Rationality and Complexity

CS598HS

Recall Nash equilibrium

The joint strategy (x,y) is a Nash Equilibrium if x is a best response to y <u>and</u> y is a best response to x

Our old friend

	C	D
C	3,3	0,4
D	4,0	1,1

"In real life, we do not always behave in a selfishly antisocial way, and we often give up an advantage in order to behave in a cooperative manner. Much work in game theory has been devoted to explaining this apparent paradox."

The n-repeated game

The repeated game strategy space

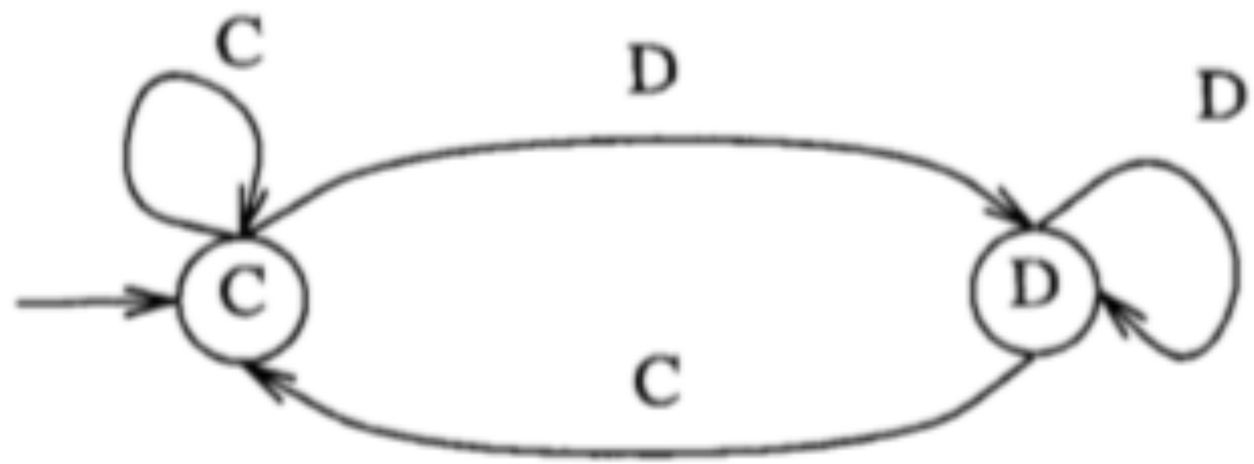
For an n round repeated prisoner's dilemma game, a pure strategy is specified as

$$\{f_1, \cdots, f_n\}$$

$$f_i : \{C, D\}^{i-1} \times \{C, D\}^{i-1} \rightarrow \{C, D\}$$

	C	D
C	3,3	0,4
D	4,0	1,1

Implementation as automata



How would we describe a mixed strategy?

	C	D
C	3,3	0,4
D	4,0	1,1

In a 4 round PD, is tit-for-tat a best response to tit-for-tat?

For *n* round PD, if we say strategy automata must have between [2, n) states, tit-for-tat is always an equilibrium

Lemma 1

If both players are allowed 2^n states, then the only equilibrium is to always defect

Theorem 1

For every $\epsilon > 0$ in the $\boldsymbol{n}$-round prisoner's dilemma played by two automata where at least one of them has a subexponential number of states, there is a mixed equilibrium with an average payoff of at least $(3 - \epsilon)$

Proof: intuition

First, force the other player to memorize something at the start to fill up memory they might use to be devious otherwise. Then, cooperate for a period of time and then prove to each other that you memorized what you were supposed to. Punish any deviation by always defecting

Proof: algorithm setup

Given $\epsilon > 0$ and a number of states $\boldsymbol{s} > \boldsymbol{n}$ let $\boldsymbol{d}$ be the smallest integer satisfying $\boldsymbol{d}2^{\boldsymbol{d}+1}(1 + \frac{1}{\epsilon}) \geq \boldsymbol{s}$, randomly choose a "business card"from $\{C,D\}^{\boldsymbol{d}}$

mixed strategy

Proof: algorithm

1) Each player reports/plays their *d*-character “business card” in the first *d* rounds

2) Each player plays *d* steps ironing out any score discrepancies introduced by business card exchange

3) In a loop: The players cooperate for a number of steps

4) Each player reads back both business cards XORd together. End loop

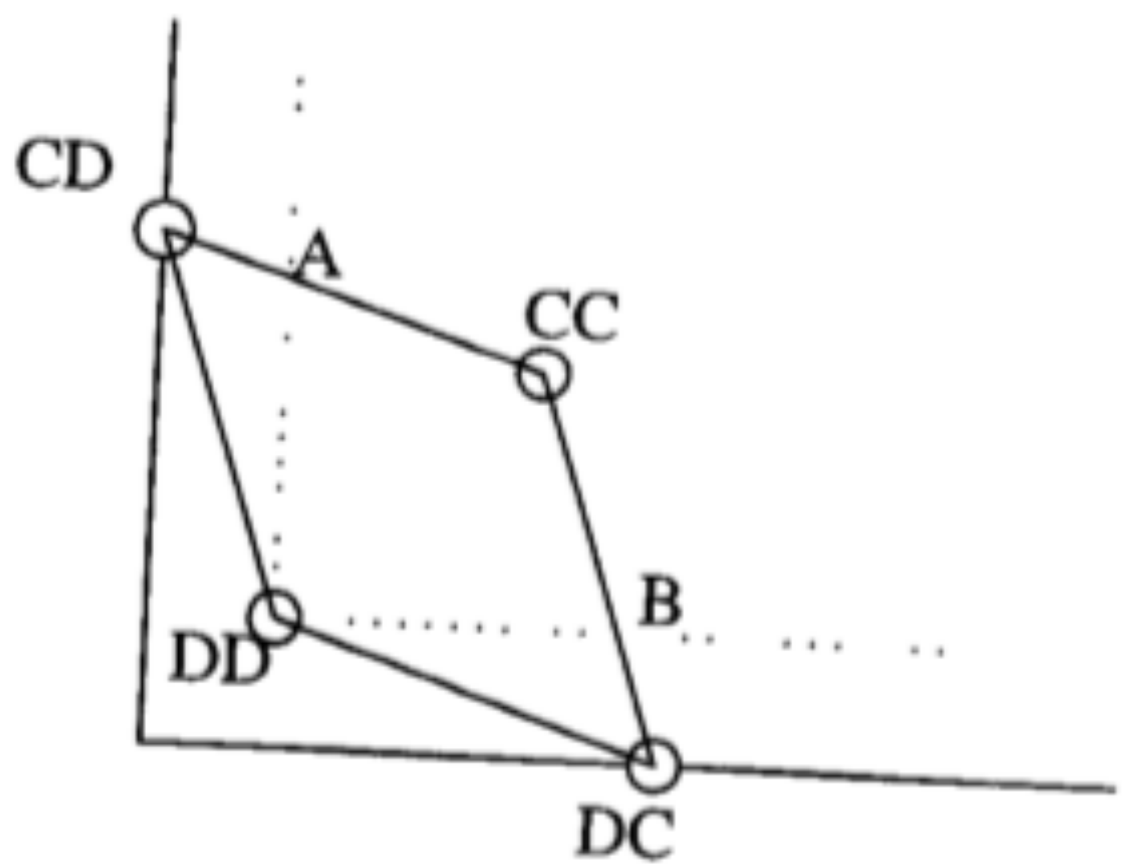
Proof: conclusion

There is no strategy in an *n* round game obeying the state bound *s* which is a better response to this strategy than itself

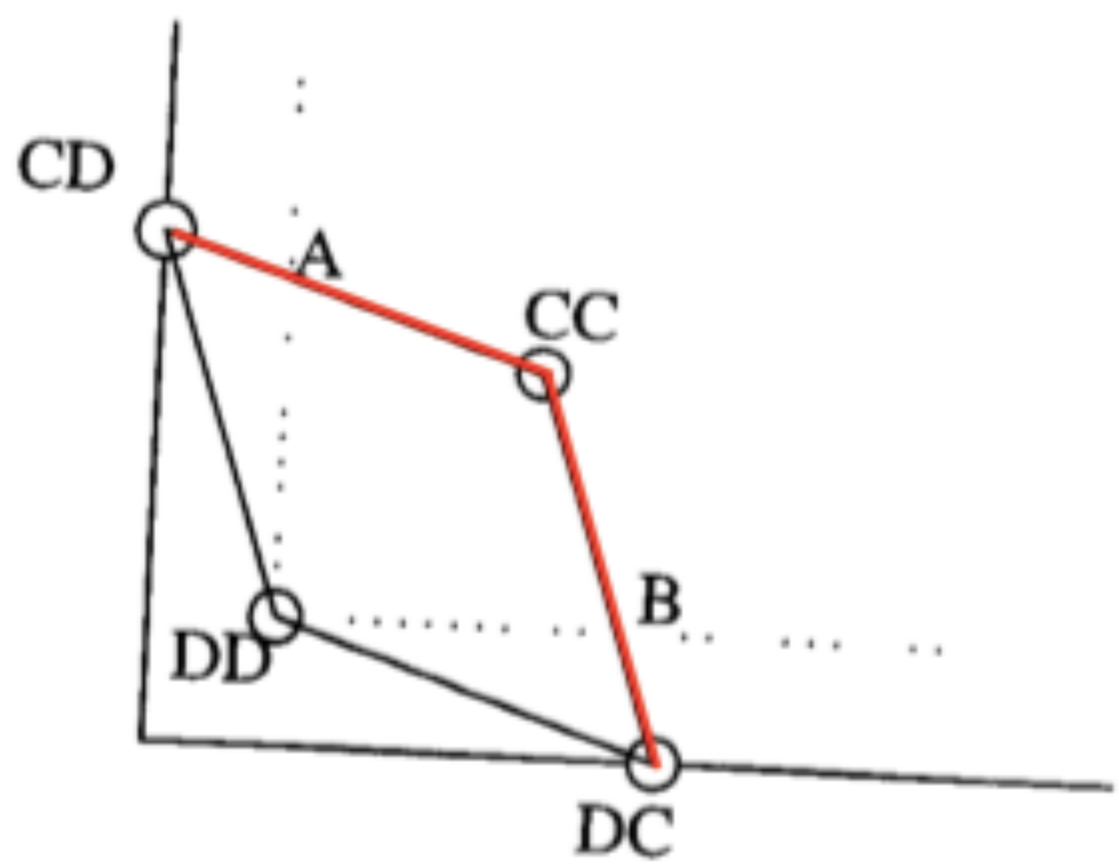
“For all sub exponential complexities, there are equilibria that are arbitrarily close to collaborative behavior"

What about other games?

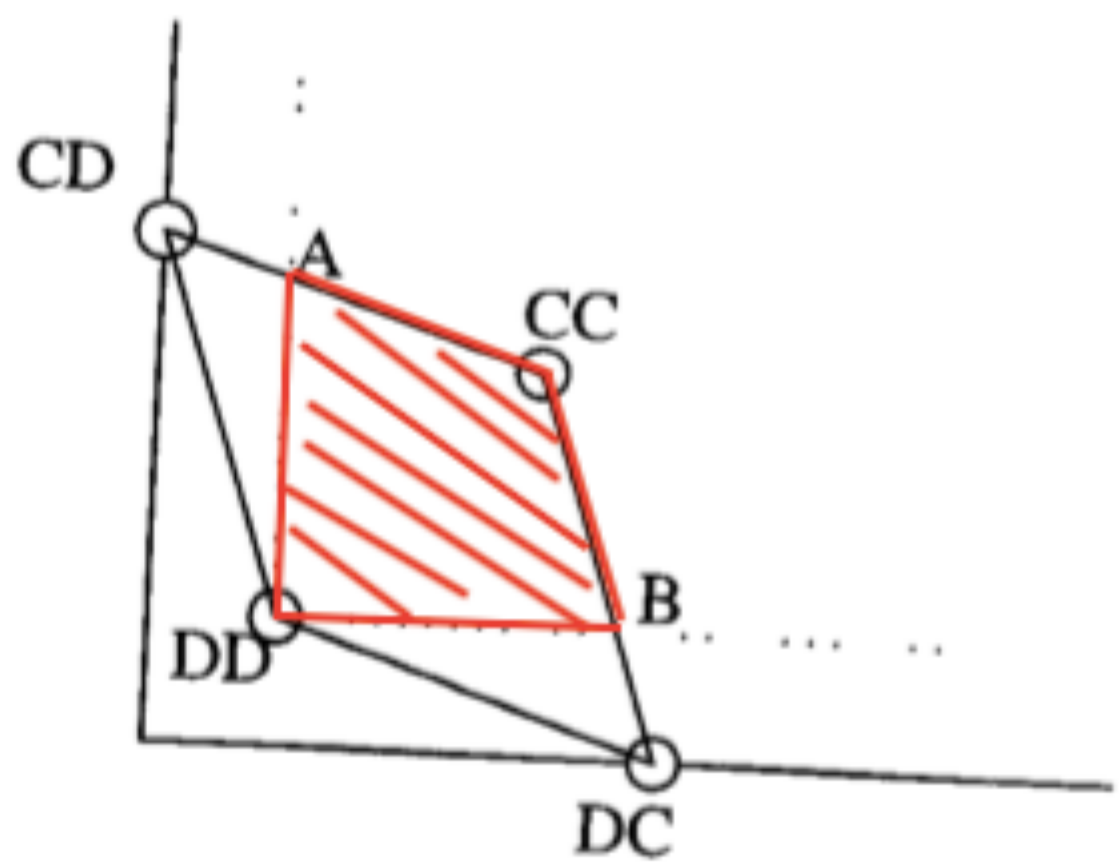
Payoff geometry



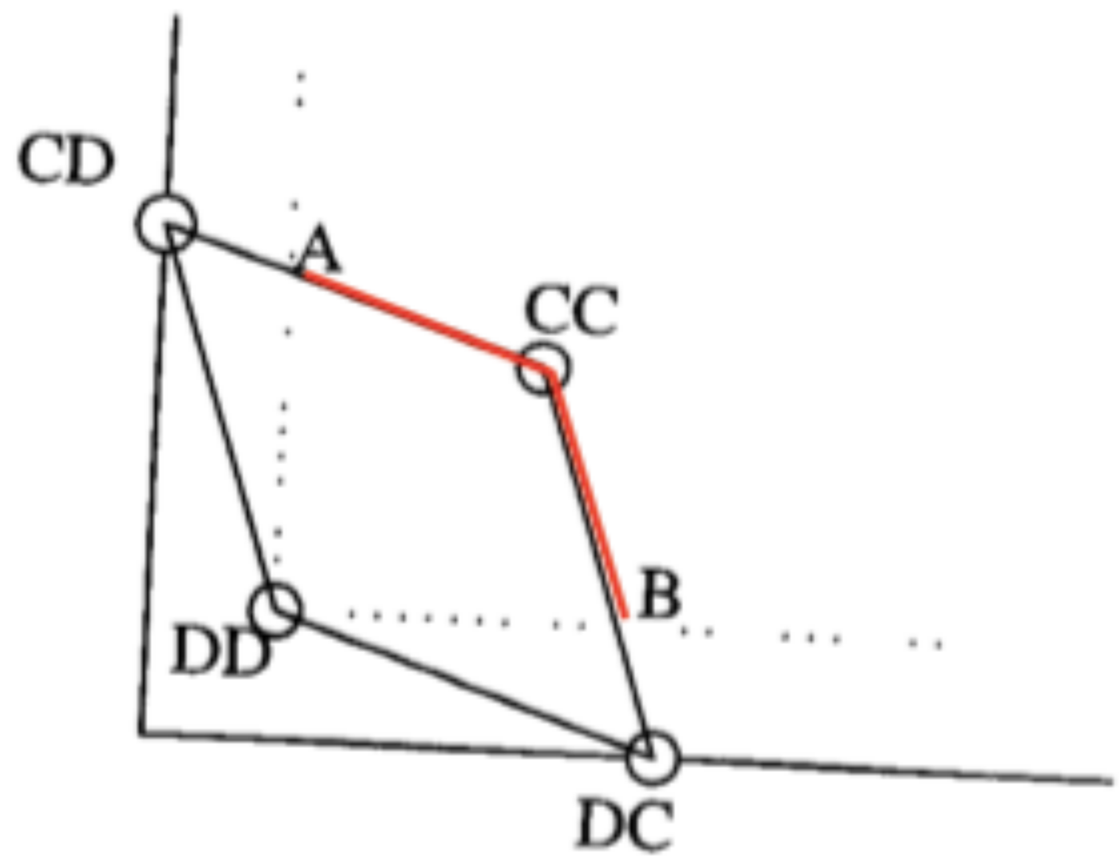
Payoff geometry: Pareto



Payoff geometry: IRR



Payoff geometry: Pareto-IRR



Theorem 2

For an arbitrary game G if $p = (p_1, p_2)$ is an individually rational Pareto optimal point, then for every ϵ, there is a subexponential bound on automata size such that an automata smaller than the bound exists for both players corresponding to a mixed equilibria with average payoff at least $p_i - \epsilon$ for each player in the n repeated game of G

Another complexity notion

Game schemes

A game scheme *g* is a polynomially computable function from 3 strings to 2 integers g(z,x,y) = (a,b)

z encodes the game, x player 1's strategy, y player 2's -> a is player 1's payoff, b is player 2's

Complexity of game theory questions

Decision problem	**Equals complexity class**
There exists a strategy *y*, which given *x* and *z* has a payoff at least *b*	NP
There exists an equilibrium which pays player 1 at least *b* for the zero-sum game *z*	EXP
There exists an equilibrium in game *z* which pays both player 1 and player 2 at least *b*	NEXP

Meta strategies