# Incentives, Gamification, & Game Theory: An Economic Approach to <u>Badge Design</u>

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## Before we start



# Badge

Badges, or equivalent rewards such as top-contributor lists that are used to recognize a **user’s contributions** on a site, clearly appear to be **valued by users** who actively pursue and **compete** for them.

# 1 Related Works

## Related Works

# 1.Perspective:

Individual contribution verses Contributor overall contribution

[Jain and Parkes 2008; Chen et al. 2009; Ghosh and McAfee 2011; Ghosh and Hummel 2011; Ghosh and McAfee 2012; Ghosh and Hummel 2012, 2013]

# 2.Participants:

Fixed number verses endogenous entry

[Chawla et al. 2012]

# 3.Interaction:

With the site verses among users

[Anderson et al. 2013]

## Relating Works

**1. Tournaments:**

Incentivizing users to pay effort

**2. Career choices:**

Jobs as contest for promotion – noise, indirect observation

# 2 Research Questions

## RQ1: Absolute & Relative

*What incentives are created by* ***mechanisms*** *induced by an* ***absolute standard*** *that must be met to earn a badge, and what incentives are created by a* ***relative standard***

## RQ2: How Relative?

*How exactly should competitive standards be specified: **independent** of actual contributors or **fraction** of the number of actual contributors?*

*Given that participation is **voluntary** so that these are not equivalent quantities.*

## RQ3: Absolute & Relative

*What happens if users' value from winning a badge depends on the* ***scarcity*** *of the badge? Do equilibria even exist in this setting where the value to winning a badge is determined endogenously by the* ***number of other winners***

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# Model

Definitions and Assumptions

## An Agent

Ability

Effort

Cost

Output

Payoff

Reservatio
n Value

## An Agent

**Ability**

Effort

Cost

Output

Payoff

Reservation Value

Agents are indexed by their ability over an atomless probability distribution

$$A \in [\underline{a}, \bar{a}] \subset \Re$$

- perfectly competitive framework

## An Agent

Ability

**Effort**

Cost

Output

Payoff

Reservation Value

Agents are **strategic**.
Effort is **costly**.

$$N \in (0, \infty)$$

## An Agent

Ability
Effort
**Cost**
Output
Payoff
Reservation Value

Cost is an increasing function denoted as:

$$C(N)$$

## An Agent

Ability
Effort
Cost
**Output**
Payoff
Reservation Value

-Ability independent output

$$X = N$$

-Ability dependent output

$$X = AN$$

Ability
Effort
Cost
**Output**
Payoff
Reservation Value

- System a noisy observation

$$Y = X\varepsilon$$

Where is a random noise drawn from a distribution with CDF.

**Observed Output**

## An Agent

Ability
Effort
Cost
Output
**Payoff**
Reservation Value

- the value of the badge is: $v$
- The probability an agent wins the badge given the effort $N$ is denoted as: $p_{win}$
- The payoff:

$$\pi = vp_{win} - C(N)$$

Expected value of the badge

Cost of putting N amount of effort

Observed Output

## An Agent

**Ability**
**Effort**
**Cost**
**Output**
**Payoff**
**Reservation Value**

Agents are **rational**.
They are voluntary, a strategic choice,
s.t. agent will *not participate* when

$$\max_N \pi(N) < \omega$$

The maximum amount of payoff

whereis the reservation value

**Observed Output**

## Some Math

In other words, we can say that output constitutes of:

$$y = n + a + \varepsilon$$

Observed output of a user is effort ability and noise

## Recall this three badges



## Recall this three badges




**Absolute standards mechanisms**

## Recall this three badges


**Absolute** standards mechanisms


**Relative** standards relative to fraction of **actual** contributor

## Recall this three badges


**Absolute** standards mechanisms


**Relative** standards relative to fraction of **actual** contributor


**Relative** standards relative to fraction of **potential** contributor

## Mechanisms

M1. **Absolute standard mechanisms**: $\mathcal{M}_\alpha$

when ALL contributors are rewarded a badge when observed output $Y$ exceeds some set standard $\alpha$

M2. **Relative standard mechanisms**: $\mathcal{M}_\rho$ where $\rho \in (0,1)$

Note that since not all agents contribute, $\rho$ can be:

M2a. The fraction of *actual* contributor $\mathcal{M}^c_\rho$. It rewards the top $\rho$ fraction of the contributors.

M2b. The fraction of *potential* contributor $\mathcal{M}^p_\rho$. It rewards a mass $\rho$ of contributors.

4.1

# Results

Incentives created by absolute standards

## Theorem 3.2

THEOREM 3.2 (EQUILIBRIUM EXISTENCE AND PARTICIPATION). *Consider the mechanism* $\mathcal{M}_\alpha$.

(1) *An equilibrium exists for all values of the standard* $\alpha$.
(2) *There is a threshold standard* $\alpha_{\max}$ *such that all agents participate with non-trivial effort when* $\alpha \leq \alpha_{\max}$, *and there is no participation for all* $\alpha > \alpha_{\max}$.
(3) *The highest payoff an agent can obtain when the absolute standard is* $\alpha_{\max}$, $\pi(n^*, v, \alpha_{\max})$, *is* $w$.

# Theorem 3.2

THEOREM 3.2

*(1) Participants will make a decision: whether they will participate, and if so, how much effort*

*(2) If the standard too easy to achieve, all agents will participate. Vice versa.*

*(3) The maximum standard for a participant to attend and provide max effort is when the payoff of the agent's matches his/her reservation function*

## Theorem 3.3 & 3.5

THEOREM 3.3 (EQUILIBRIUM EFFORT). *The optimal effort $n^*(\alpha)$ (assuming participation) is* non-monotone *in $\alpha$ with a unique maximum at $\alpha_{opt}$. Further, $n^*(\alpha)$ is increasing for $\alpha \leq \alpha_{opt}$ and decreasing for $\alpha \geq \alpha_{opt}$.*

THEOREM 3.5 (EQUILIBRIUM MASS OF WINNERS). *Let $m^*(\alpha) = 1 - F(\alpha - n^*(\alpha))$ denote the equilibrium mass of winners when the site chooses a standard $\alpha$.*

(1) *The mass of winners in equilibrium decreases monotonically with increasing $\alpha$: for $\alpha < \alpha_{\max}$,*

$$\frac{\partial m^*(\alpha)}{\partial \alpha} < 0.$$

(2) *The fraction of winners at $\alpha_{\max}$, the highest absolute standard at which agents participate, satisfies $m^*(\alpha_{\max}) > 0$, while the fraction of winners converges to one as the standard $\alpha$ diverges to $-\infty$.*

## Theorem 3.3 & 3.5

THEOREM 3.3 & 3.5

*(1) If the task is too easy, the user would use less their his/her optimal effort.*

*(2) If the task is too hard, since the user would not gain as much value as intended. The harder it gets, the less participants will attend*

*(3) Thus, there exist an optimal set standard.*

## In short

Designers should be cautious about setting the optimal standard when designing badges.

4.
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# Results

Incentives created by relative standards

## Lemma 4.1, Theorem 4.2, 4.3

LEMMA 4.1 (IMPLEMENTATION VIA $\alpha$). *Consider the mechanism $\mathcal{M}_\rho$ for any value of $\rho \in (0,1)$. There exists a pure-strategy equilibrium in $\mathcal{M}_\rho$ if and only if there exists a pair of values $(\alpha^*(\rho), n^*)$ that simultaneously satisfy the following two equations:*

$$v f(\alpha^*(\rho) - n^*) - c'(n^*) = 0 \tag{4}$$

$$1 - F(\alpha^*(\rho) - n^*) = \rho, \tag{5}$$

*and the inequality $\pi(n^*, \alpha^*(\rho)) \geq w$. That is, if there exists an equilibrium of $\mathcal{M}_\rho$ at $\rho$, there is a standard $\alpha^*(\rho)$ such that an agent obtains a reward if and only if her observed output exceeds $\alpha^*(\rho)$, i.e., if $y \geq \alpha^*(\rho)$.*

## Lemma 4.1, Theorem 4.2, 4.3

THEOREM 4.2. *(Existence) There exists a unique pure-strategy equilibrium in the mechanism $\mathcal{M}_\rho$ for all values of $\rho \in [\rho_{\min}, 1)$, where $\rho_{\min} = m^*(\alpha_{\max})$ is the fraction of winners in the absolute standard mechanism $\mathcal{M}_\alpha$ when $\alpha = \alpha_{\max}$.*

THEOREM 4.3 (PARTIAL EQUIVALENCE BETWEEN ABSOLUTE AND RELATIVE STANDARDS). *There is a range of values $(-\infty, \alpha_{\max}]$ and $[\rho_{\min}, 1)$ for which there is an equivalence between $\mathcal{M}_\alpha$ and $\mathcal{M}_\rho$ in the following sense: for every $\alpha \in (-\infty, \alpha_{\max}]$ there is a unique value of $\rho \in [\rho_{\min}, 1)$ such that agents choose exactly the same equilibrium effort under $\mathcal{M}_\alpha(\alpha)$ and $\mathcal{M}_\rho(\rho)$, and vice versa.*

## Lemma 4.1, Theorem 4.2, 4.3

LEMMA 4.1, THEOREM 4.2, 4.3

*(1) If the fraction is non-zero for relative standards, there will always be participants. Recall that if the set standards were too high in the absolute case, no participants would attend.*

*(2) For every absolute standard that have participants, there will be an identical ratio in the relative standards case that yields the same results.*
*That is, you can always design a relative standard badge that matches identically with a absolute standard badge.*

## Lemma 4.4, 4.5

LEMMA 4.4 (NONEXISTENCE OF EQUILIBRIA IN $\mathcal{M}_\rho^c$). *Consider the relative standards mechanism $\mathcal{M}_\rho^c$, which rewards the top $\rho$ fraction of all* contributors. *There exists no equilibrium in $\mathcal{M}_\rho^c$ for $\rho < \rho_{\min} = m^*(\alpha_{\max})$.*

LEMMA 4.5 (MIXED-STRATEGY EQUILIBRIA IN $\mathcal{M}_\rho^p$). *Consider the relative standards mechanism $\mathcal{M}_\rho^p$, which rewards the top $\rho$ fraction of the population. There exists a mixed-strategy equilibrium for all $\rho \in (0, \rho_{\min}]$ with non-zero participation probability $p$ and non-zero effort $N$.*

## Lemma 4.4, 4.5

LEMMA 4.4, 4.5

*(1) If there is <u>no fix fraction</u> of potential participants, there is an interval of this fraction that does not yield an equilibrium.*

*(2) If there is <u>a fix fraction</u> of potential participants, even though the strategies that contributors would take is <u>mixed strategies, there is still an equilibrium.</u>*

*(3) The two different kind of relative standard mechanism behaves identical outside this "unstable fraction"*

## In short

Designers should prefer relative badge design, especially Top X model, when he/she is unsure of the optimal standard.

# 4.3 Results

Value depending on fraction of winners

## Theorem 5.1, 5.2

THEOREM 5.1. *Consider the mechanism $\mathcal{M}_\alpha$, and suppose an agent's value from winning is* $v(m)$.

(1) *A* unique *equilibrium exists for every* $\alpha$.
(2) *If* $\alpha \geq \alpha_{\max}$, *then there is no participation in equilibrium. If* $\alpha < \alpha_{\max}$ *then there is participation in equilibrium.*
(3) *There exists a solution* $m^{a**}$ *to* $m^a = 1 - F(\alpha - n^*(\alpha, v(m^a)))$. *If* $\pi(\alpha, n^*(\alpha, v(m^{a**})), v(m^{a**})) \geq w$, *then the equilibrium is a pure-strategy equilibrium where all agents participate; otherwise, the equilibrium is in mixed strategies.*

THEOREM 5.2. *Suppose that the value of winning depends on the fraction of the population which wins. Consider the relative standards mechanism* $\mathcal{M}^p_\rho$, *which rewards the top* $\rho$ *fraction of the population. If* $\rho \leq \rho_{\min}(\rho)$ *then there exists a mixed-strategy equilibrium with non-zero participation probability* $p$ *and non-zero effort. If* $\rho > \rho_{\min}(\rho)$ *then there exists a pure-strategy equilibrium.*

## Theorem 5.1

THEOREM 5.1, 5.2

*If the value of the badge depends on the number of badge winners,*

*(1) Both* $\mathscr{M}_\alpha$ *and* $\mathscr{M}_\rho$ *exists equilibria for all values of* $\alpha$ *and* $\rho$.

## Lemma 5.3

LEMMA 5.3. *If $v(\cdot)$ is strictly convex, then uncertainty about the mass of winners with a correct mean increases equilibrium effort. If $v(\cdot)$ is strictly concave, then uncertainty about the mass of winners with a correct mean reduces equilibrium effort.*

LEMMA 5.3

*If the user does not know the total number of winners,*

*(1) The more information you have, the less effort you require*

*(2) Vice versa*

*However, in most real word scenario, the more you know about the number of participants having a badge, the more information you have*

## In short

How the participants value the badge affects strongly with the final outcome

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# Thoughts

Value depending on fraction of winners

## Summary

1. Absolute standards
2. Relative standards
3. Value change on scarcity

## Critics

1. There is only one badge in this experiment.
   Badge-badge relationship? / number of badge

2. Noise capturing method.

3. Assuming everyone value of badge equally

4. Does not provide the way to calculate optimality of badge design

## Critics

5. It seems like people often design absolute standard badges?

# Facebook Badges

New Rising Star Badge

This badge recognizes new members who ma
engaging posts within their first month.

See All Rising Stars

You're a Conversation Starter

You're great at making posts that people find valuable. Your conversation starter badge will show up next to your name on comments and posts.

New Visual Storyteller Badge

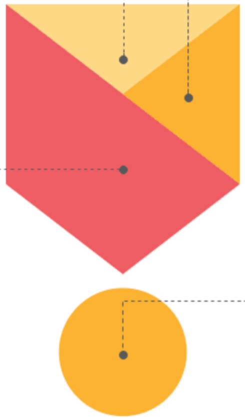
This badge recognizes people who consistently share images or videos that people value.

See All Visual Storytellers

# Airbnb Badge

## How to Become a Superhost

Every host has their own style, but these are the Superhost basics.



### Hosting Experience

Superhosts develop their hospitality style by welcoming guests frequently. You must host 10 trips within the last year.

### High Response Rate

Being responsive shows guests they have your support. Superhosts maintain a 90% response rate or higher by responding to guests quickly.

### 5-Star Reviews

Superhosts provide listings that inspire enthusiastic reviews. At least 80% of your reviews need to be 5 stars.

### Commitment

Superhosts don't cancel confirmed reservations unless there are extenuating circumstances.

## Pokémon Badge

## Reddit Badge

## Related Works

1. Immorlica et.al “Social Status and Badge Design”
   - Optimal mechanism is a leaderboard with a cutoff
   - if status valuations are concave: coarse status partition
   - if status valuation are convex: partition user in status class

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## Questions for discussion

1. What are other factors that can affect the incentives of badges for users?

2. Are game badges different from gamification badges?

3. Alternative badge designs

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