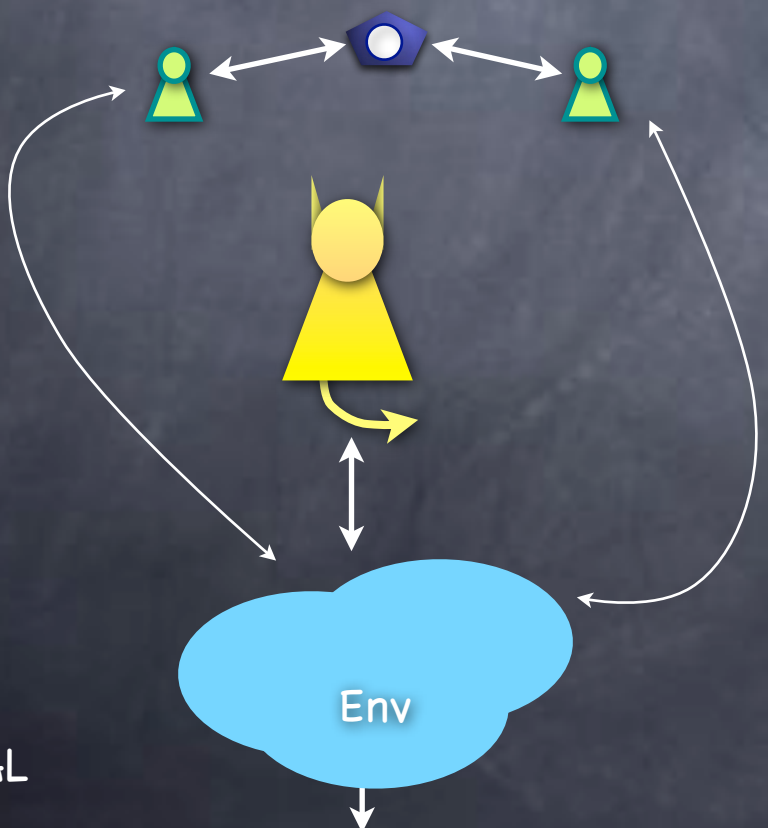

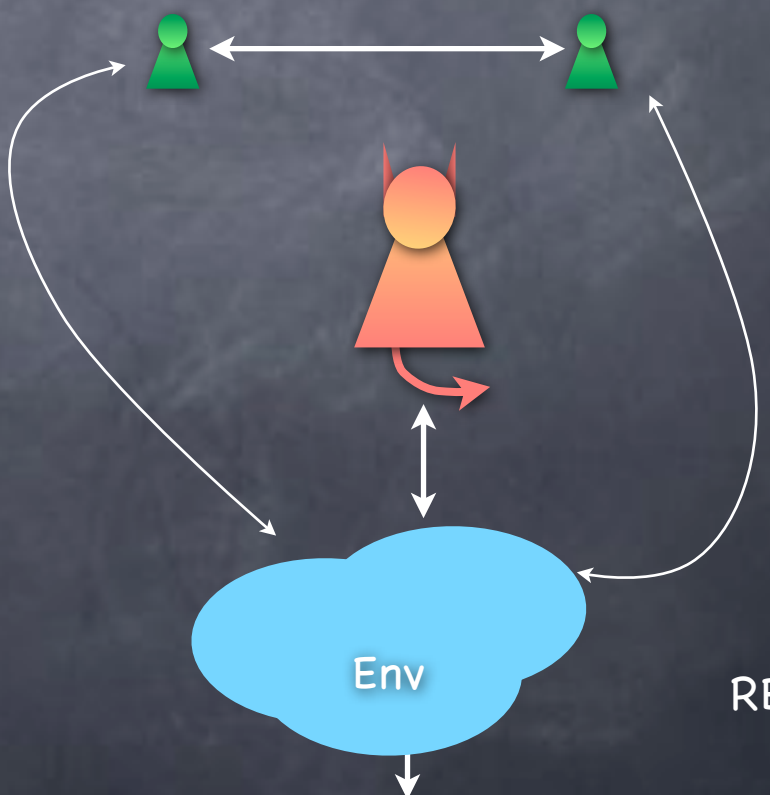

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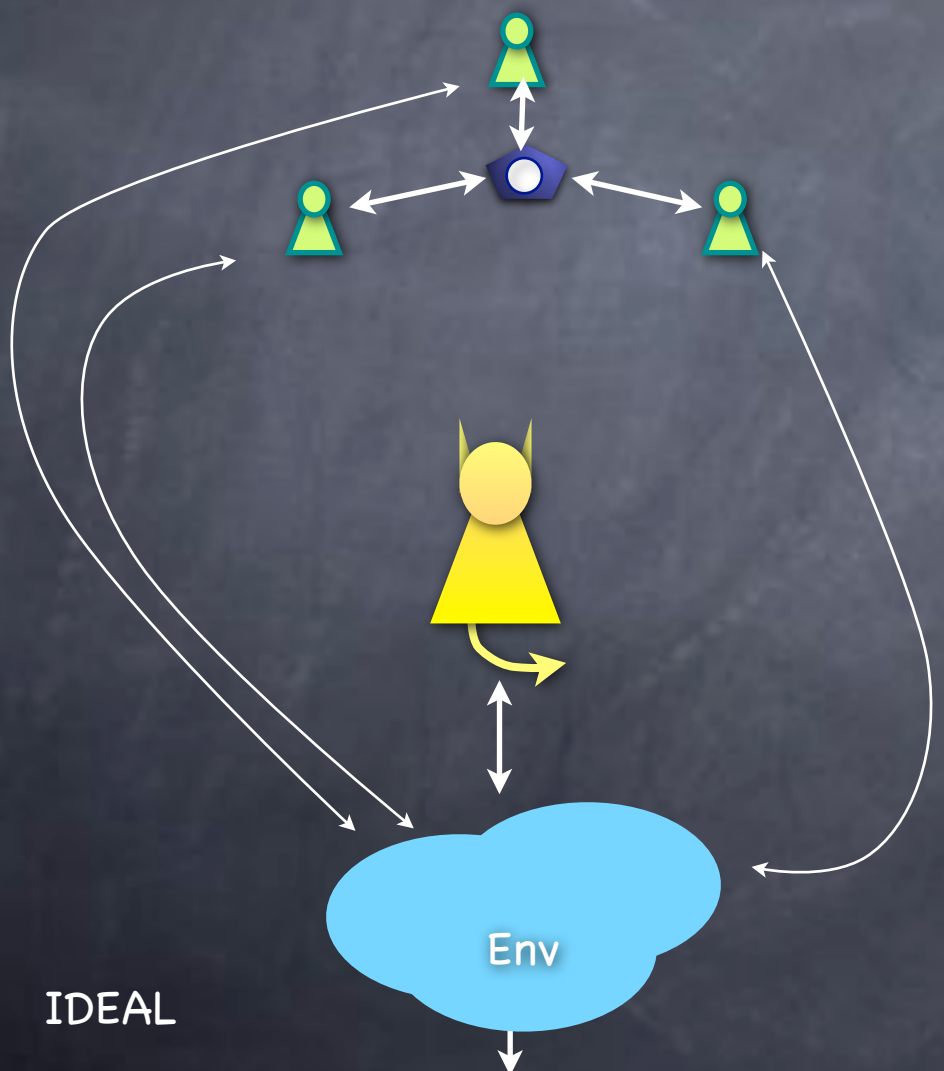

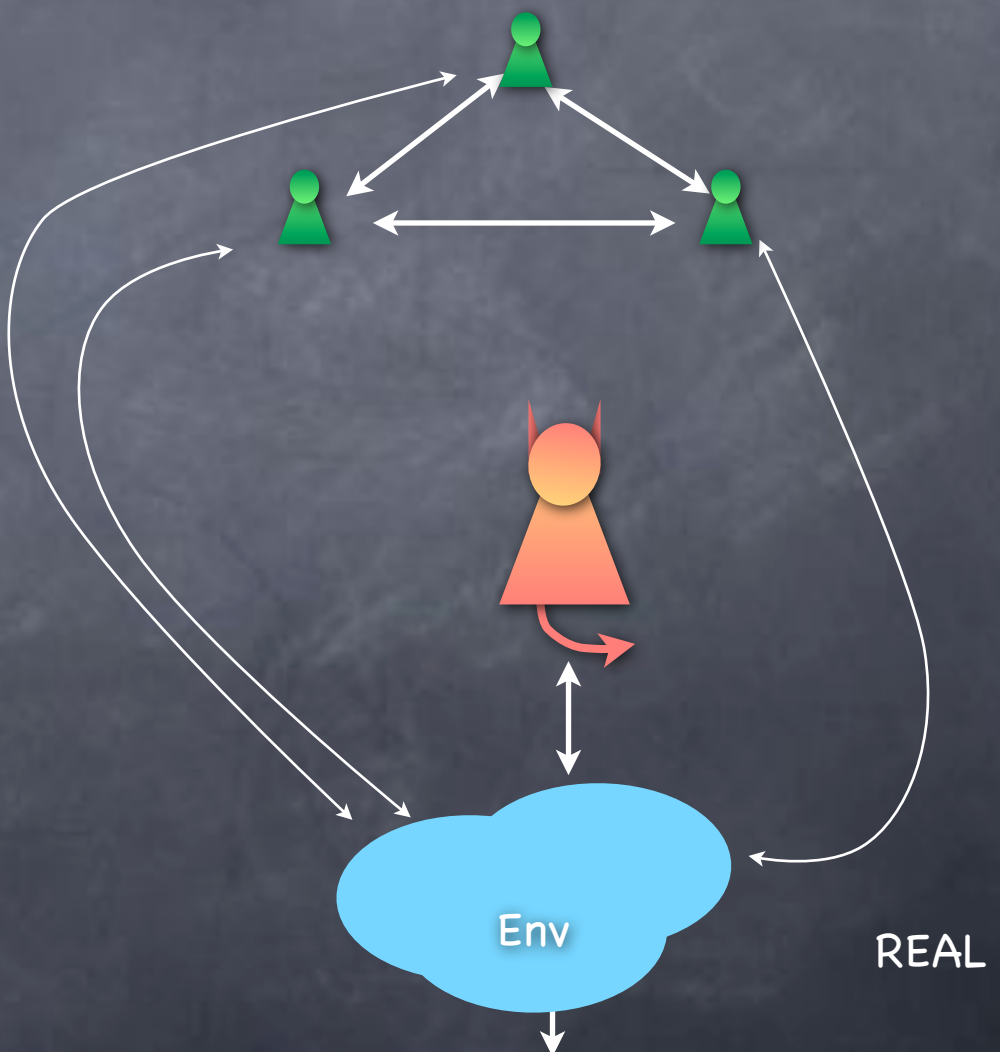

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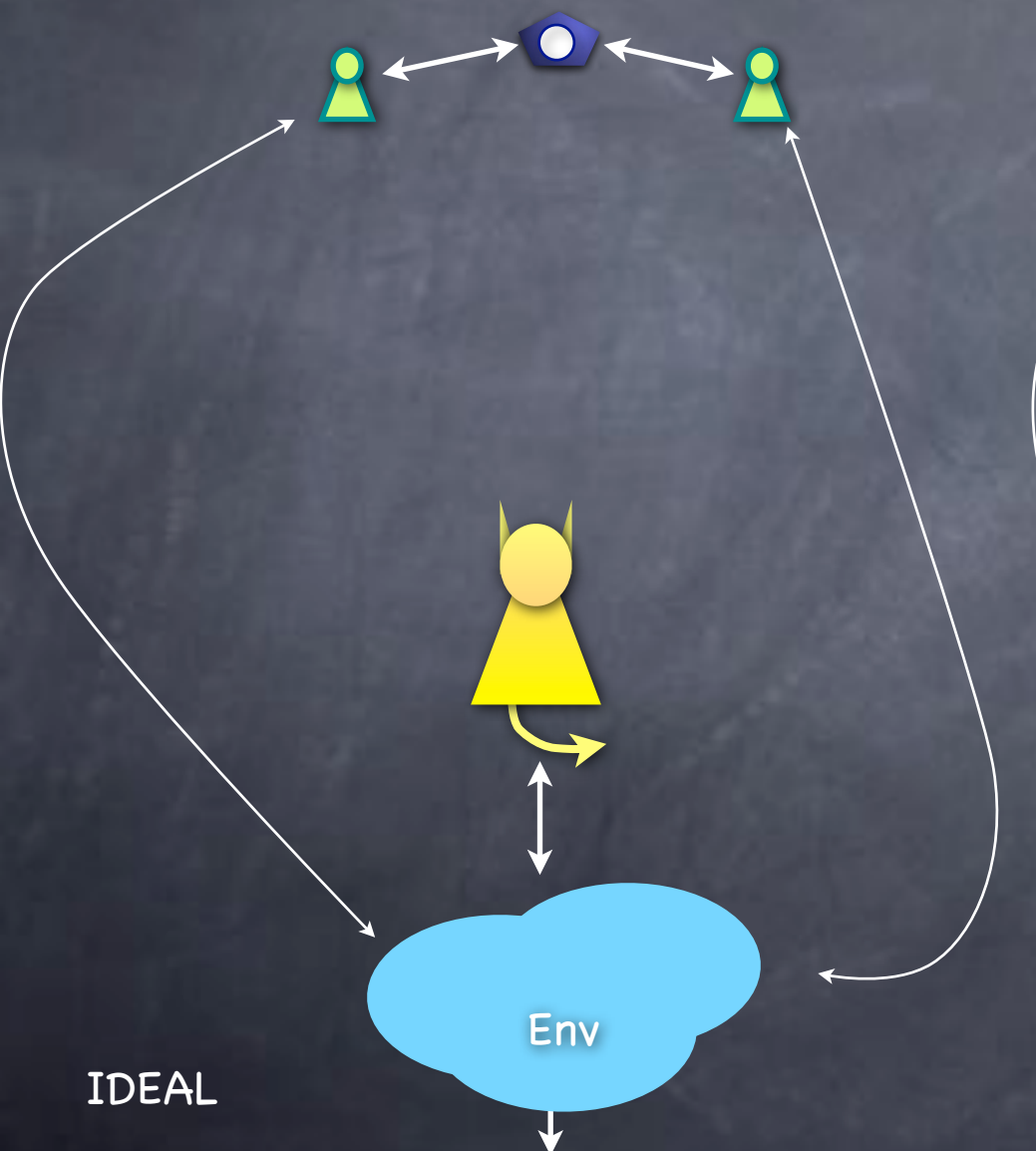

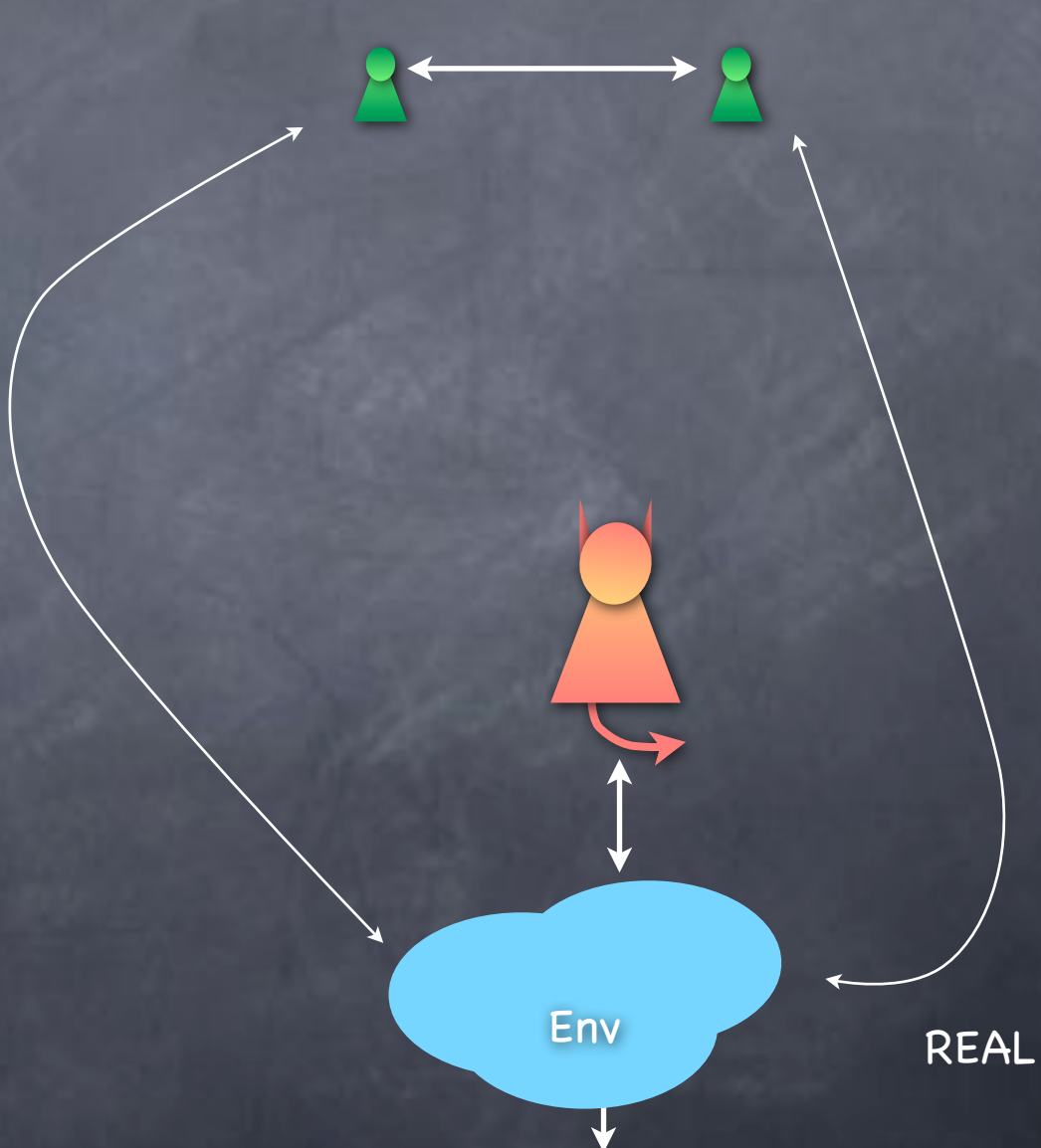

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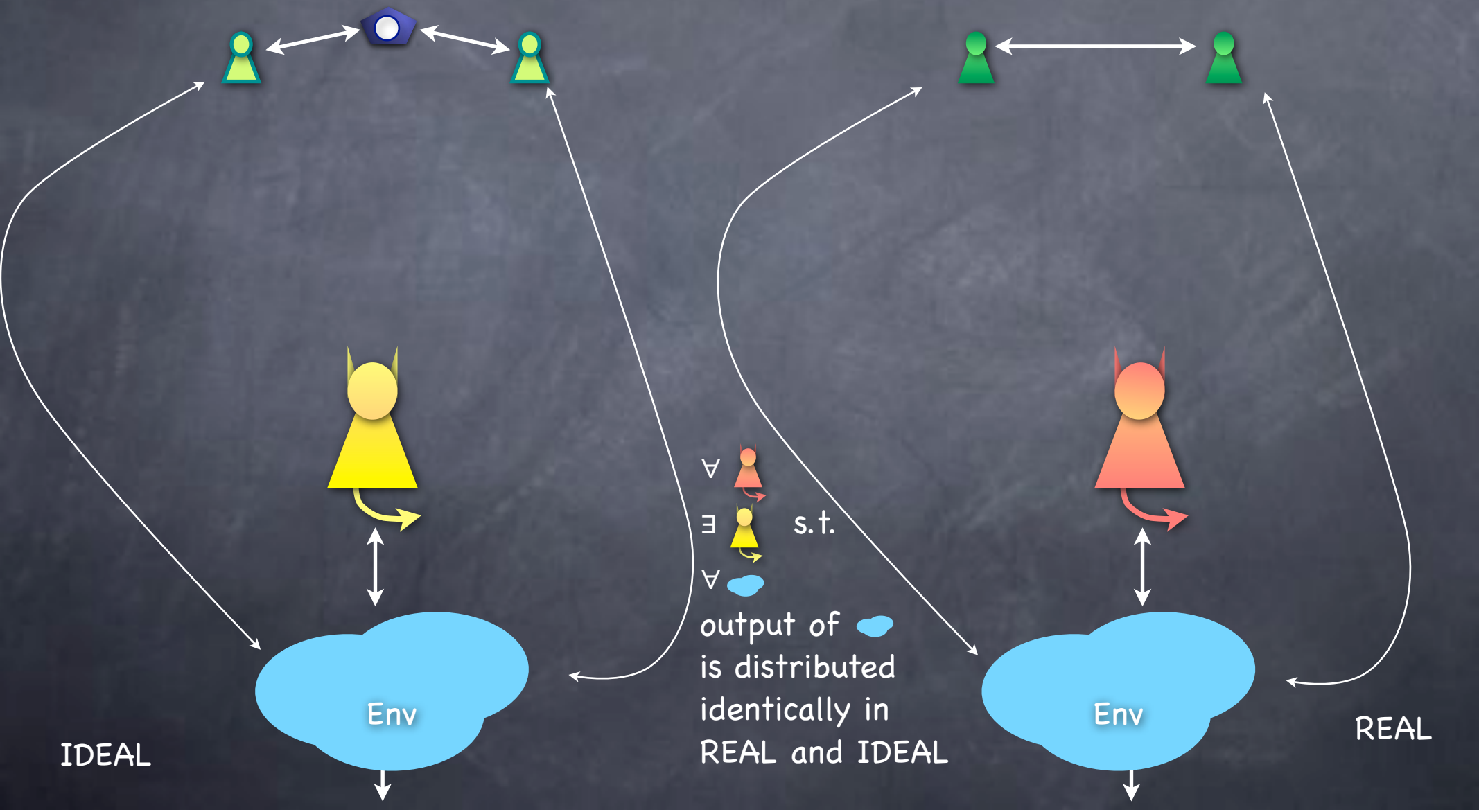

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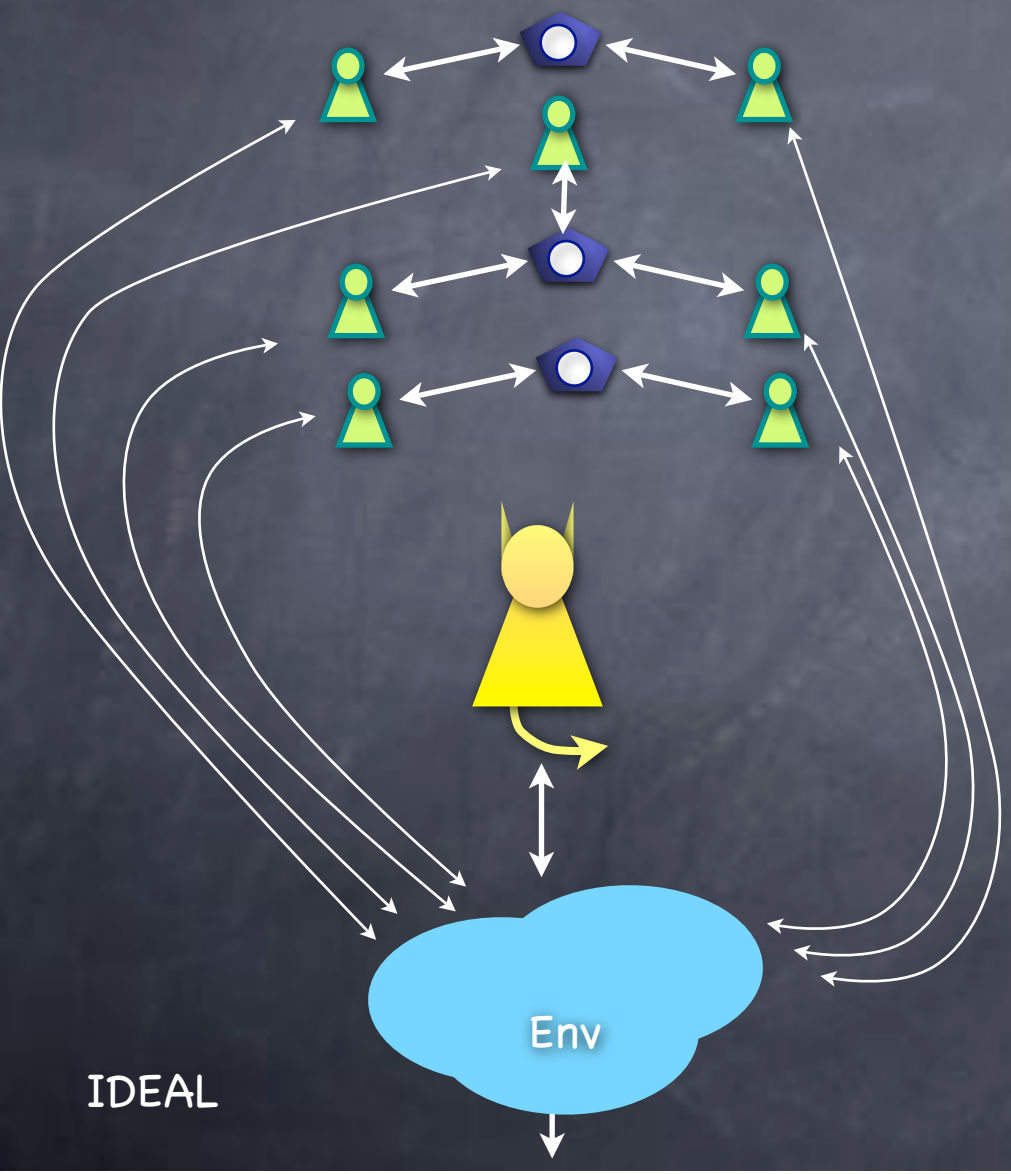

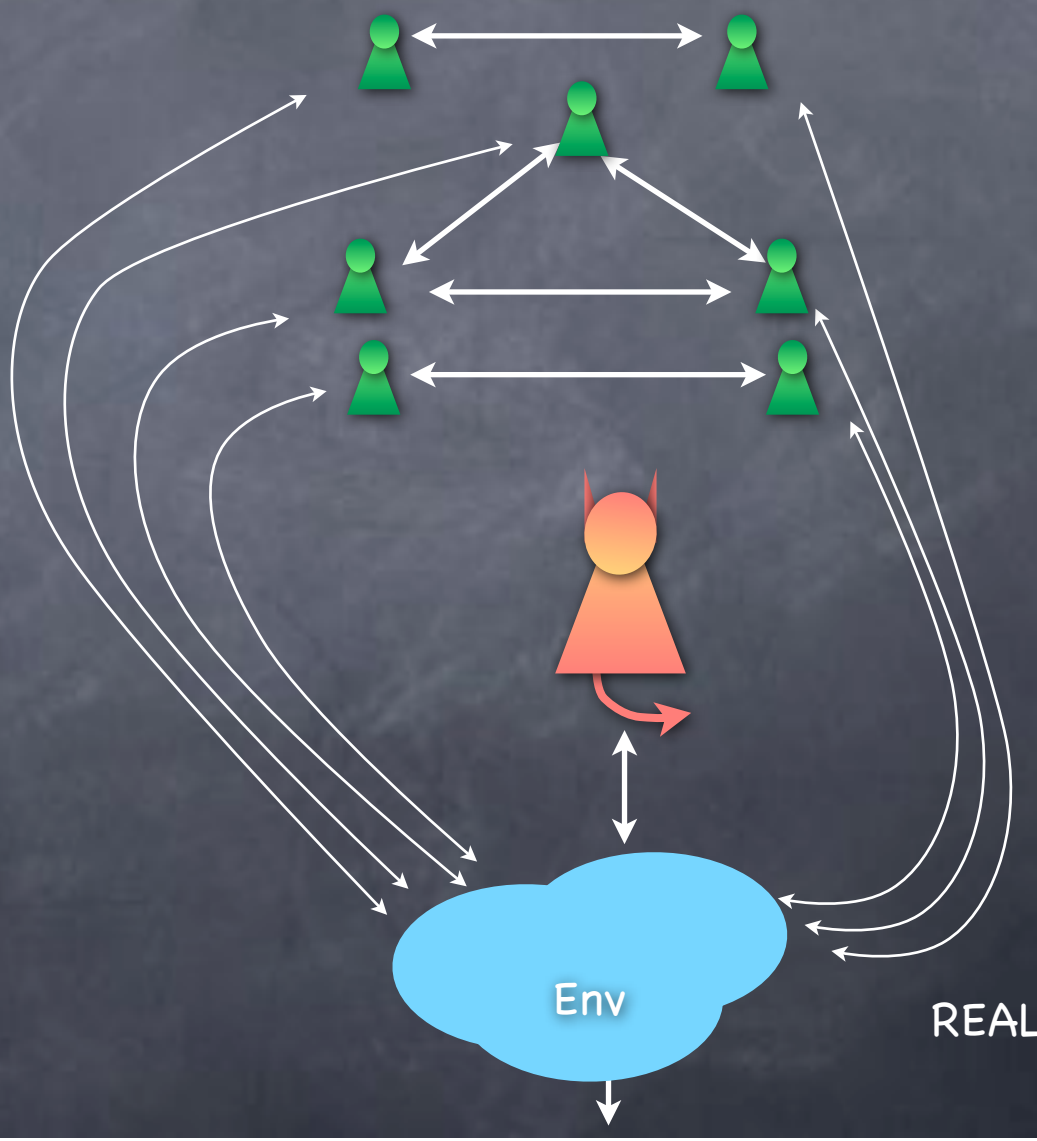

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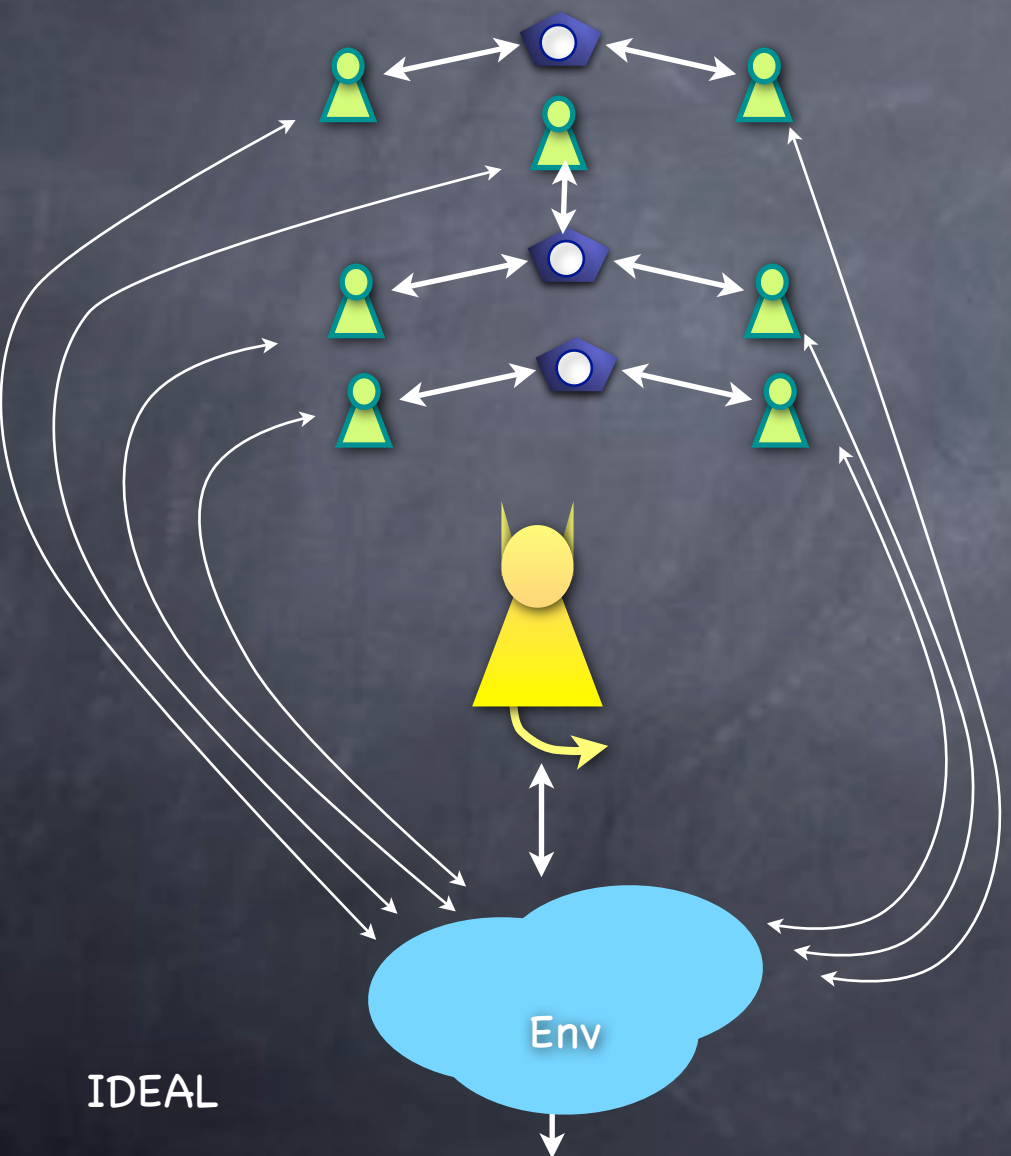
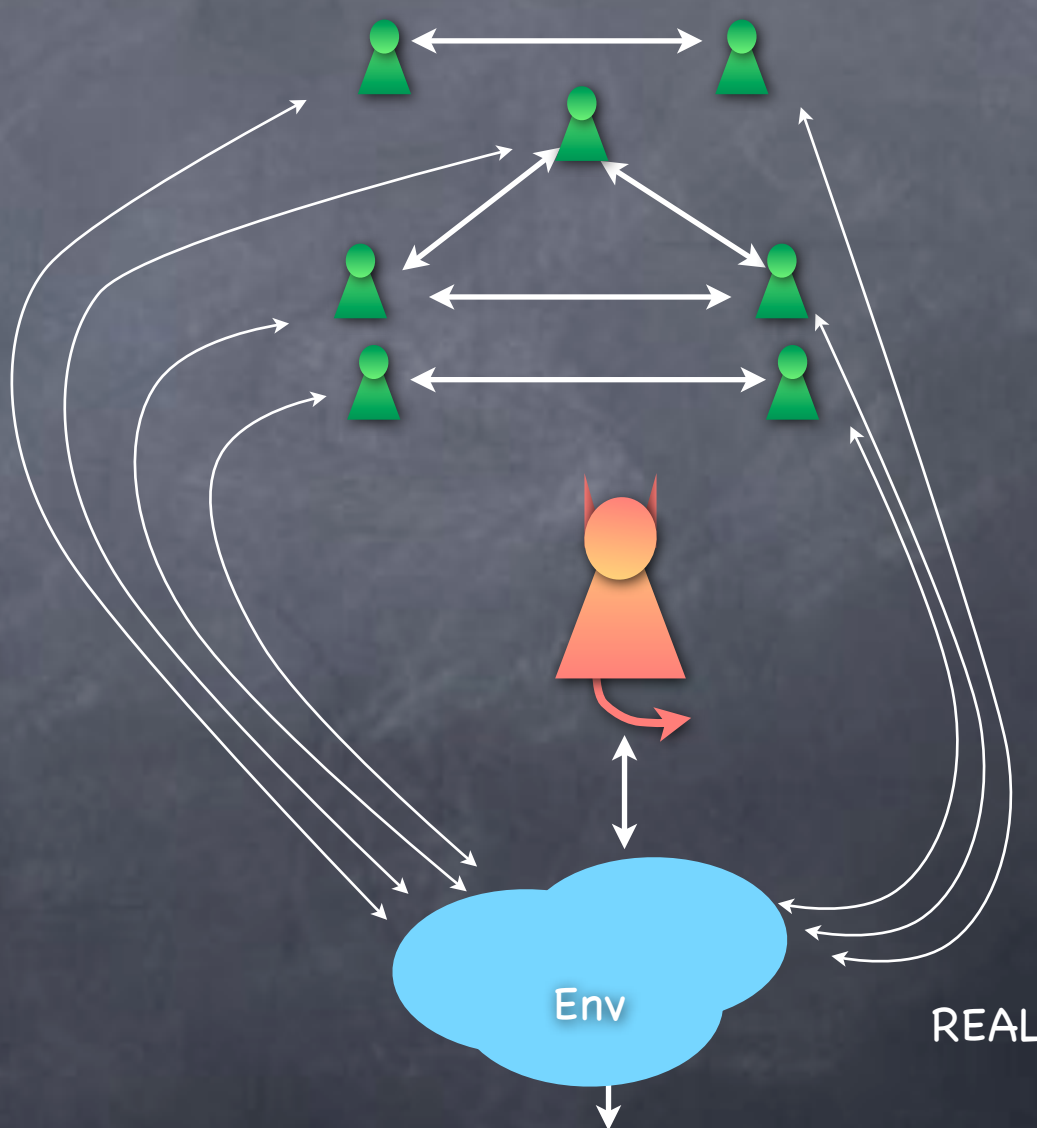
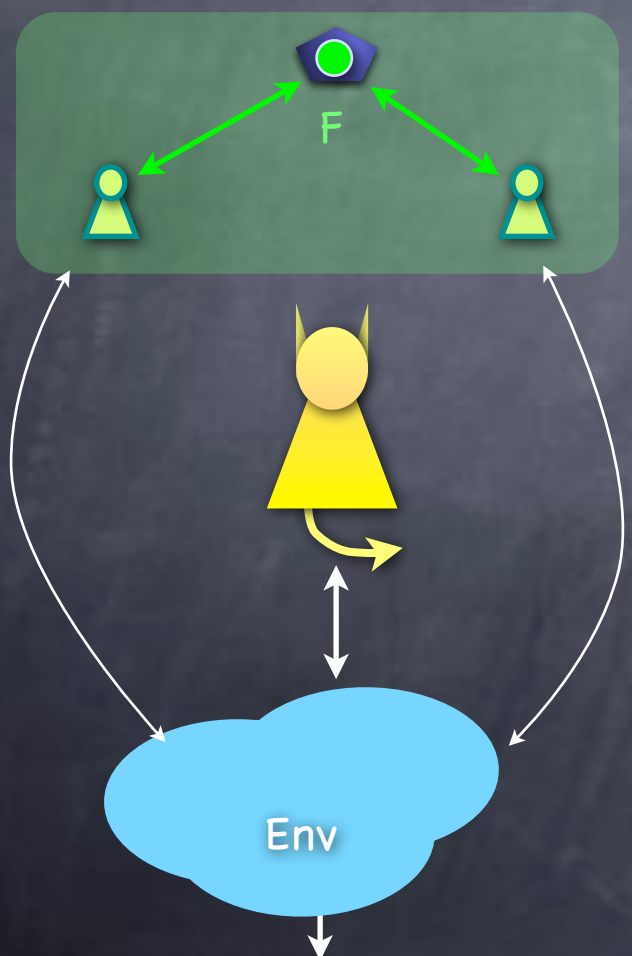




Universal Composition - 2



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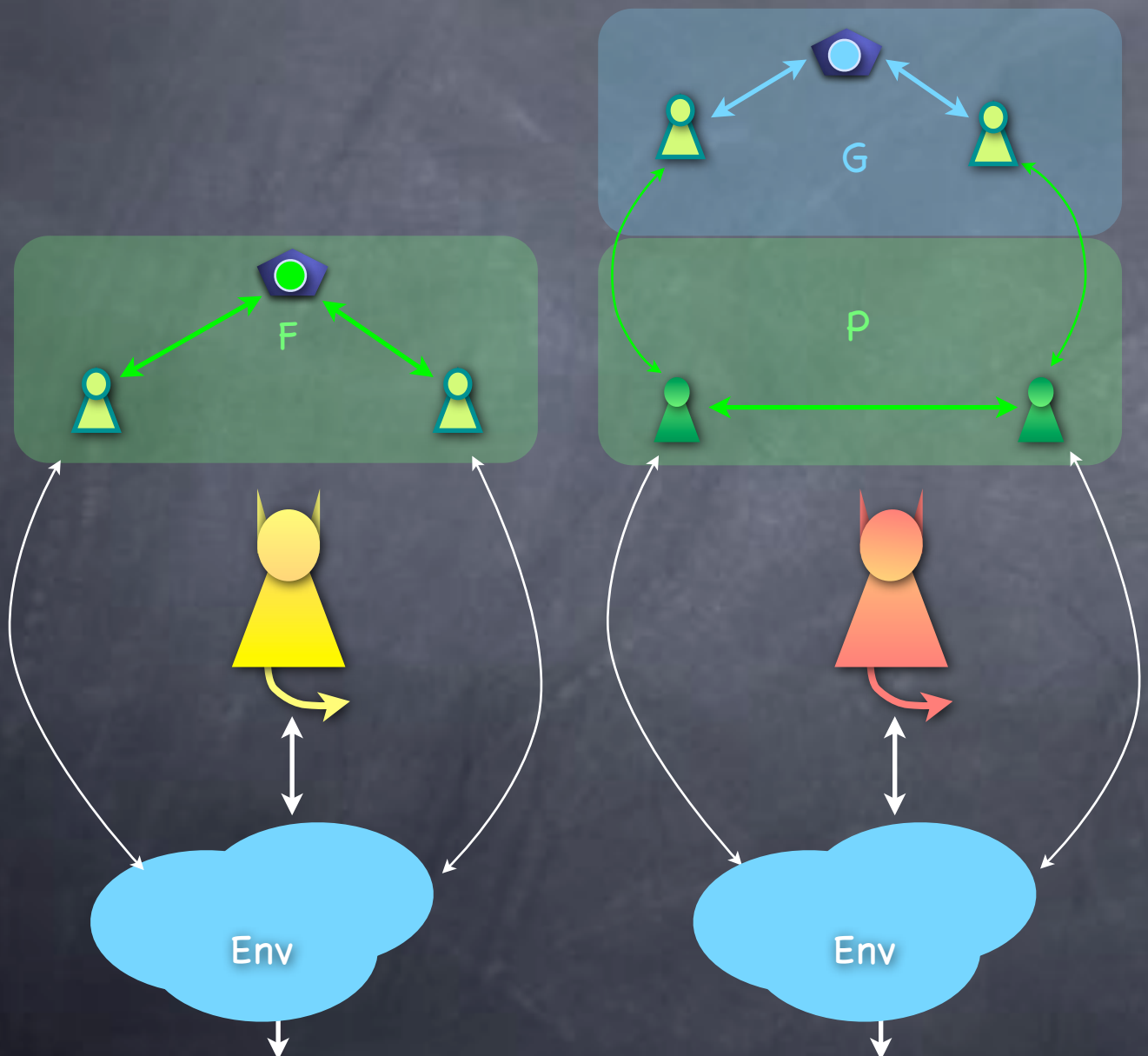

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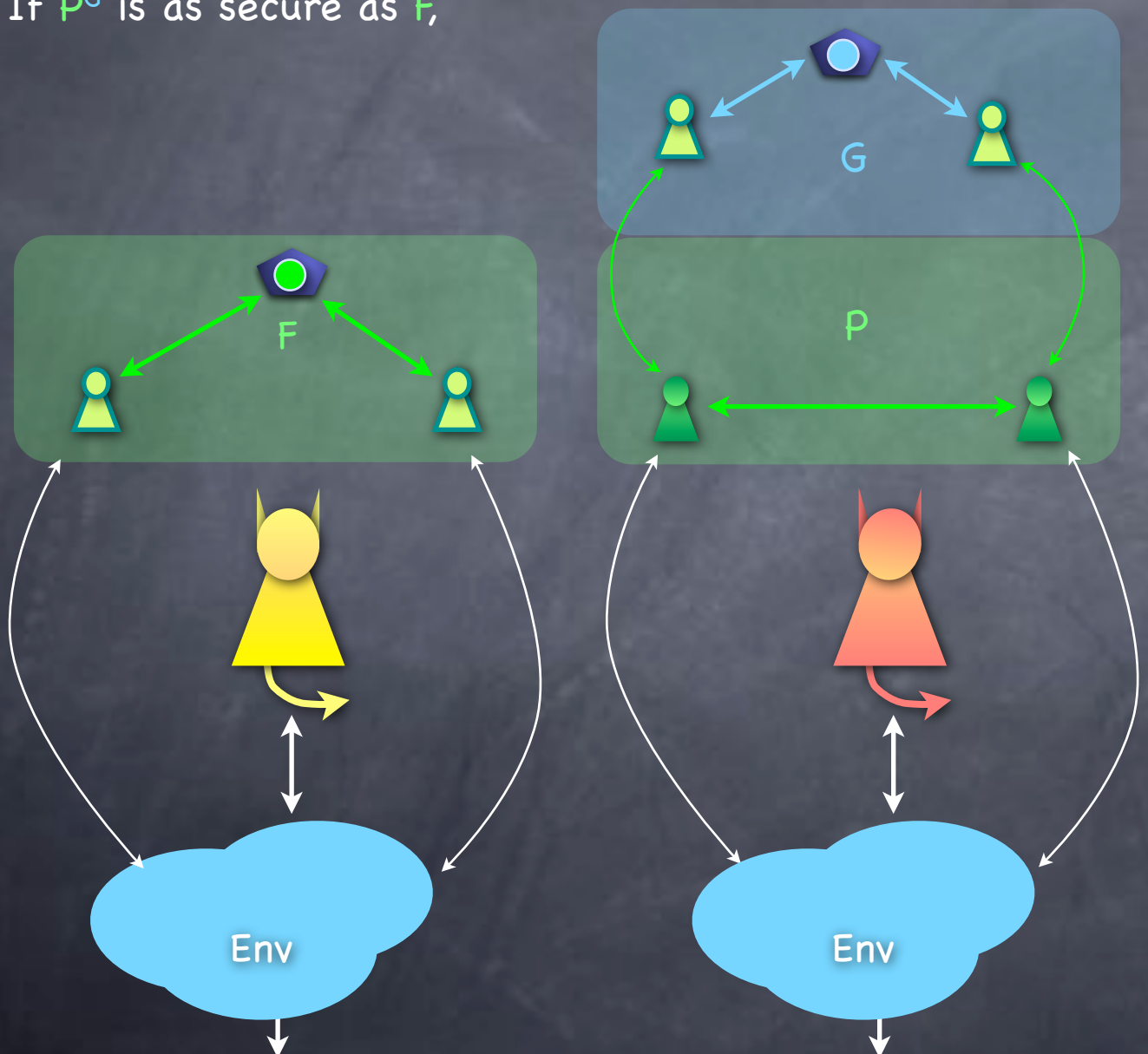

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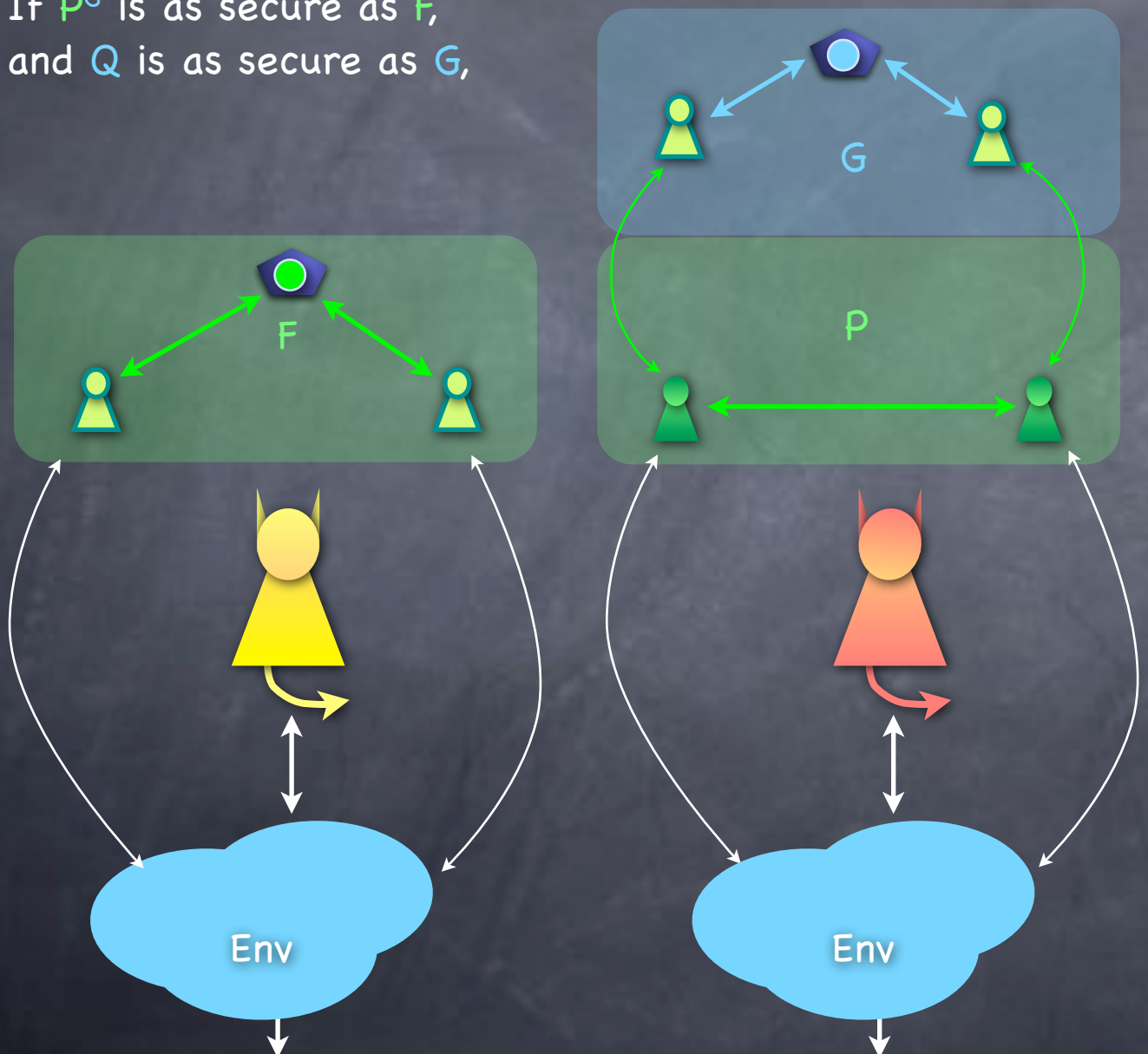

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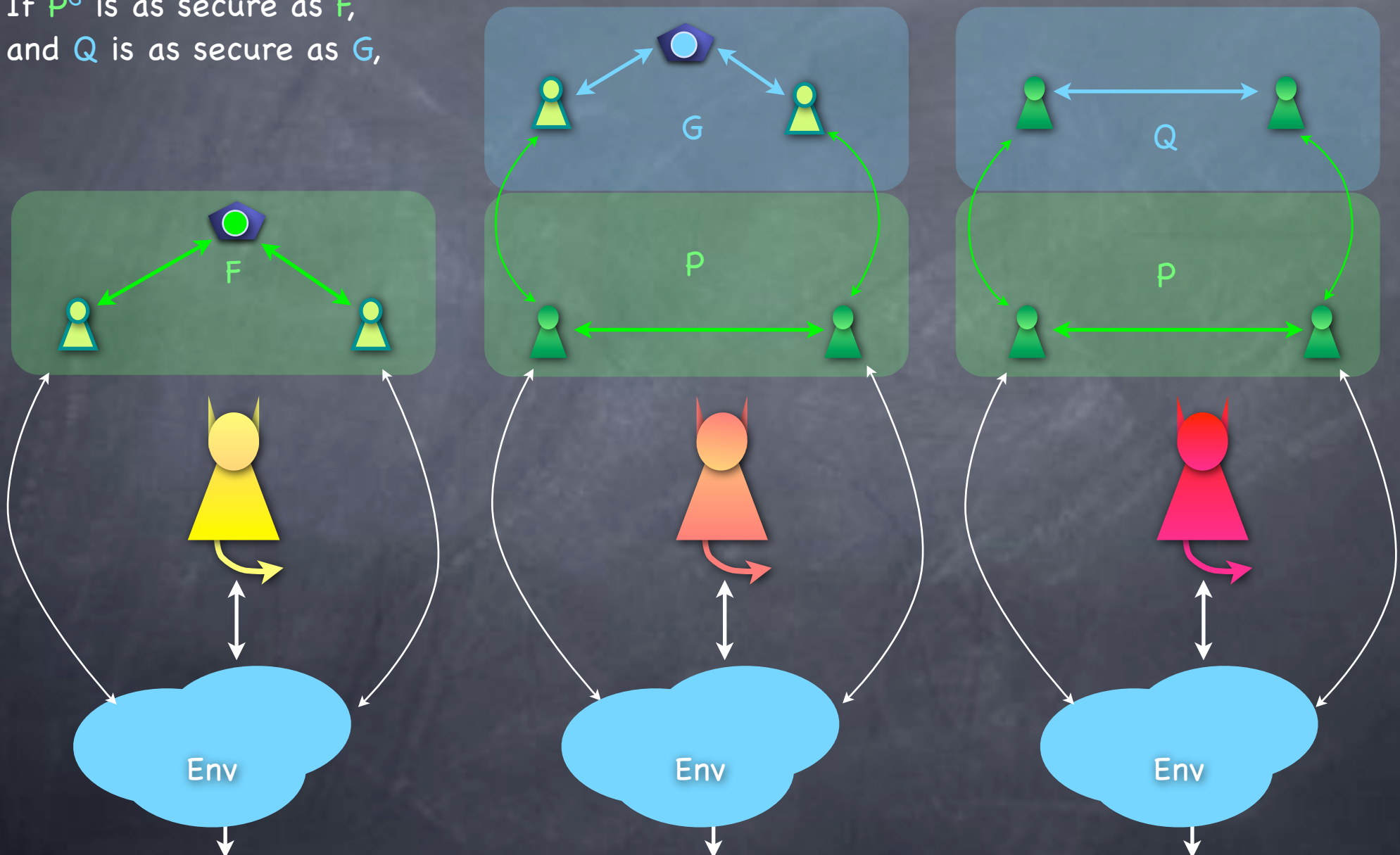

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If P^G is as secure as F,
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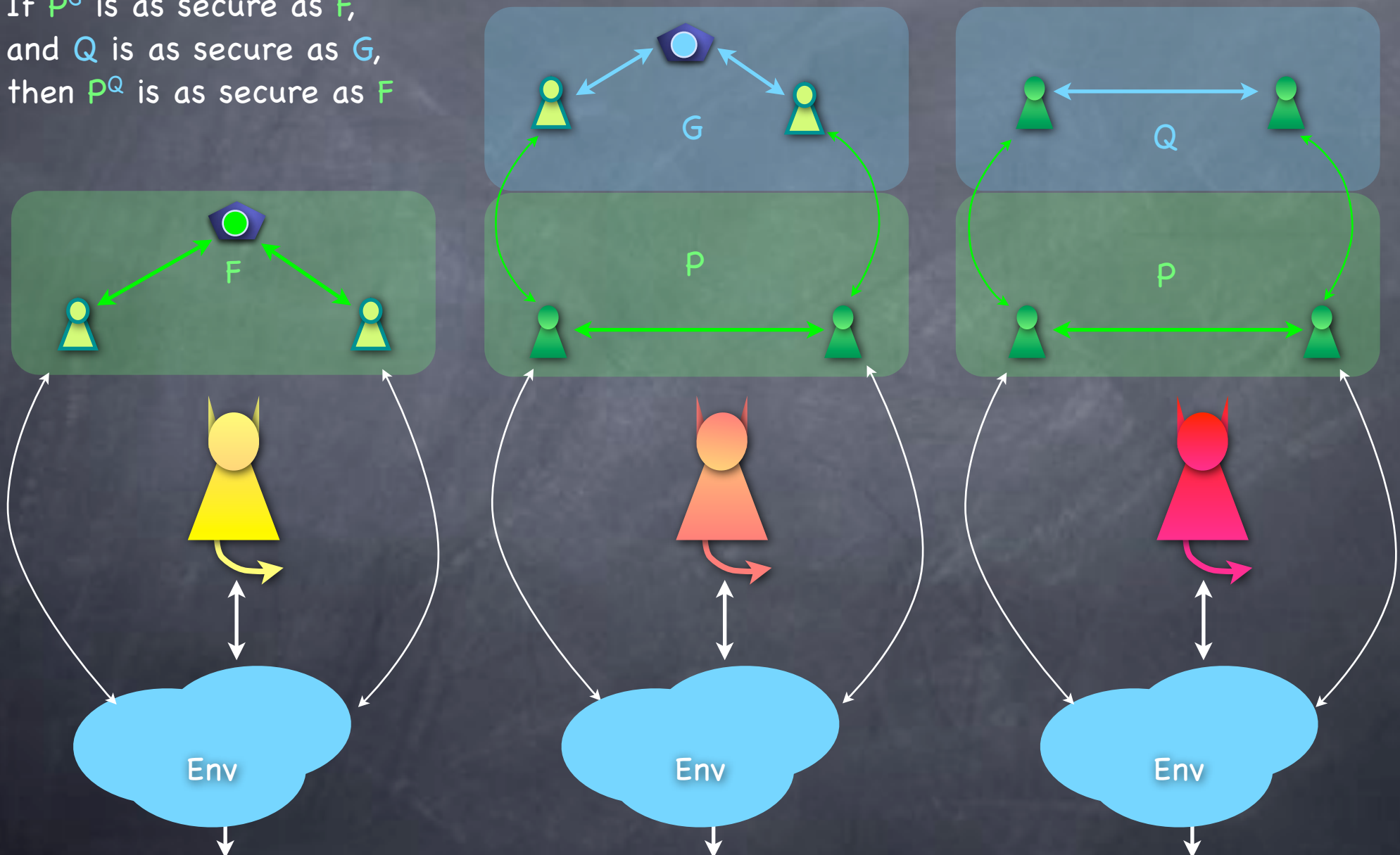

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- Gives a modular implementation of the IDEAL world

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  - But by itself may not imply universal composition: e.g. with PPT REAL world, unbounded IDEAL (simulator or functionality)

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  - Even when considering only one component, other components could be present in the environment
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  - Also, UC by itself does not imply a meaningful security (nor require an environment)

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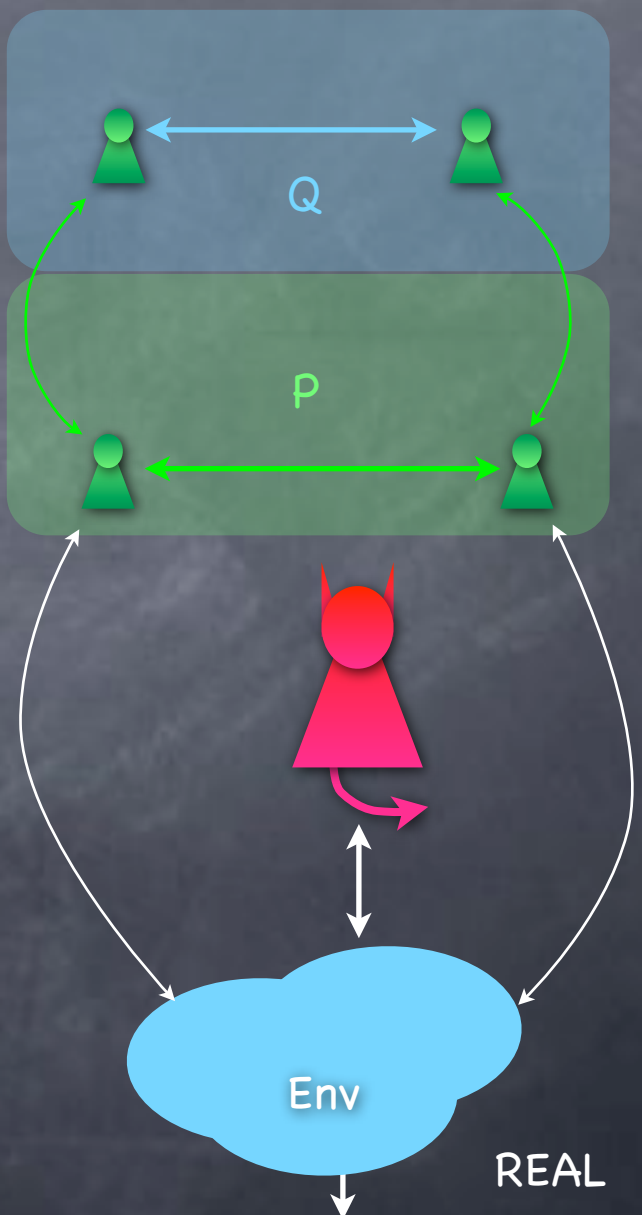
- Considering an arbitrary environment is anyway necessary for the security guarantee to be useful
  - But by itself may not imply universal composition: e.g. with PPT REAL world, unbounded IDEAL (simulator or functionality)
  - Also, UC by itself does not imply a meaningful security (nor require an environment)
    - e.g. Define security of composed system as security of each individual component; Or, define everything secure.

Proving the UC theorem



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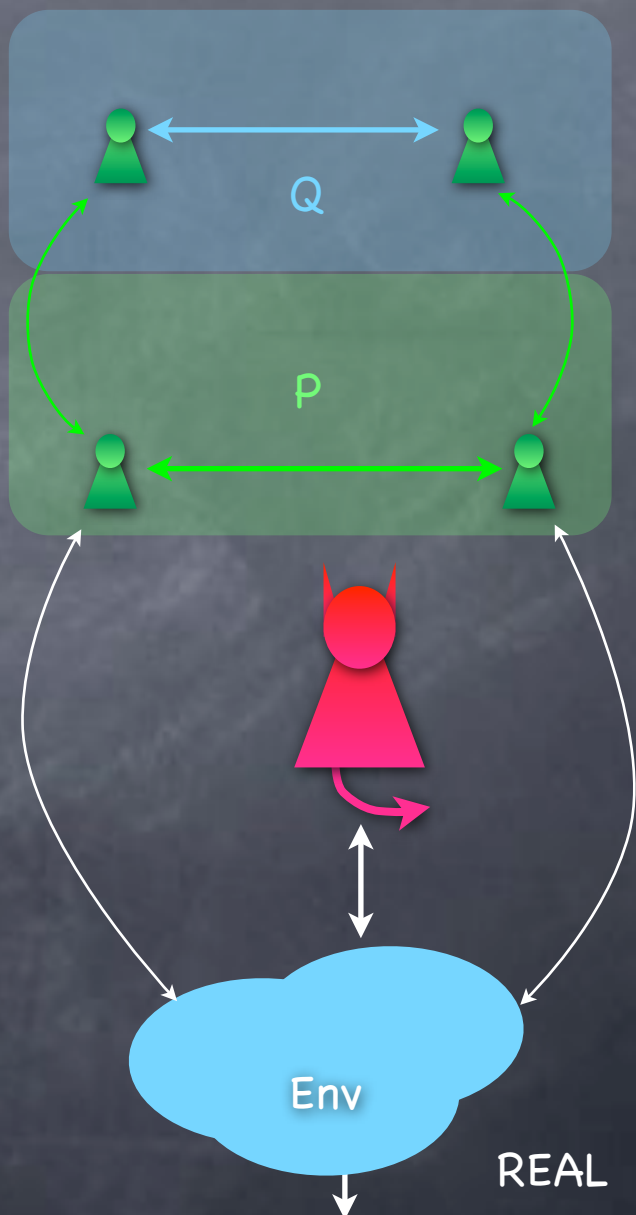

Proving the UC theorem

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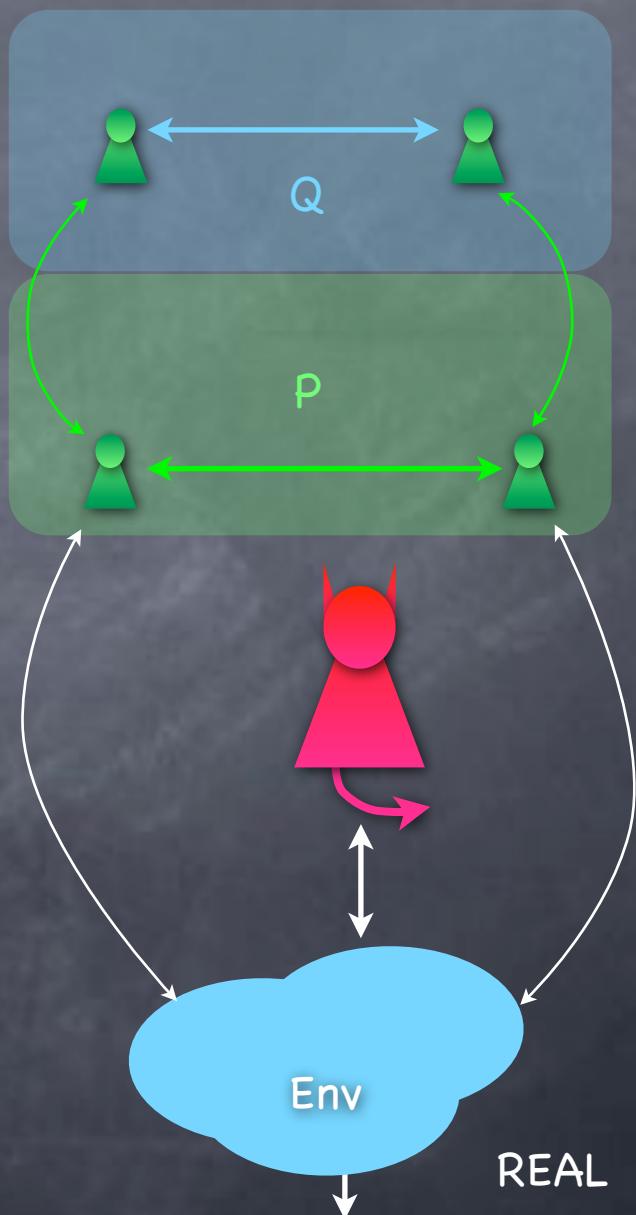

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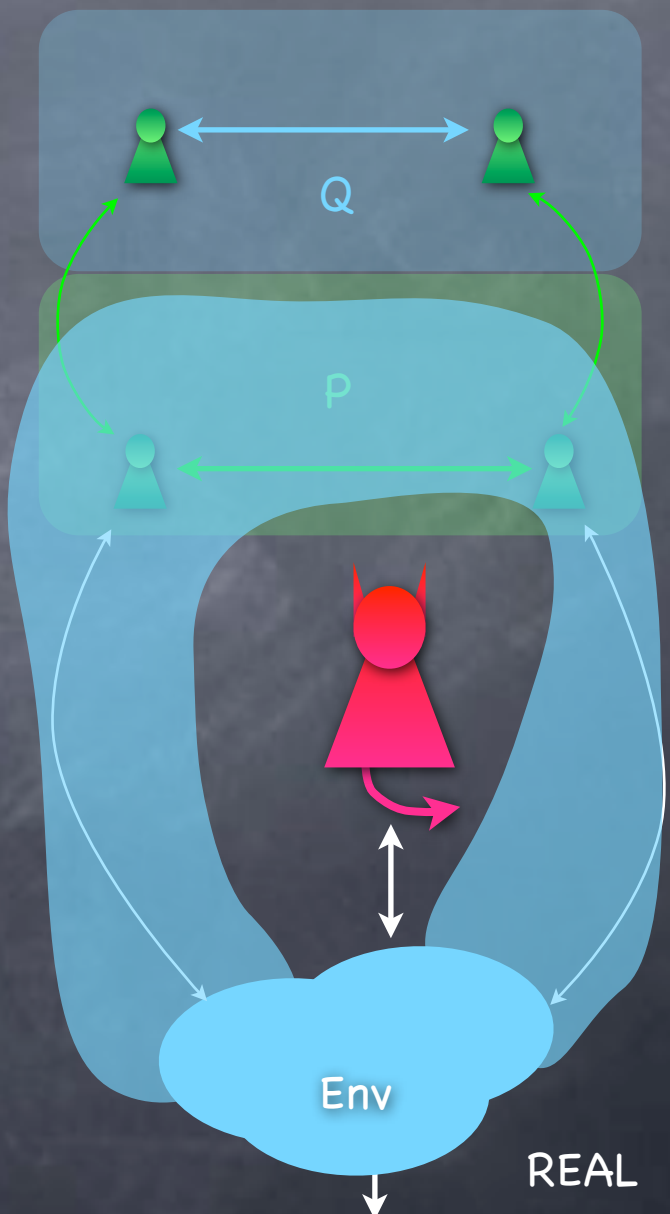

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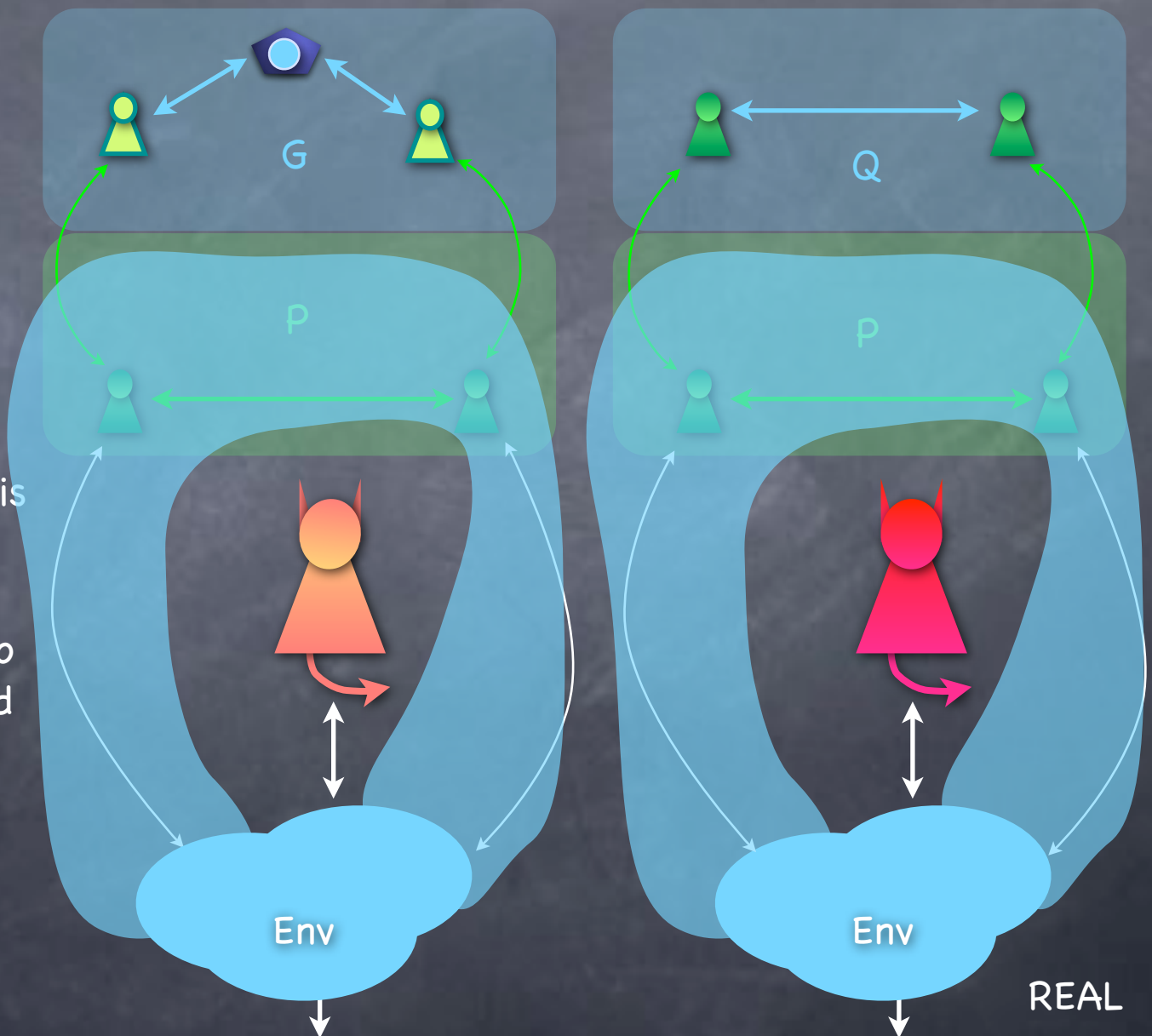

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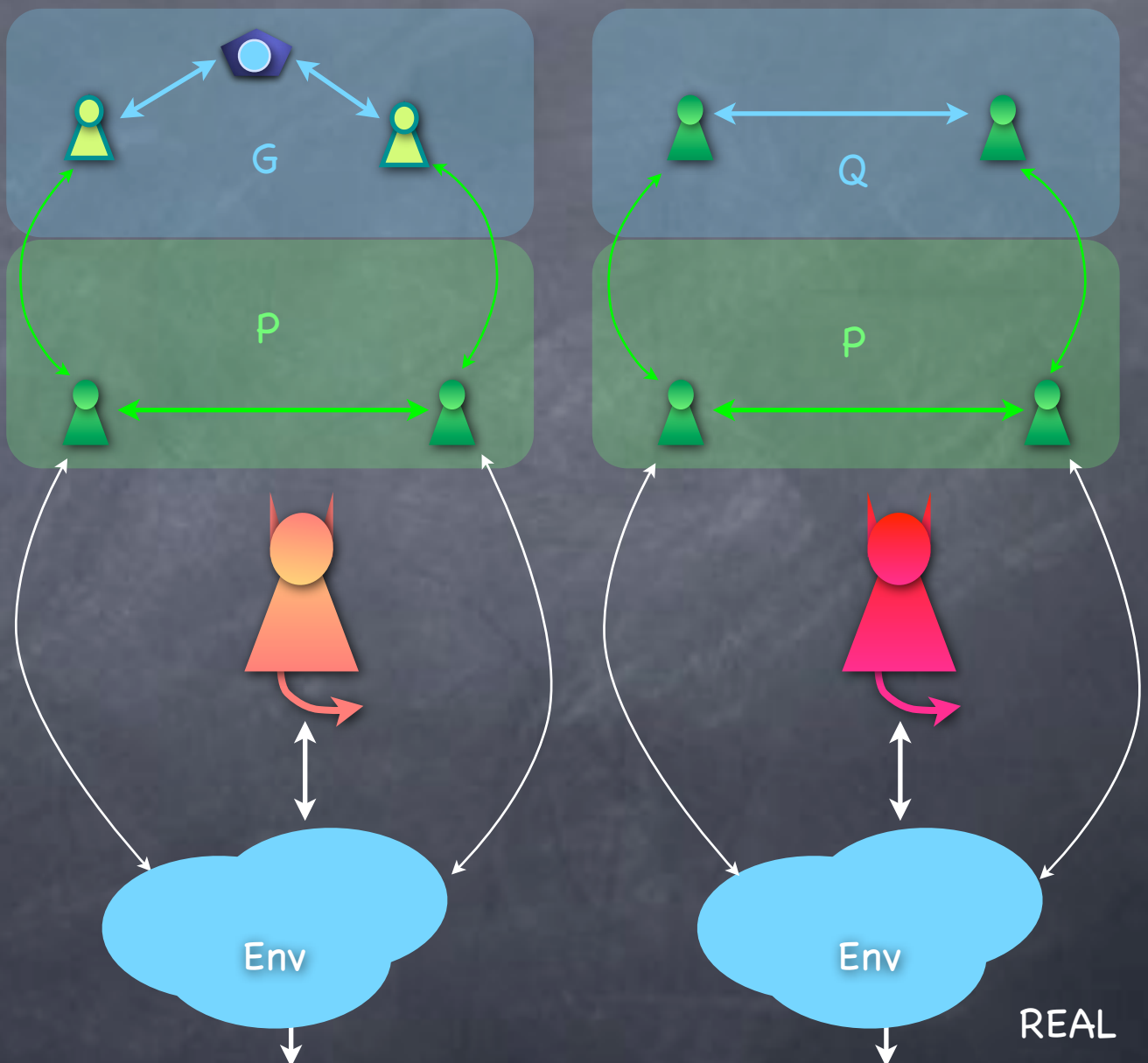

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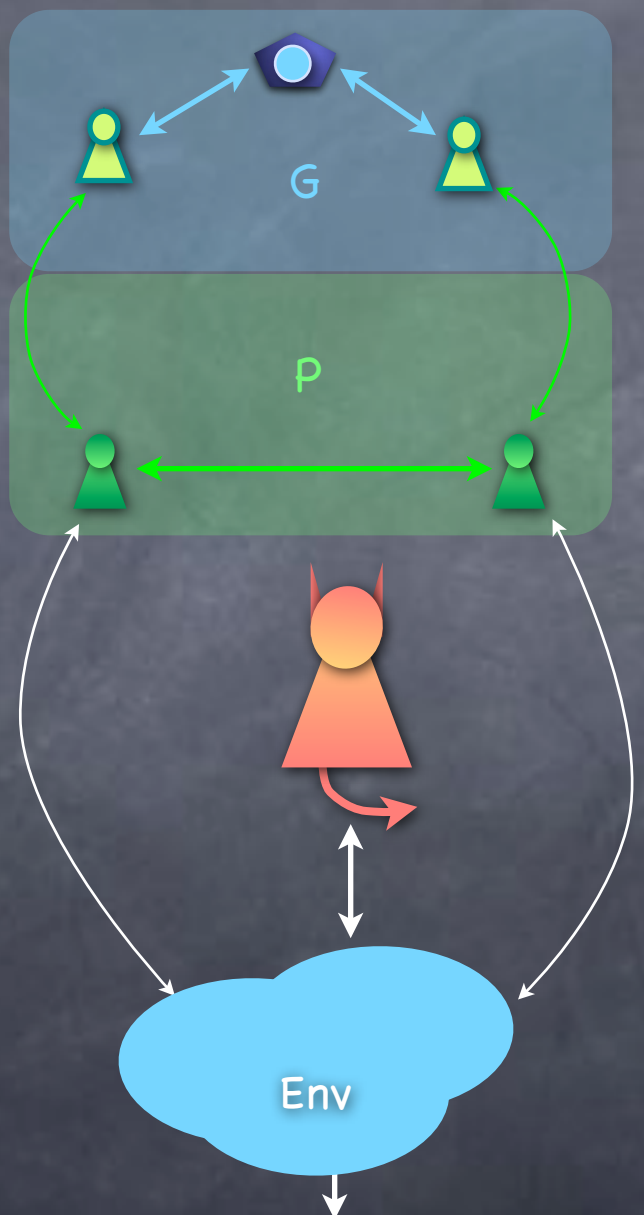

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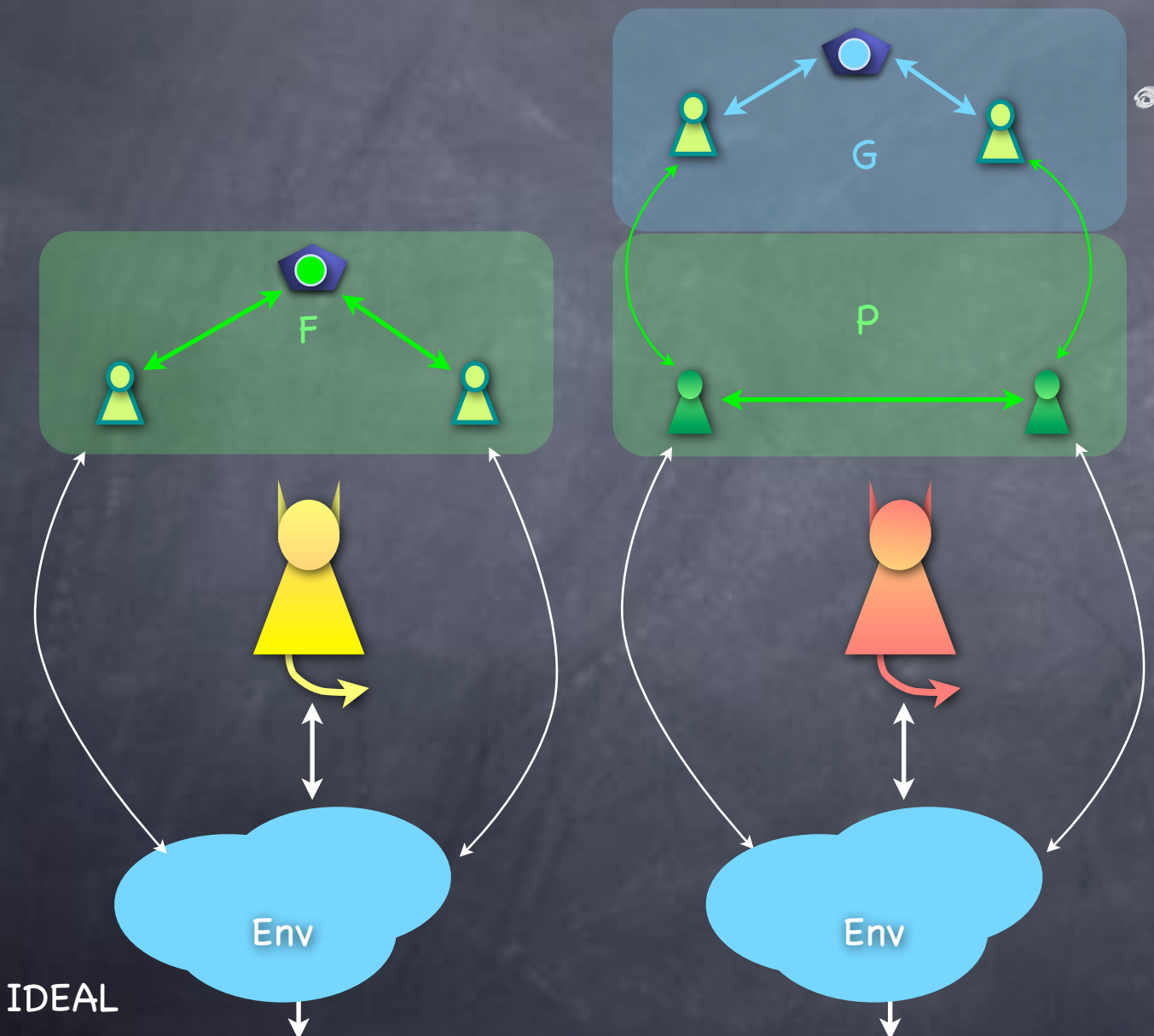

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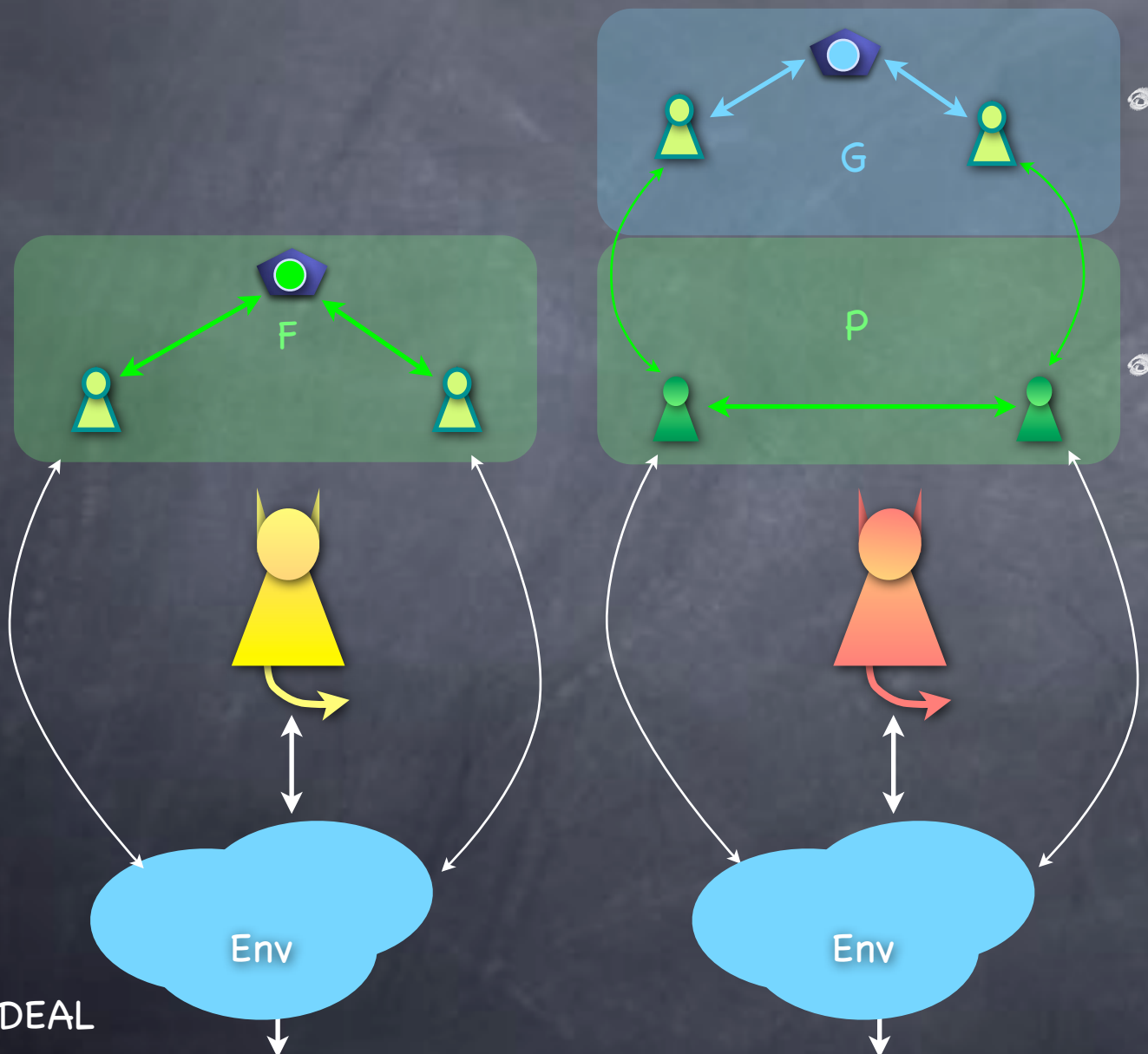

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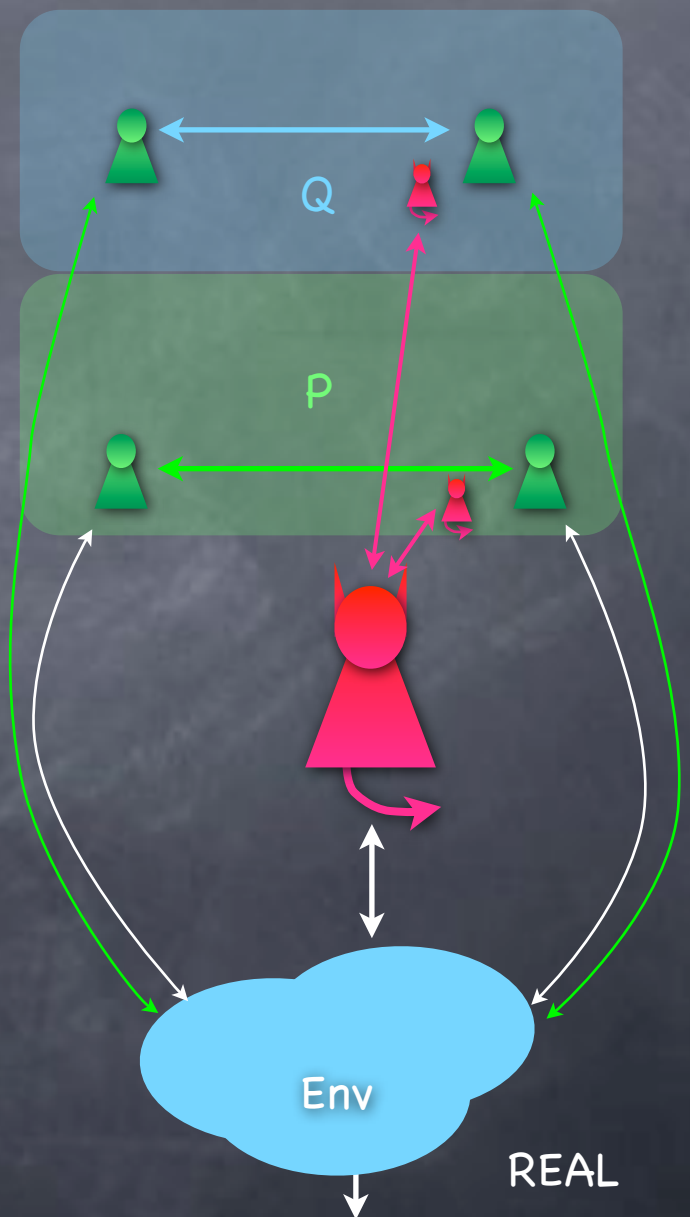

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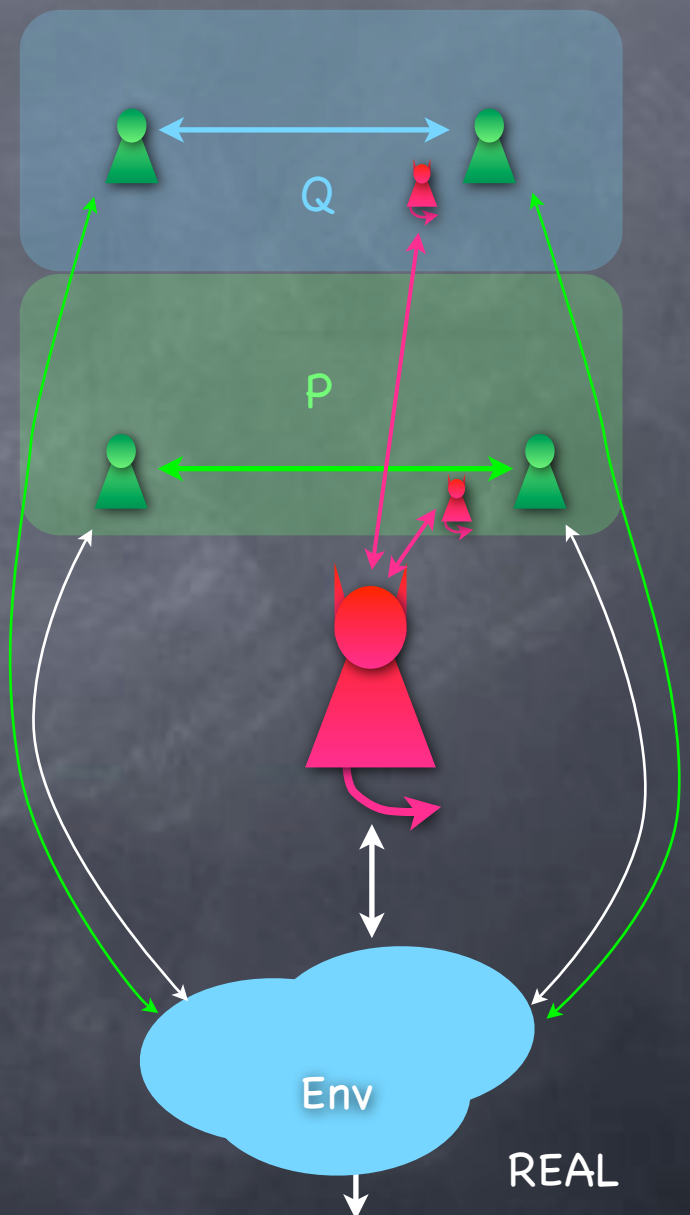

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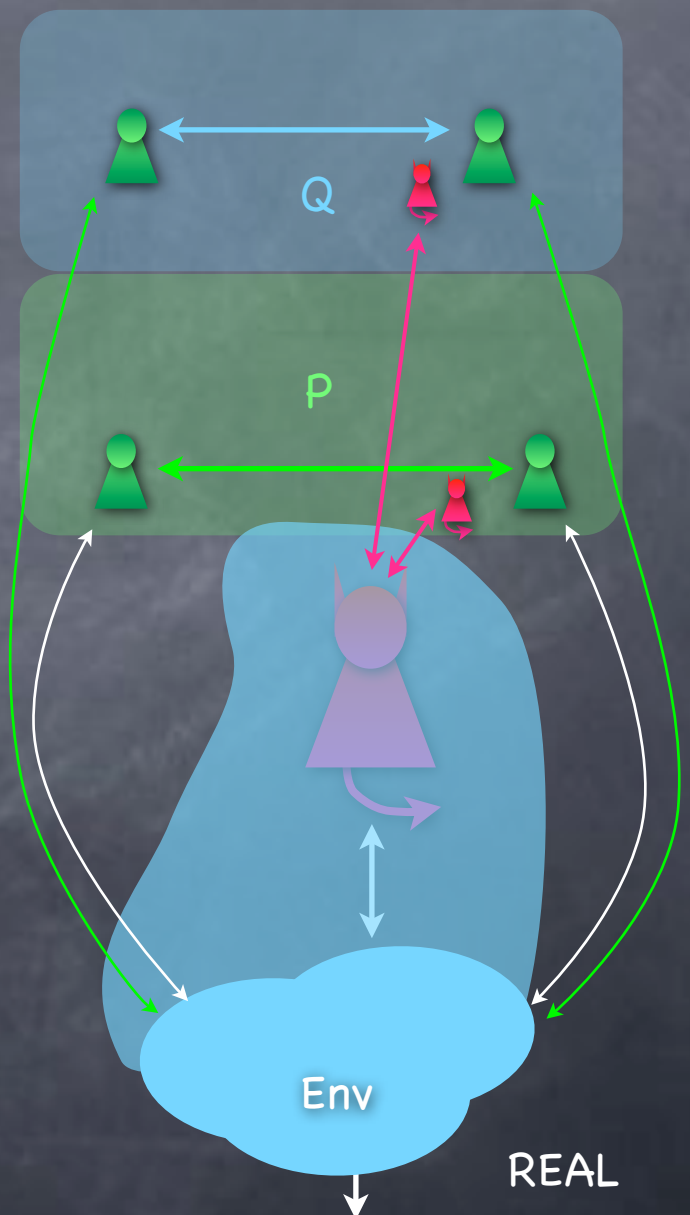

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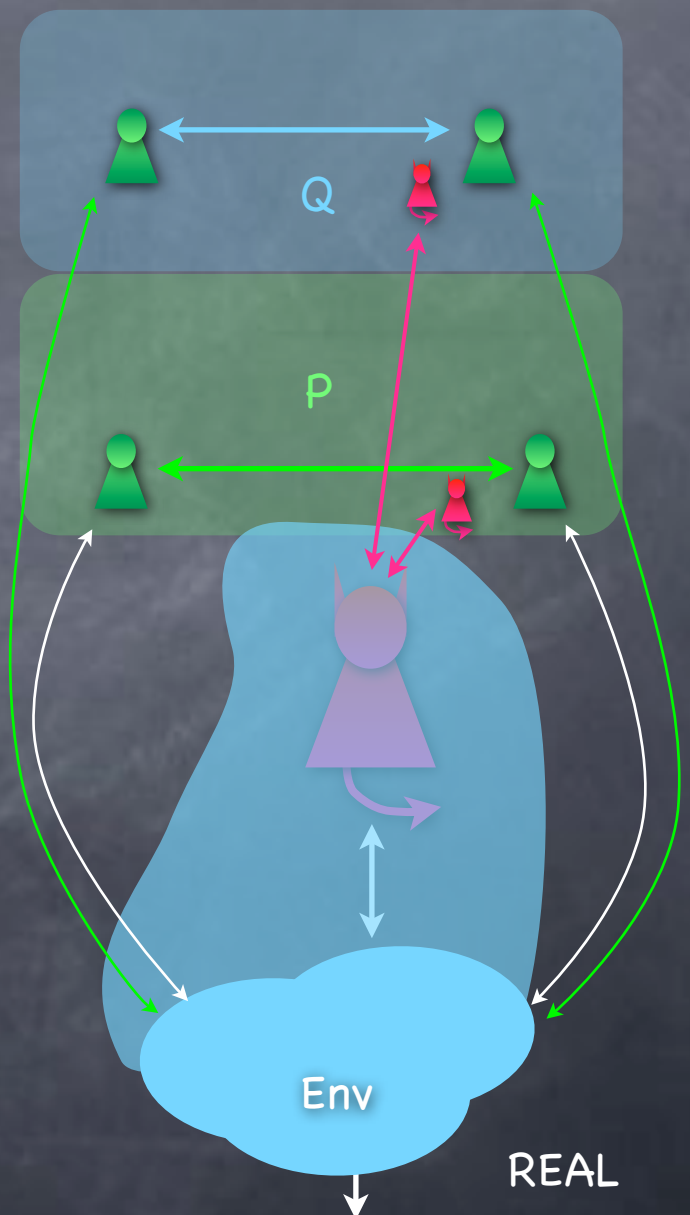

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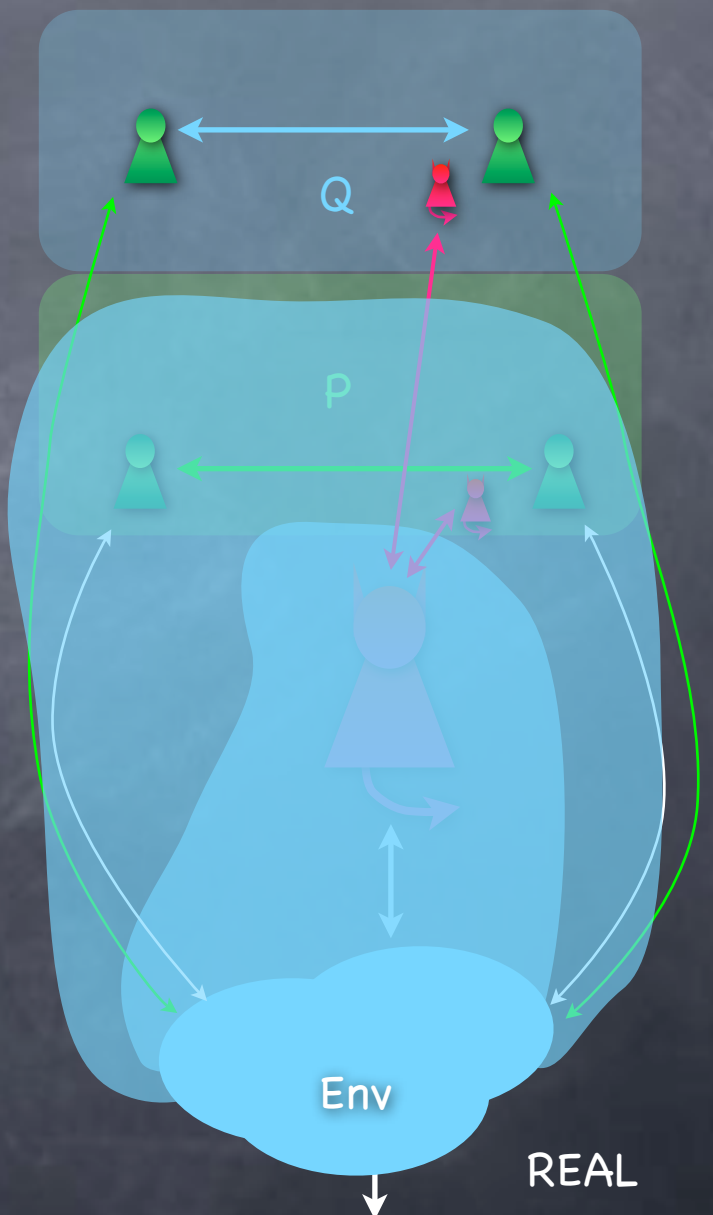

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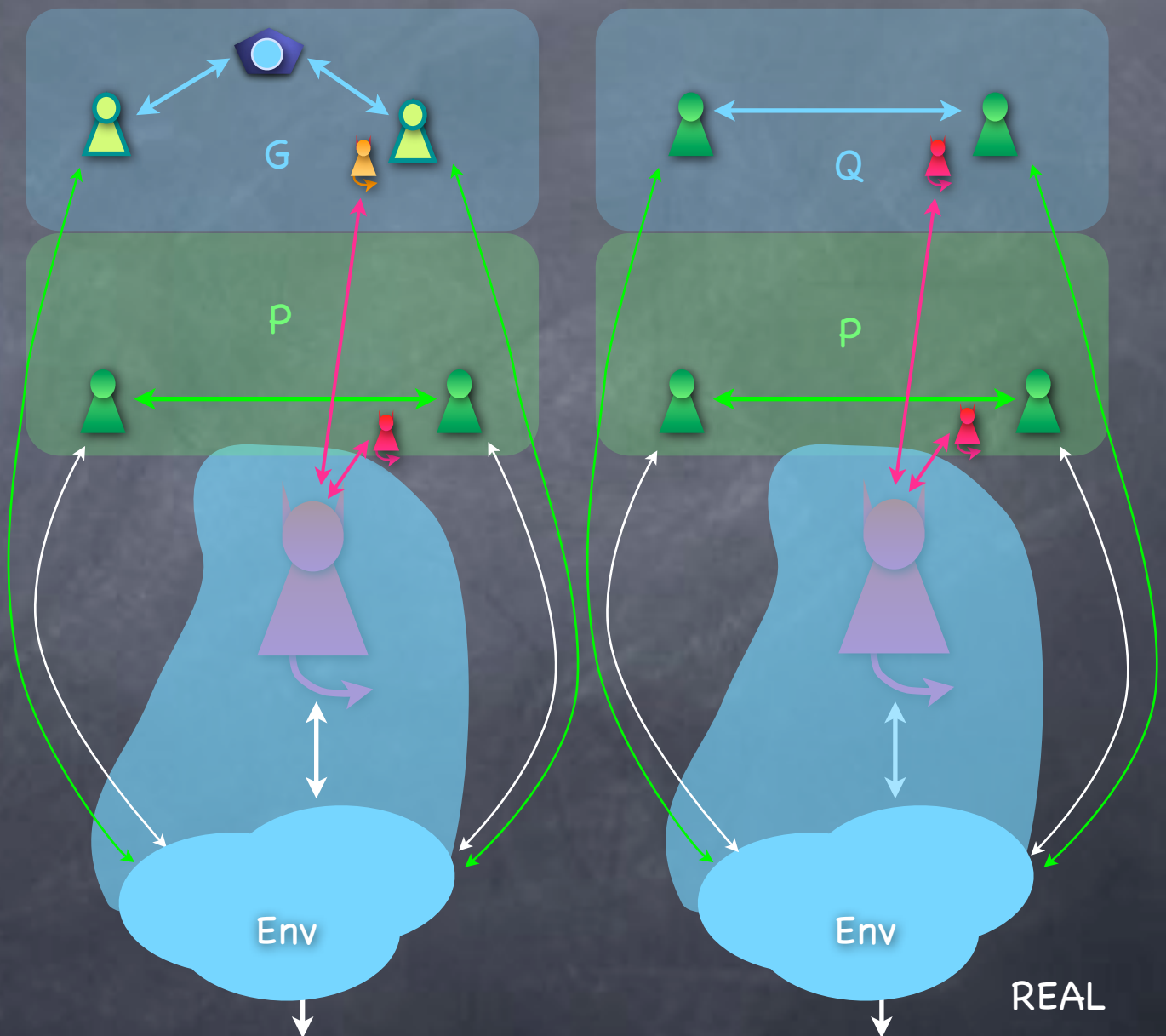

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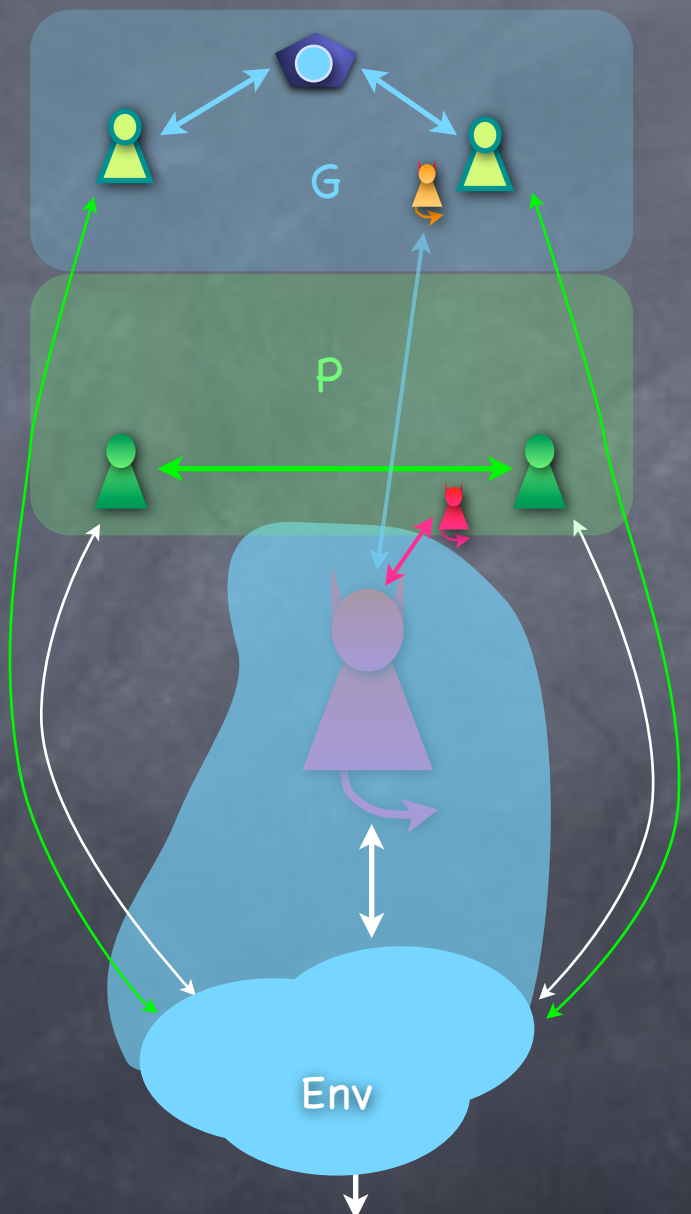

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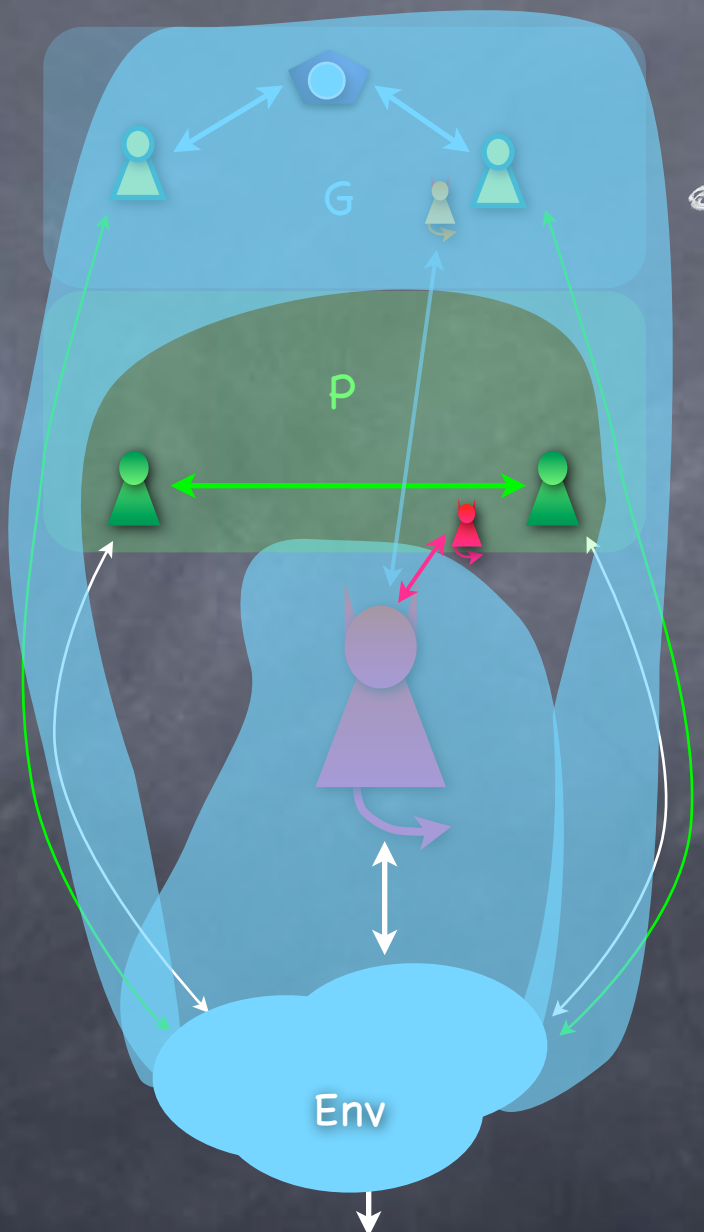

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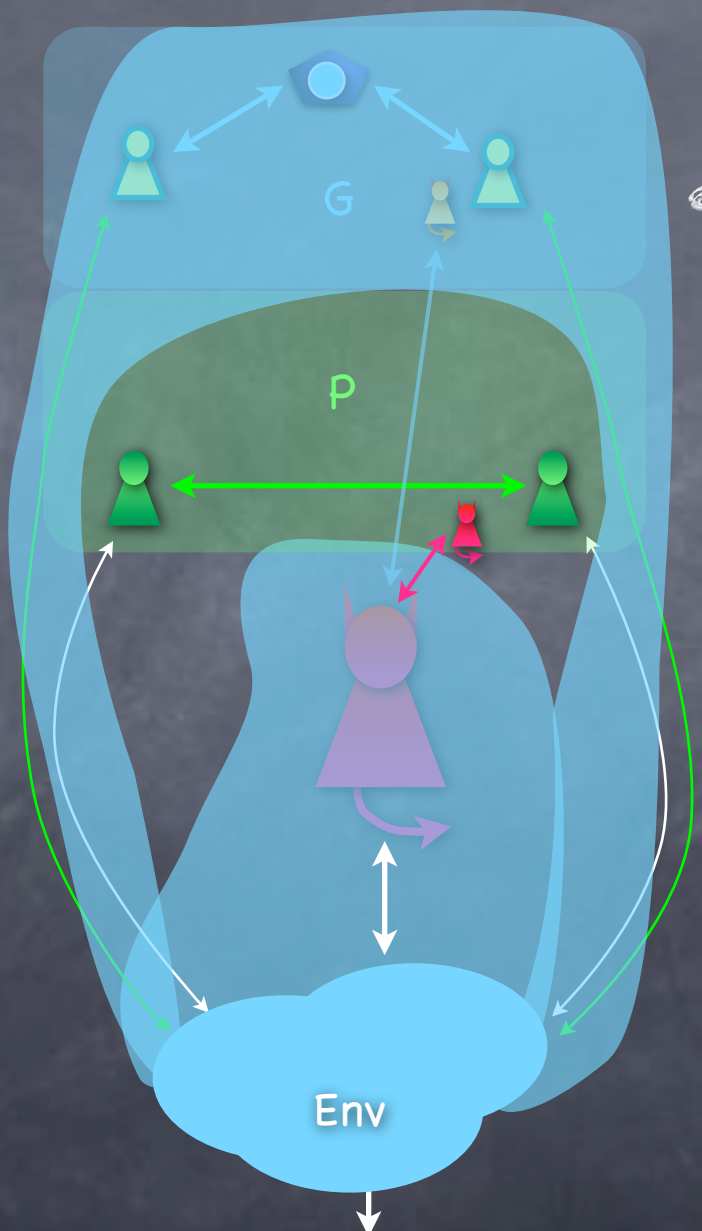

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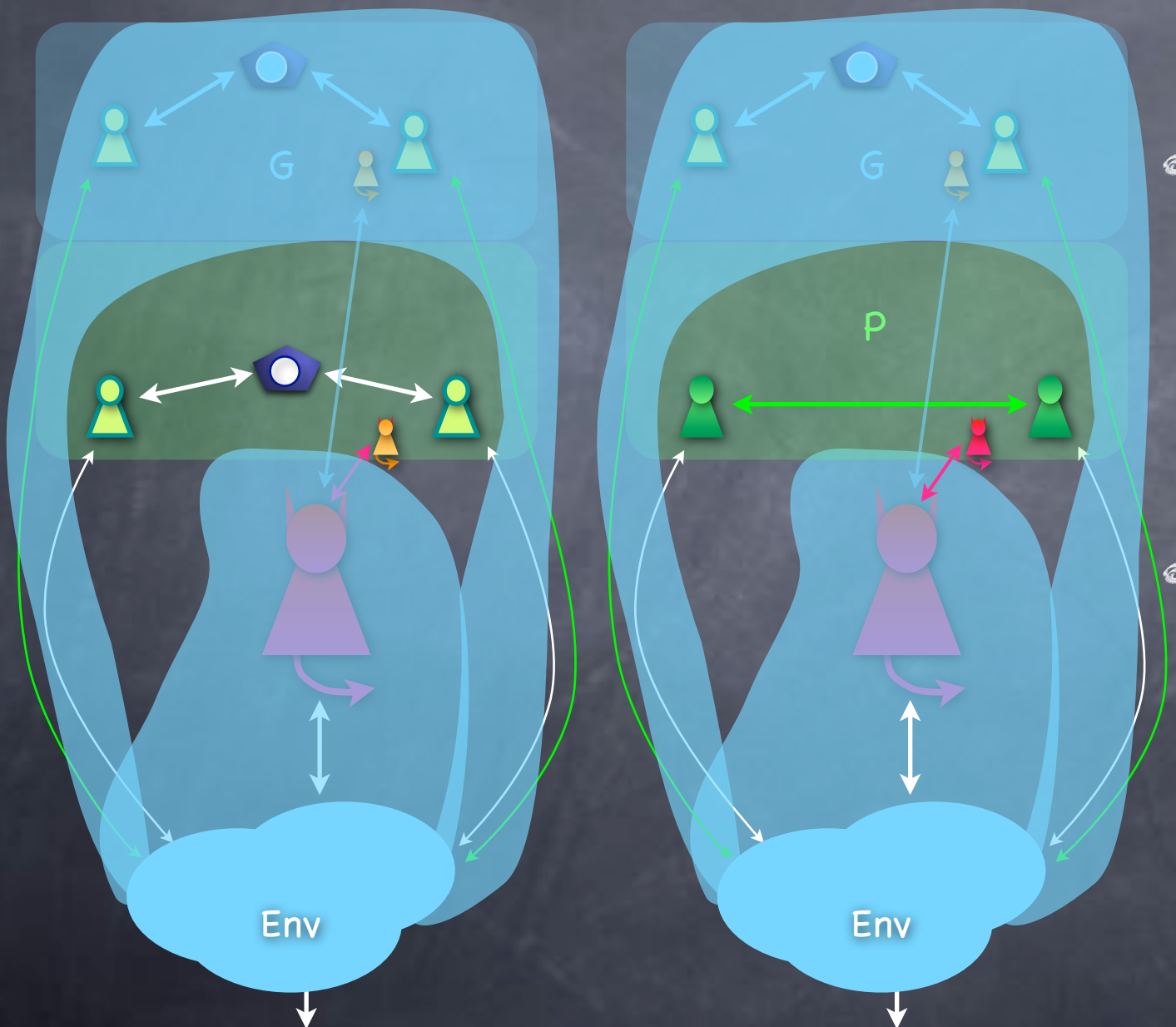

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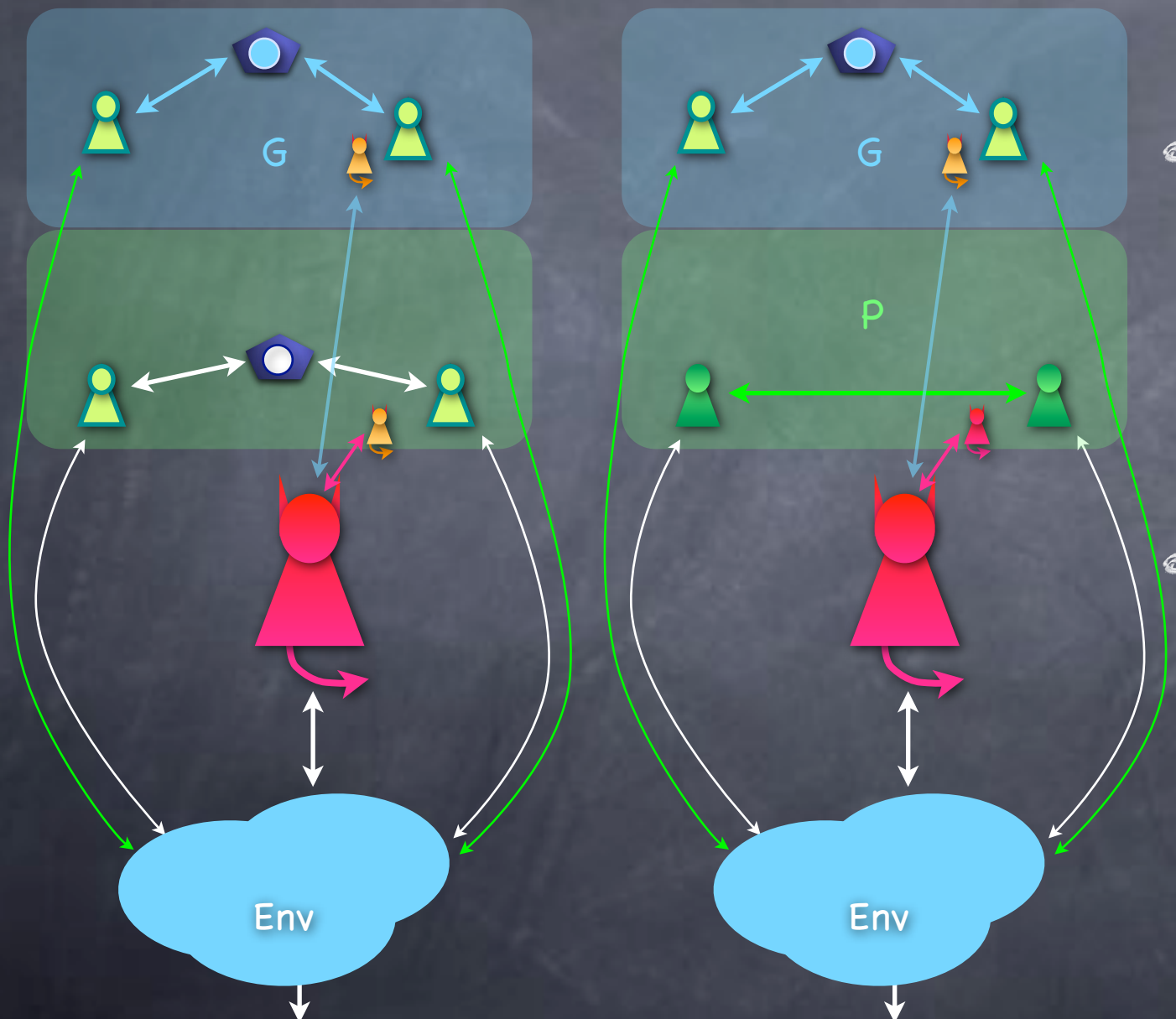

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