Reverse Auction & Routing Games.

Thursday, October 28, 2021 1:53 PM

★ last lec: Spectrum Auctions. (FCC)

item = (location, freq band) ⇒ Heterogeneous items ⇒ Combinatorial auctions.

⇒ VCG is not applicable.

★ Simultaneous Ascending Auctions (sell sing-items seperates in English auction format)

\+ package bidding.

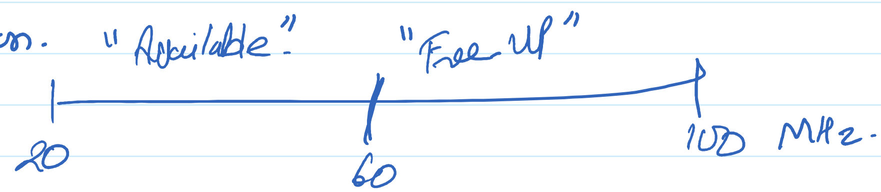
★ Since 2014: FCC does two step process

$\boxed{\text{Step 1: Reverse auction (buy spectrum)}}$

Step 2: Forward " (sell ") ✓

→ Based on "Demand" & also all are willing to sell / relinquish

At the location.



Step 1:

A: set of agents

agent i has value "v_i" for her spectrum. (Private)

b_i: bid of agent i

r_i: range it owns.

⇒ If we buy out $S \subseteq A$ then we have to pack

$(A \setminus S)$ in the "available" range.

→ We say S is a "feasible" set of winners if $A \setminus S$ can be packed in the "available" range.

Algo (Reverse Auction)

1. Init $S = A$. ($\Rightarrow A \setminus S = \emptyset$ & hence feasible)
2. While $\exists\, i \in S$, s.t. $\boxed{S' = S \setminus \{i\} \text{ is "feasible"}}$

   ↳ Remove one such i from S → (*)

   End while
3. Declare S as winners.

(*) is underdetermined, How to ensure monotone allocation rule?

→ If all agents are "equal" then remove the "highest bidder"

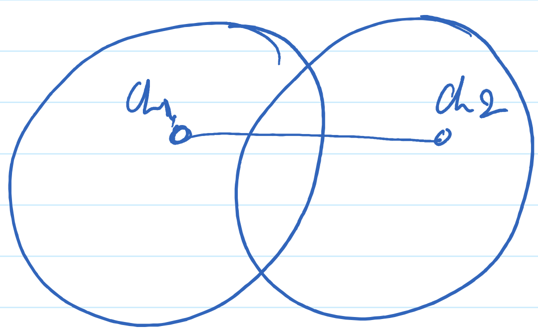
OR

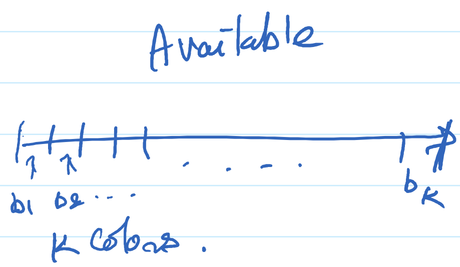
→ Per-capita highest bid

→ f(bid, r_i, what-type-of-spectrum).

☆ Re-packing $A \setminus S'$ ($S' = S \cup \{i\}$): packing + coloring.

NP-Land.



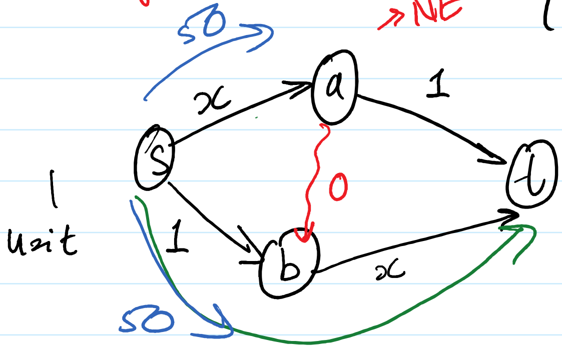


Is this graph k-colorable?

SAT solvers.

(nonatomic) Routing Games: Braess's Paradox (1968)

$\rightarrow$ NE



OPT = NE

$$cost^{pp}(50,50) = \frac{50}{100} + 1 = 1.5$$

$$cost(50,50) = 1.5 \times 100 = 150$$

$$cost^{p}(s-a-b-t) = \frac{50}{100} + \frac{51}{100} = 1.01$$

NE

$$cost^{pp}(NE) = \frac{100}{100} + \frac{100}{100} = 2.$$

99: s-a-b-t

1: s-b-t

$$cost^{b} = 1 + \frac{100}{100} = 2$$

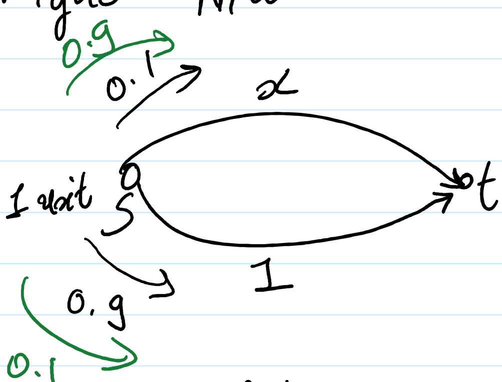
$$cost(NE) = 200$$

$$cost(OPT) = 150.$$

What is OPT? (50, 50)

$$\text{"Price-of-Anarchy"} = \frac{cost(worst\ NE)}{cost(OPT)} = \frac{\cancel{200}^{4}}{\cancel{150}_{3}} = \frac{4}{3}$$

★ Piguo N/w.



$$cost(x, 1-x) = x \cdot x + (1-x) \cdot 1$$
$$= x^2 + 1 - x.$$

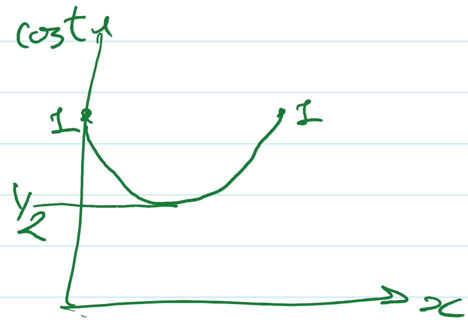
NE: No "infinitesimal" flow can change their path and reduce their cost.

0.1 vs 0.9

$\hookrightarrow$ cost 0.1 $\quad$ $\hookrightarrow$ cost 1.

$NE = (1, 0) \qquad cost(1,0) = 1$

$$OPT = \arg\min_{x} \; (x^2 + 1 - x)$$



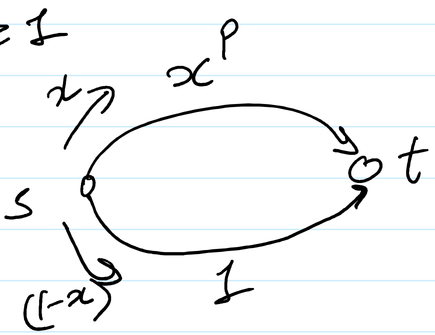
$$\frac{d}{dx} \; '' = 2x - 1 = 0$$
$$\Rightarrow x = 1/2$$

$$OPT = (1/2, 1/2) \qquad cost\,(OPT) = \frac{1}{4} + 1 - \frac{1}{2} = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$$

$$PoA = \frac{cost(NE)}{cost\,(OPT)} = \frac{1}{3/4} = \boxed{\frac{4}{3}}$$

Co-incidence?

☆ $p \geq 1$



$$cost(x, 1-x) = x \cdot x^p + (1-x) \cdot 1$$
$$= x^{p+1} + 1 - x$$

$$NE = (1, 0), \quad cost(NE) = 1$$

$$OPT = \arg\min_{x} \; x^{p+1} + 1 - x$$

$$\frac{d}{dx} \; '' = (p+1)x^p - 1 = 0$$
$$\Rightarrow x = \left(\frac{1}{p+1}\right)^{1/p}$$

$$cost\,(OPT) = \lim_{p \to \infty} \underset{\downarrow}{\left(\frac{1}{p+1}\right)^{1+\frac{1}{p}}} + 1 - \underset{\downarrow}{\left(\frac{1}{p+1}\right)^{1/p}}$$

$$\underbrace{\underset{0}{\downarrow} \qquad\qquad \underset{1}{\downarrow}}$$

$$\downarrow$$

$$0 + 1 - 1 = 0$$

$$PoA = \frac{cost(NE) \Rightarrow 1}{cost(OPT) \Rightarrow 0} \to \infty$$

Conclusion: Definitely the degree of the cost-functions matter.
Does the complexity of n/w matter?

Goal. <u>NO!!</u>

★ Set up
- A directed n/w $G = (V, E)$
- s, t special nodes in V.
- r-units of flow has to go from s to t.
- For each edge $e \in E$, cost function $c_e : \mathbb{R}_+ \to \mathbb{R}_+$

  $c_e \in \mathcal{C}$ $\downarrow$ non-dec, nonneg, continuous.

<u>Thm</u>: Given a class $\mathcal{C}$ of cost functions, Among all n/ws with edge costs from $\mathcal{C}$, "Pigou-like" n/w has the worst PoA.