Quadratic Voting

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Overview

- Introduction ☜
  - Motivation: Imperfection of democracy
  - Problem definition: Majority-minority conflicts
  - Insight: Lack of markets
- Price-theoretic model
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  - Vote pricing rule and its robustness optimality
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It has been said that

democracy is the worst

form of government

except for all the others

that have been tried.

Majority–Minority Conflicts

- Majority rule was necessary for expressing the popular will and the basis for establishing the republic.
- But majority could abuse its powers to oppress a minority.


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Insight: lack of Markets

- One-person-one-vote (1p1v) constraints each voter's influence on collective decisions and thus preventing potential Pareto-improving trade.

- Standard market system of exchange motivates individuals to reveal their true preferences.

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Price-Theoretic Model

- Consider a society of N voters i = 1,…,N where there are many binary decisions.
- Assume there are enough issues and each issue is sufficiently inconsequential that every voter has a quasi-linear “continuation value” for retaining voice credits for future votes.

Price-Theoretic Model

- Each voter is endowed with a large stock of “voice credits” that they may spend influencing the outcome of the decisions.
- Assume that voice credits have been distributed fairly in the relevant society so that social optimality is achieved by maximizing total equivalent continuation value.

Price-Theoretic Model

For some particular decision with alternative A and B,

1) Each voter i chooses a continuous number of votes v_i, v_i is positive if voter favors A and negative if voter favors B.

2) Each voter pays a cost $c(v_i)$ for her votes where c is differentiable, convex, even, and strictly increasing in $|v_i|$ to a central clearing house. C is called ***vote pricing rule***.

Price-Theoretic Model

For some particular decision with alternative A and B,

3) The value that voters receive, in units of voice credits, is $2u_i$ if voters prefer A to B. Negative value means voters prefer B to A.

4) Assume voters are price-taking, that is, all voters agree on a marginal pivotality of votes p on the decision. Under this assumption, the way a voter i chooses v_i is to maximize $2u_i p v_i - c(v_i)$.

Vote pricing rule: Robust optimality

A vote pricing rule is robustly optimal if, for every $p > 0$, N, vector u, each voter choose votes v_i^* so that $\sum_i v_i^*$ has the same sign as $\sum_i u_i$.

THEOREM :

A vote pricing rule is robustly optimal if and only if it is quadratic.

Rough Idea

- Consider the class of vote pricing rules $c(x) = x^a$ for $a > 1$.
- To maximize $2u_i p v_i - c(v_i)$, we take the derivatives and let it equal 0

$$2u_i p v_i - c(v_i)$$

By differentiation on $v_i \Longrightarrow$

$$2pu_i = a(v_i)^{a-1}$$

$$\Longrightarrow \quad v_i = \operatorname{sign}(u_i)\left(\frac{2p}{a}\right)^{\frac{1}{a-1}} |u_i|^{\frac{1}{a-1}}.$$

Rough Idea

$$v_i = \text{sign}(u_i)\left(\frac{2p}{a}\right)^{\frac{1}{a-1}} |u_i|^{\frac{1}{a-1}}.$$

- If a = 2, it leads to v_i^* proportional to u_i and satisfies the definition of robust optimality.
- For any other a, v_i^* is not proportional to u_i and cannot ensure $\Sigma_i v_i^*$ has the same sign as $\Sigma_i u_i$ for some arrangements of p and u_i.

THEOREM :

A vote pricing rule is robustly optimal if and only if it is quadratic.

Some observations

$$v_i = \text{sign}(u_i)\left(\frac{2p}{a}\right)^{\frac{1}{a-1}} |u_i|^{\frac{1}{a-1}}.$$

- Consider the extreme case of a -> 1 (voting cost approaches linear), the power on u_i approaches infinite, which means voters with slightly greater values will vote infinitely more. This leads to dictatorship of the most intense voter.

- Consider the extreme case of a -> infinity, the power on u_i approaches 0, which means voters buy exactly 1 vote. This leads to 1p1v.

Quadratic Voting

As is shown, Quadratic Voting pricing rule is the optimal intermediate point between dictatorship and majority rule. Voters intend their own gain while the interests of the whole society advance.

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Practical Promise: difficulty

- Price-taking assumption is unlikely to hold in game theoretic models.
- Collusion, value distribution, imperfectly rational voters.

Practical Promise: experiment #1

- Voters' values are drawn independently and identically from a known value distribution and act as rational, risk-neutral expected utility maximizers.
- Under appropriate conditions and in all symmetric Bayes-Nash equilibria in large populations the price-taking assumption approximately holds for almost all voters.
- The welfare losses from QV decay generically at a rate 1/N as the population grows. In another parallel to markets, this is the same rate of convergence of private goods markets toward efficiency as competition grows.

Practical Promise: experiment #2

- Considers the baseline model in smaller populations for a range of distributions numerically.
- The welfare lost from QV relative to the optimum is very small (rarely more than a few percentage points)while that loss under 1p1v may easily be near 100 percent.

Practical Promise: experiment #3

- Allow for collusion, uncertainty about the value distribution, and the possibility that voters are not perfectly rational and consequentialist.

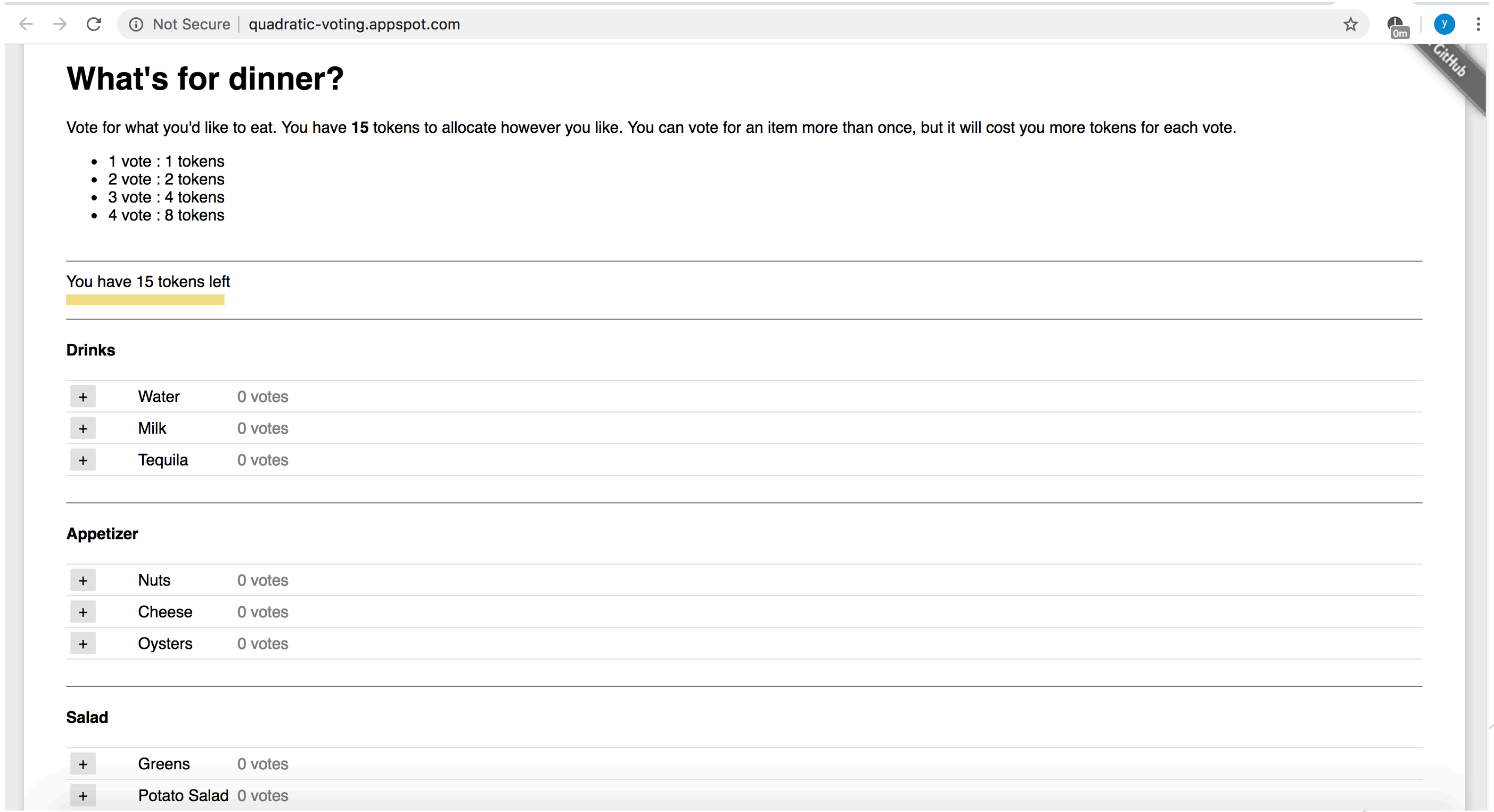
- In some of these settings, QV performs better than in our baseline and it rarely has more than a very small welfare loss.

Practical Promise: conclusion

- None of the experiments are perfect. Some settings are stylized, Specific experimental. Some are approximate or numerical.
- Together, the paper believes the experiments suggest significant promise.
- Higher-stakes applications to politics or corporate governance remain far more speculative and not advisable without further experimentation.

Applications

- Quadratic Voting application for deciding what to eat for dinner.



COLLECTIVE DECISION ENGINES

CDE has embodied QV for market research as an algorithmic mechanism in an intuitive user interface, brought to life in a new mobile tool called weDesign.

weDesign engages respondents in an online web or phone application by which they make trade-offs that reveal their preferences and the passion behind those preferences, thus combining the best of quantitative and qualitative methodologies. The weDesign system provides a powerful new tool for anyone looking for deeper and more rigorous insights into the hivemind of any target audience. It incorporates contemporary mobile UX (as opposed to conventional market research tools), which enables the respondent to intuitively understand the underlying math using metaphors from the physical world (rubberbands, triangles, etc.) that obey quadratic rules.

Survey respondents have a budget that they allocate quadratically to buy votes for product and service attributes that they might like. The limited budget and quadratic rule cause respondents to answer survey questions in the same careful way that they allocate their household budget among different goods and service. Rather than blowing their whole budget on the one issue that is most important to them, they purchase influence in proportion to how important different categories are.

weDesign forces survey respondents to answer survey questions using the same decision-making process that they use when they create household budgets and shop for goods. This unique approach to market research promises to revolutionize the industry.

BACK

Quadratic Voting: A New Way to Govern Blockchains for Enterprises

Sherman Lee Contributor

I write about deep tech, crypto, and artificial intelligence.

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- from a trustful distributed system toward a trustless decentralized network as the underlying technology evolves.
- Vitalik Buterin of Ethereum believes quadratic voting to be a leading candidate to address governance problems blockchain communities face.