Spring 2016, CS 583: Approximation Algorithms

Homework 3

Due: 3/18/2016

Instructions and Policy: Each student should write up their own solutions independently. You need to indicate the names of the people you discussed a problem with; ideally you should discuss with no more than two other people.

Read through all the problems and think about them and how they relate to what we covered in the lectures. Solve as many problems as you can. Please submit your solutions to at least 4 problems.

Please write clearly and concisely - clarity and brevity will be rewarded. Refer to known facts as necessary. Your job is to convince me that you know the solution, as quickly as possible.

Problem 1 In the Rectangle Independent Set problem (RIS) we are given a collection of n axis-parallel rectangles $\mathcal{R} = \{R_1, \ldots, R_n\}$ in the plane and the goal is to find a maximum cardinality subset of the rectangles that do not overlap. In this problem you will derive an $\Omega(1/\log n)$-approximation. Given a horizontal line L let $\mathcal{R}_a$ be the set of rectangles in $\mathcal{R}$ that lie above L, and let $\mathcal{R}_b$ be the set of rectangles that lie below L, and let $\mathcal{R}_c$ be the set of rectangles that intersect L.

- Describe a polynomial-time algorithm that finds an optimum solution for rectangles in $\mathcal{R}_c$.
- Prove that, given $\mathcal{R}$, there is a line L that can be found in polynomial-time such that $|\mathcal{R}_a|$ and $|\mathcal{R}_b|$ are both at most $\lceil n/2 \rceil$.
- Use the above two parts to design a divide and conquer style algorithm that achieves the desired $\Omega(1/\log n)$-approximation.
- Does the algorithm extend to the weighted case?

Problem 2 Problem 2.1 from Williamson-Shmoys book.

Problem 3 Problem 2.10 from Williamson-Shmoys book.

Problem 4 For the same problem as in the preceding one consider a local-search algorithm that start with an arbitrary set $S \subseteq E$ of k elements and does a swap if it improves the value. Prove that this gives a 1/2-approximation. *Hint: Set up an (arbitrary) matching between a local optimum S and an optimum solution S^* and consider the swaps corresponding to this matching.*

Problem 5 Problem 9.2 from Shmoys-Williamson book. However, there is a slight typo in the data given for the problem. Assume, that for all facilities, the opening cost is 2. The distances c_{ij} remain as given in the problem.

Problem 6 Let $G = (V, A)$ be a directed graph with arc weights $c : A \to \mathcal{R}^+$. Define the density of a directed cycle C as $\sum_{a \in C} c(a)/|V(C)|$ where $V(C)$ is set of vertices in C.

1. A cycle with the minimum density is called a minimum mean cycle and such a cycle can be computed in polynomial time. How?

   Hint: Given density λ, give a polynomial-time algorithm to test if G contains a cycle of density $< \lambda$. Now use binary search.

2. Consider the following algorithm for ATSP. Given G (with c satisfying asymmetric triangle inequality), compute a minimum mean cycle C. Pick an arbitrary vertex v from C and recurse on the graph $G' = G[V - C \cup \{v\}]$. A solution to the problem on G can be computed by patching C with a tour in the graph G'. Prove that the approximation ratio for this heuristic is at most $2H_n$ where $H_n = 1 + 1/2 + \ldots + 1/n$ is the nth harmonic number.

Problem 7 For Metric-TSP consider the nearest neighbour heuristic discussed in class. Prove that the heuristic yields an $O(\log n)$ approximation. (Hint: use the basic idea in the online greedy algorithm for the Steiner tree problem from Lecture 1 (Spring 2011)). **Extra Credit:** Give an example to show that there is no constant c such that the heuristic is a c-approximation algorithm.