Spring 2011, CS 598CSC: Approximation Algorithms

Homework 3
Due: 03/30/2011 in class

Instructions and Policy: Each student should write up their own solutions independently. You need to indicate the names of the people you discussed a problem with; ideally you should discuss with no more than two other people.

Solve as many problems as you can. I expect at least three.

Please write clearly and concisely - clarity and brevity will be rewarded. Refer to known facts as necessary. Your job is to convince me that you know the solution, as quickly as possible.

Problem 1 Problem 2.1 from Shmoys-Williamson book.

Problem 2 Problem 5.4 from Vazirani book.

Problem 3 Problem 4.1 from Shmoys-Williamson book.

Problem 4 We consider the non-metric facility location problem. We are given a set of clients $\mathcal{D}$ and a set of facilities $\mathcal{F}$. Each facility $i \in \mathcal{F}$ has a cost $f_i \geq 0$ for opening it. There is a non-negative cost $c(i, j)$ to connect client j to facility i. The goal is to open a set of facilities and connect each client to an open facility so as to minimize the sum of the facility opening cost and the connection costs of the clients. In the non-metric version we do not assume that the $c(i, j)$ values form a metric so they can be arbitrary non-negative numbers.

- Show that an $\alpha(|\mathcal{D}|)$-approximation for non-metric facility location implies an $\alpha(n)$-approximation for the set cover problem with n elements.
- Given an $O(\log |\mathcal{D}|)$-approximation for the non-metric facility location via the LP relaxation we discussed in class with y_i variables for the facilities and $x_{i,j}$ variables for connecting clients to facilities. You can use the following hints.
  - Let $\bar{y}, \bar{x}$ be an optimum solution for the LP realxation. For each client j define $\alpha_j = \sum_i c(i, j)x_{i,j}$ to be the LP connection cost.
  - Reduce the problem to set cover (via the standard LP relaxation for it) as follows. For each client j restrict it to be connected to only those facilities i where $c(i, j) \leq 2\alpha_j$.

Problem 5 Recall the Maximum Independent Set (MIS) problem for the intersection graphs of disks in the Euclidean plane: Given a set of disks in the plane, construct a graph by creating a vertex for each disk, and connecting two vertices by an edge if the corresponding disks intersect. Assume as before that all disks have unit radius. We say that a solution X to the Independent Set problem is s-optimal if we cannot get a larger independent set by removing at most s vertices from X and adding at most $s+1$ vertices from $V-X$. Consider the following local search algorithm for Independent Set in unit disk graphs: Start with an arbitrary solution, and as long as the current solution is not s-optimal, find a larger independent set by removing at most s vertices and adding at most $s+1$. Prove that there is a (small) constant s such that the local search algorithm gives a constant-factor approximation. Try to make the approximation ratio as small as you can.