The role of compatibility in the diffusion of technologies through social networks

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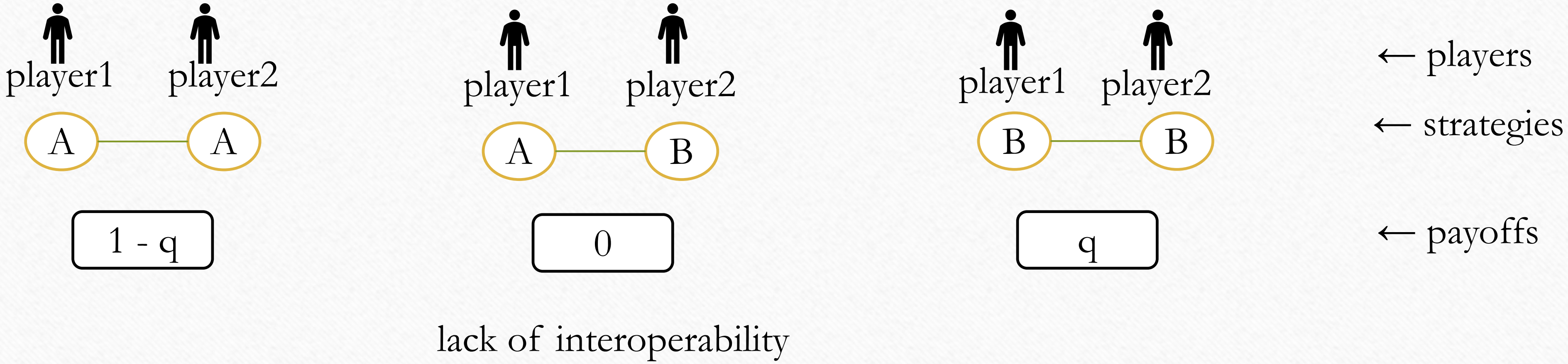
Background

- Diffusion in social network
  - Process in which new ideas, behaviors or practices diffuse through populations
  - Emergence of social norms
  - Adoption of new technologies

Background

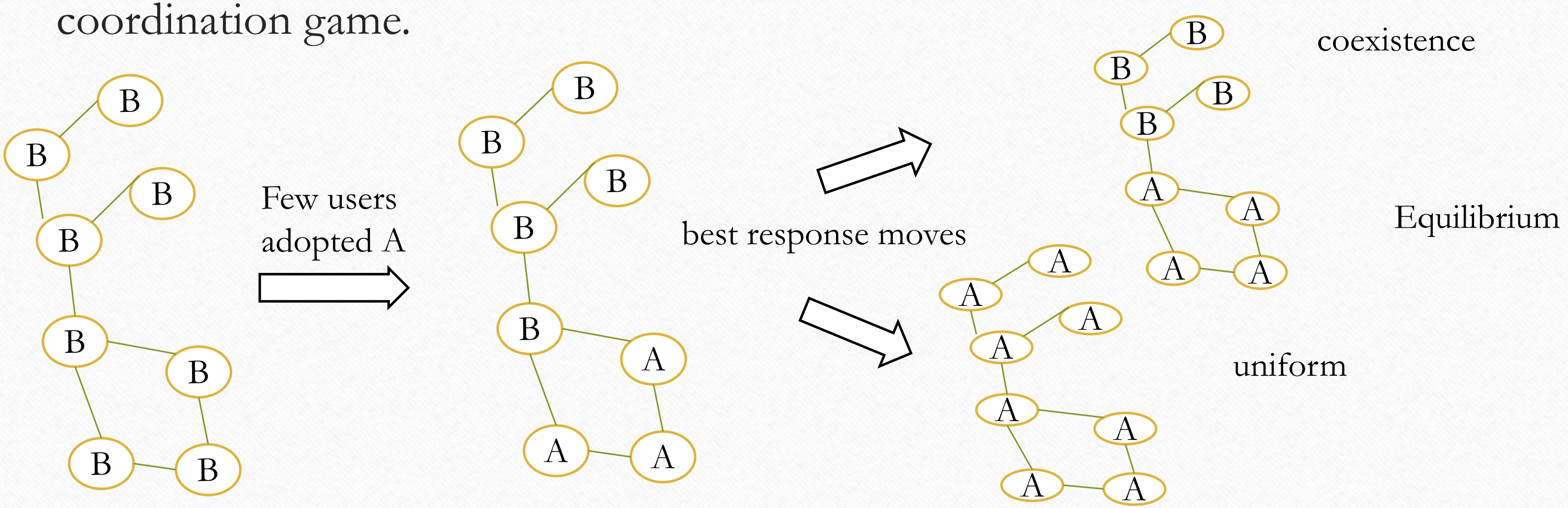
- Coordination game in social networks



If q < ½ means that A is better technology than B since A-A payoff > B-B payoff

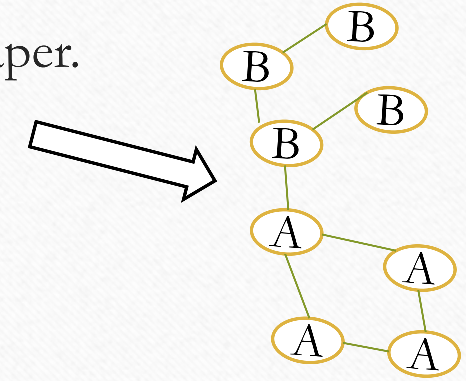
This work

- This work focuses on analyzing the dynamics of reaching equilibrium in the coordination game.



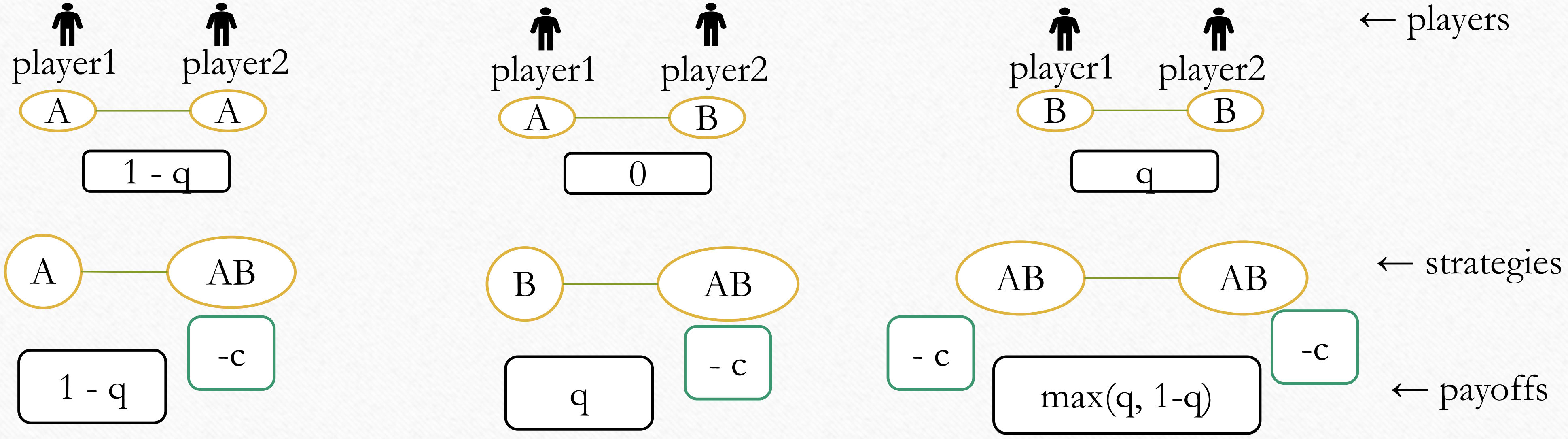
Key observation:

- coexistence is the typical outcome
  - detailed analysis of the coexistent boundary is the focus of the paper.



- individuals can become bilingual
  - people speak multiple languages
  - people have accounts on multiple online systems

Game with bilingual behavior (AB strategy)

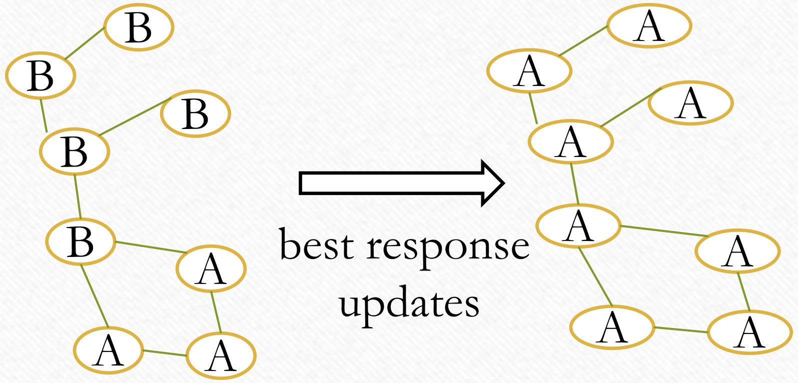


c: fixed cost penalty paid by adopter of AB

two parameters: **q, c**

Definitions

- Assume underlying graph is infinite and each node has degree Δ
- A can become *epidemic*
- r = c / Δ; penalty per edge cost

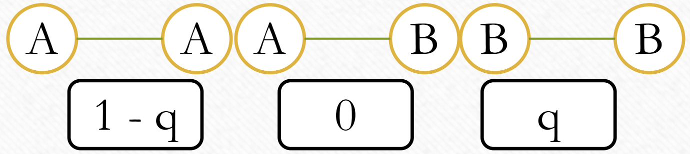


Start from state in which finite set S adopt A

A has become *epidemic*

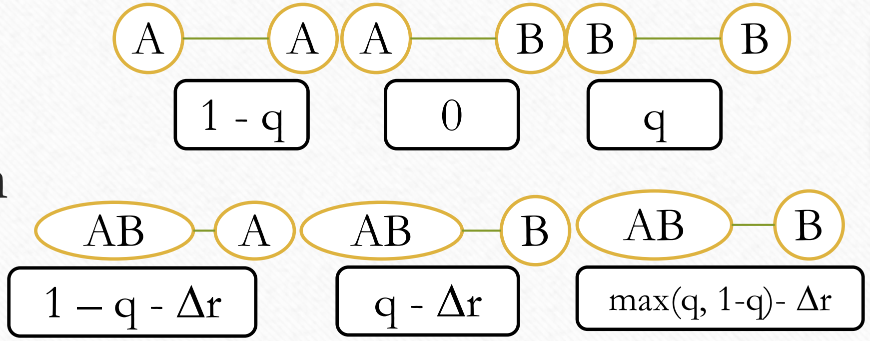
Contagion Threshold - non-bilingual model

- supremum of *q* for which A can become epidemic in G.
- sup $q^*(G) = ½$
- Alternatively, for $q > ½$, there is no graph G in which A can become epidemic.
- If $q > ½$, the already existing B-B edges has better payoff and no individual will switch to A.



Contagion Threshold - for bilingual strategy

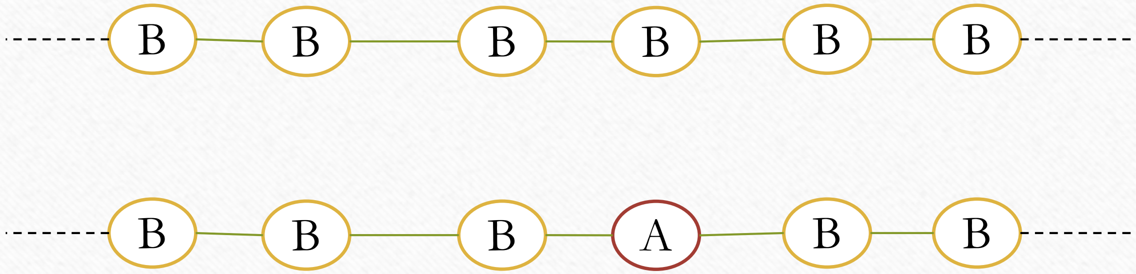
- There are two parameters in this game: ***q, r***



- So, instead of a contagion threshold we have an epidemic region in a two dimensional space.

Epidemic Region for Infinite line

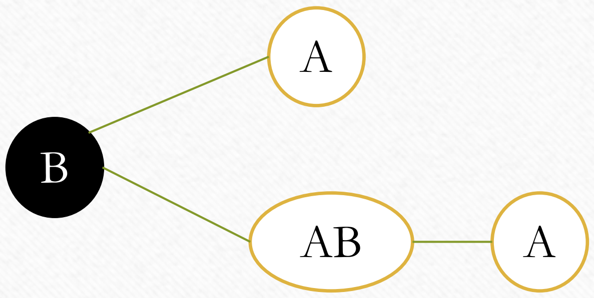
- Infinite Line:
  - Δ=2; all nodes have degree 2



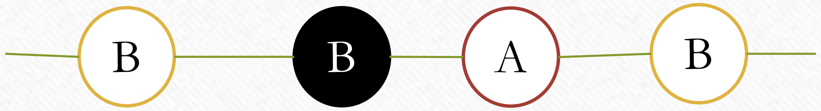
Adoption of A

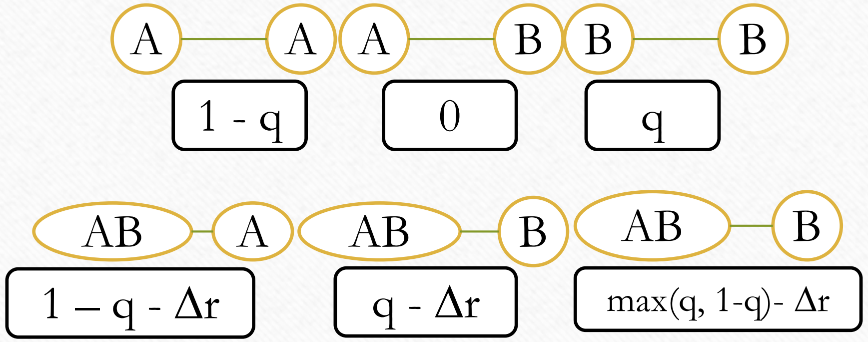
Epidemic Region for Infinite line

- Two ways for A to be epidemic:
- **Case1:** B nodes directly switch to A
- **Case2:** B nodes switch to AB then A



Epidemic Region for Infinite line



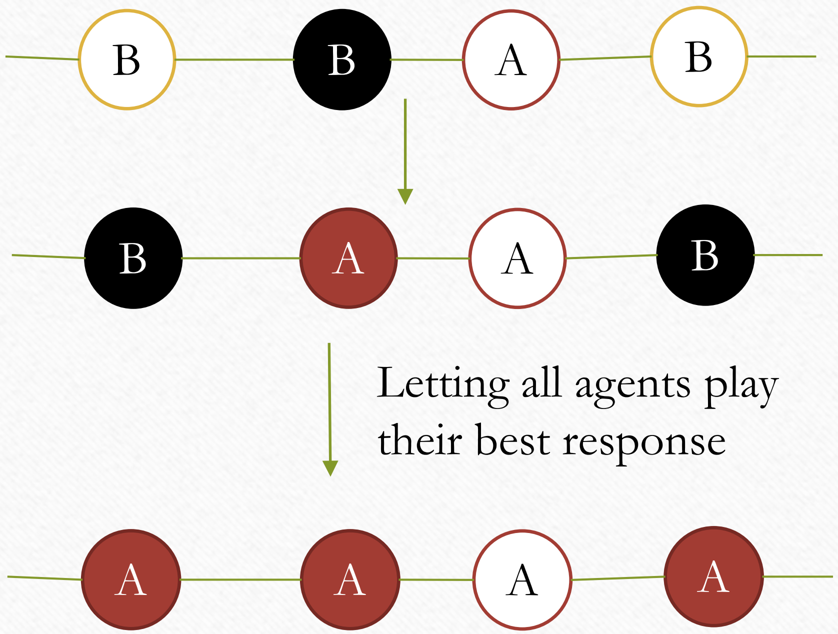


B switch to	Payoff before switch	Payoff after switch	Utility
A	B-B + B-A q + 0	B-A + A-A 0 + 1-q	**1 - q - q**
AB	B-B + B-A q + 0	B-AB + AB-A q + 1 – q - 2r	**1 - 2r - q**

Case 1: Direct A

- For B to switch to A,
  - Utility A should be positive:
    - $1 - 2q \geq 0$
    - $\boldsymbol{q \leq 1/2}$
  - Utility A is greater than utility AB:
    - $1 - q - q \geq 1 - 2r - q$
    - $\boldsymbol{q \leq 2r}$

Case 1: Direct A



Conditions

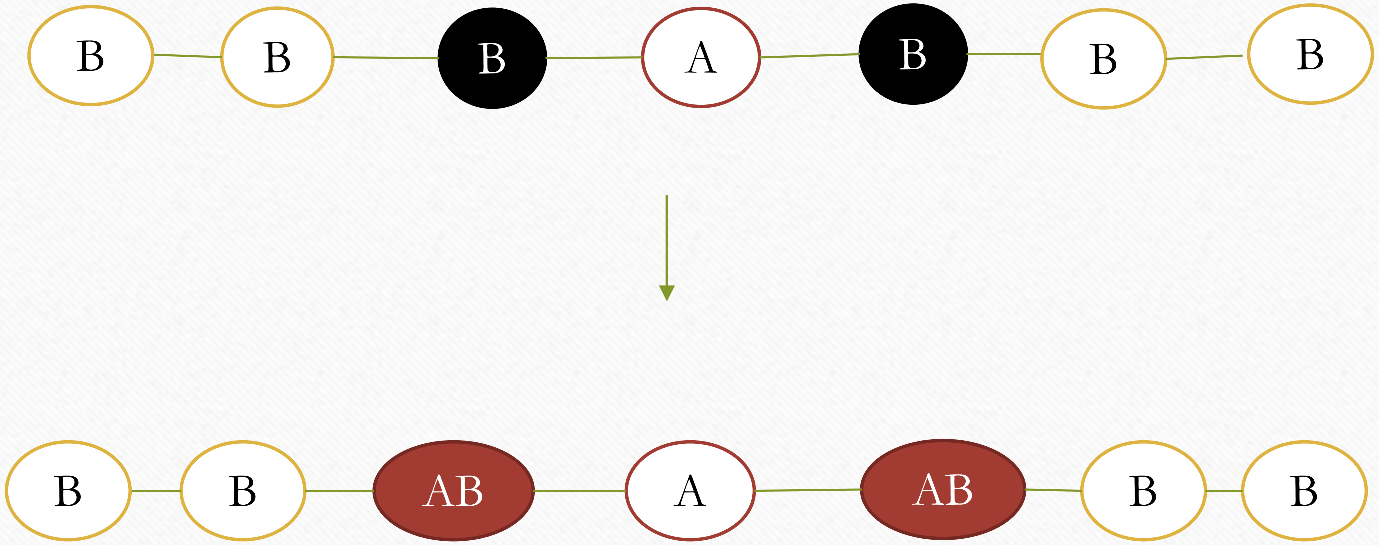
$\boldsymbol{q \leq 1/2}$

$\boldsymbol{q \leq 2r}$

Case 2: Via AB

- For B switch to AB,
  - Utility AB should be positive:
    - $1 - 2r - q \geq 0$
    - $\mathbf{1 - q \geq 2r}$
  - Utility AB is greater than utility A:
    - $1 - q - q \leq 1 - 4r - q$
    - $\mathbf{q \geq 2r}$

Case 2: Via AB

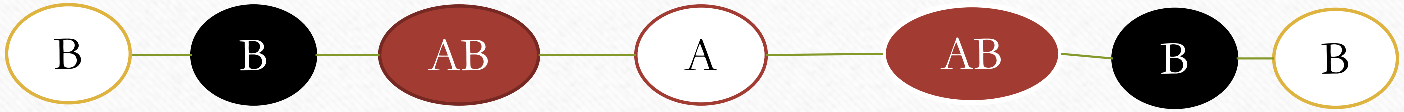


Conditions

$\mathbf{1 - \ q \geq 2r}$

$\boldsymbol{q \geq 2r}$

Case 2: Via AB



B switch to	Payoff before switch	Payoff after switch	Utility
A	AB-B + B-B q + q	AB-A + A-B 1 - q	**1 - q - 2q**
AB	AB-B + B-B q + q	AB-AB + AB-B max(q, 1-q) + q – 2r	**max(2q, 1) -2r - 2q**

Case 2: Via AB

- For B to switch to AB.
  - Utility AB should be positive:
    - $\max(2q, 1) - 2r - 2q \geq 0$
    - $1 \geq 2r + 2q$
  - Utility AB should be greater than utility A:
    - $\max(2q, 1) - 2r - 2q \geq 1 - q - 2q$
    - $1 - 2r \geq 1 - q$
    - $q \geq 2r$

Conditions

$1 - q \geq 2r$

$q \geq 2r$

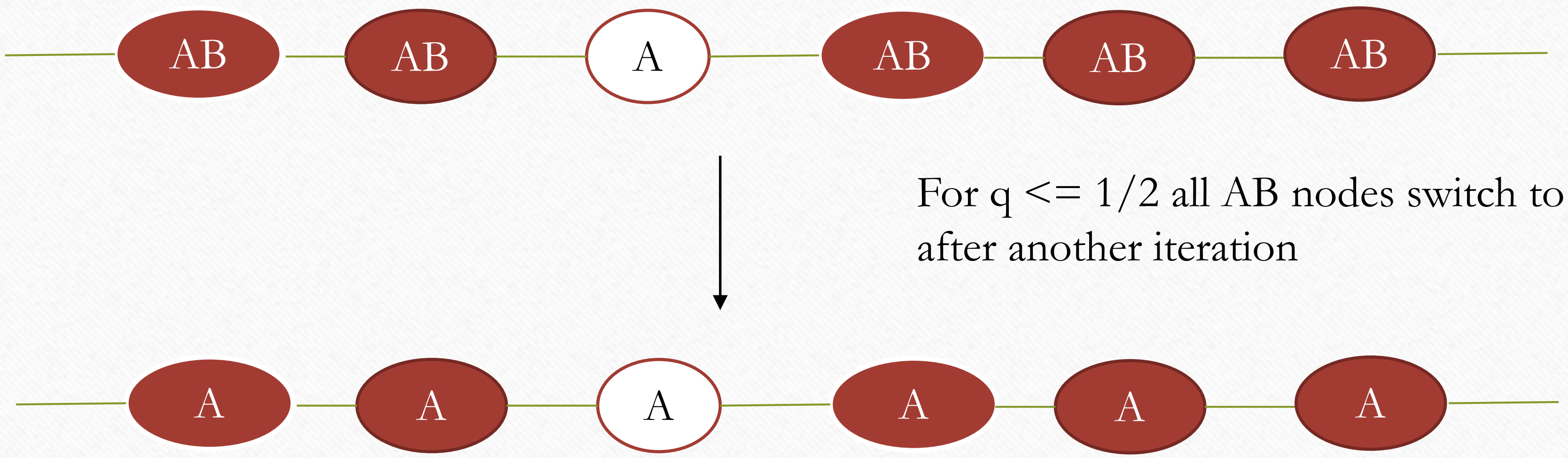
since r > 0 and for inequality to hold

$\max(2q, 1) - 2q \geq 2r \geq 0$

$\max(2q, 1) \geq 2q$

$\max(2q, 1) = 1$ and $q < 1/2$

Case 2: Via AB



For q <= 1/2 all AB nodes switch to A after another iteration

Epidemic region for infinite line

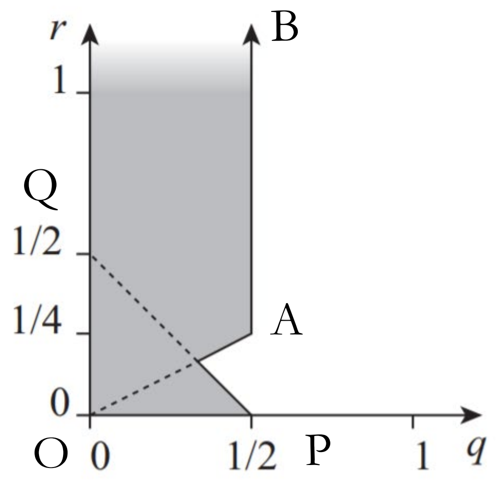


Figure 1: The region of the (q, r) plane for which technology A can become epidemic on the infinite line.

- Direct A (Region OAB)
  - $q \leq 1/2$
  - $q \leq 2r$
- Via AB (Region OPQ)
  - $1 \geq 2r + 2q$
  - $q \geq 2r$
  - $q \leq 1/2$

Epidemic Regions for other graphs

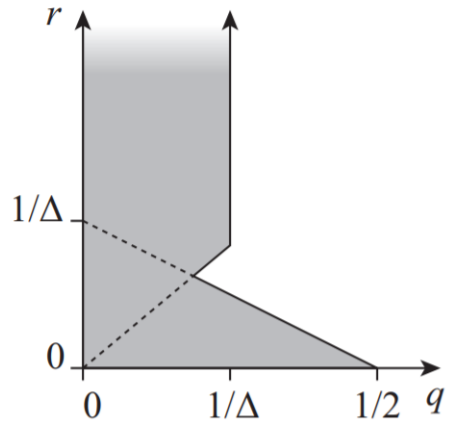


Figure 4: Epidemic regions for the infinite Δ-regular tree

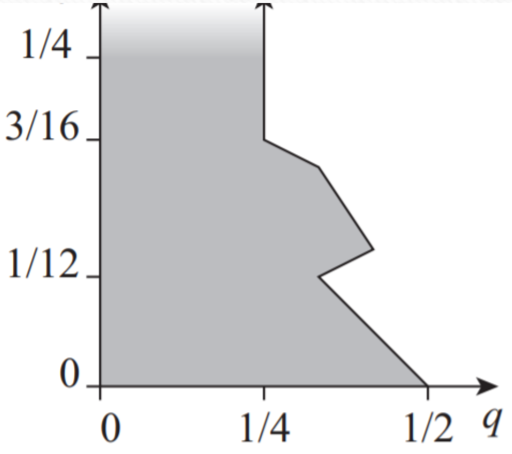


Figure 3: Epidemic regions for the infinite grid

Interpretation for epidemic region

- r is very small
  - Cheap to adopt AB, and on another update nodes switch to A (better)
- r is very large
  - Adoption cost of AB is very high, so nodes having A, B neighbors will switch to A
- r is intermediate
  - Allows a boundary of AB between adopters of A and B.

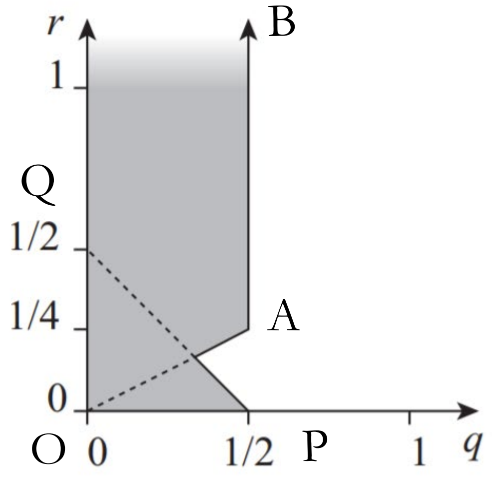


Figure 1: The region of the (q, r) plane for which technology A can become epidemic on the infinite line.

Interpretation for epidemic region

- The interesting insight is that worse technology can still survive if the cost of bilingual is in the middle interval.

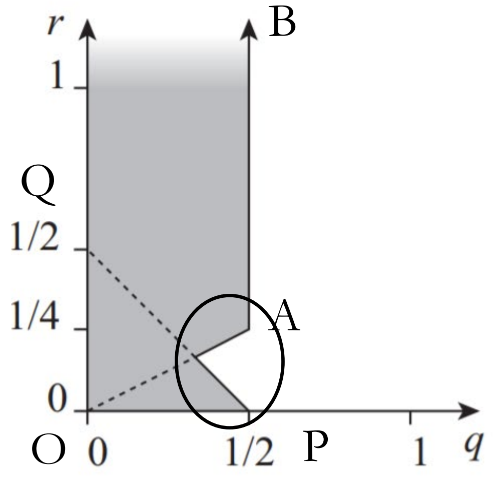


Figure 1: The region of the (q, r) plane for which technology A can become epidemic on the infinite line.

Characterization:

- Contagion games have well defined and stable equilibria.
  - If she adopts technology A she never discards it and once discards technology B, she never readapts it. Thus, after infinite best responses, each converge to a single strategy.
- Independent of the order of best responses.
  - Outcome is same for all schedules.

Blocking Structures

- A cannot become epidemic if (G, q, r) possess a certain blocking structure.
- The inequalities are linear in (q, r) so the epidemic region is the union of bounded or unbounded polygons.

DEFINITION 4.5. *Consider a contagion game (G, q, r). A pair (S_{AB}, S_B) of disjoint subsets of $V(G)$ is called a* blocking structure *for this game if for every vertex $v \in S_{AB}$,*

$$\deg_{S_B}(v) > \frac{r}{q}\Delta,$$

and for every vertex $v \in S_B$,

$$(1-q)\deg_{S_B}(v) + \min(q, 1-q)\deg_{S_{AB}}(v) > (1-q-r)\Delta,$$

and

$$\deg_{S_B}(v) + q\deg_{S_{AB}}(v) > (1-q)\Delta,$$

where $\deg_S(v)$ denotes the number of neighbors of v in the set S.

Blocking Structures

- At r->∞, inequality 1 will fail
- At r->0, inequality 2 will fail
- At q->1, inequality 2 will fail
- At q->0, inequality 3 will fail
- So, for few extreme cases A will become epidemic

DEFINITION 4.5. *Consider a contagion game* (G, q, r). *A pair* (S_{AB}, S_B) *of disjoint subsets of* $V(G)$ *is called a* blocking structure *for this game if for every vertex* $v \in S_{AB}$,

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where $deg_S(v)$ *denotes the number of neighbors of* v *in the set* S.

Modeling Compatibility and Interoperability

- Interoperability
  - positive benefit (x) in A-B interactions ($x<q<1-q$)
- Extension to three technologies

Interoperability

- The q, r terms can be rescaled in terms of x and on rescaling by addition or division the behavior of the game remains unaffected.
- x can only strict the blocking structure; if A is epidemic for G; then A is epidemic for G'.

Three technologies-Compatibility

- Suppose B and C are incumbent technologies currently in equilibrium
- If a better technology A appears, can they save themselves?
  - If B and C increases their compatibility by a calculated amount, they can resist an epidemic of A
- There are cases when compatibility is harmful to both parties.

Are the following assumptions realistic?

- All nodes have same degree but real world networks follow power law distribution.
- Every neighbor has the same influence in this model but in real-world, relationship strength might be a factor.