Mechanism Design

Thursday, October 7, 2021 1:56 PM

Designing "rules of the game" to achieve desired outcomes.

☆ Auctions

- Single seller / auctioneer.

Single item to sell. (indivisible)

- set of agents / bidders / players : $1, \dots, n$

Bayesian / incomplete info.

agent i has value v_i for item. Private information

☆ Sealed Bid Auction :

① Auctioneer solicits bids from the agents

agent i submits bid b_i in a "sealed envelope"

b_i need not be v_i

② After looking at the bids auctioneer decides

"Winner" & "Payment"

- Winner = The highest bidder $\arg\max_i b_i$

- Payment P =

auctioneer's revenue.

Goal of auctioneer : max s.w. = give item to $\arg\max_i v_i$

⇓

Utility of agent i = $v_i - P \geq 0$ if i is the winner

$= 0$ o.w.

Winner = Highest bidder

Payment Rules

	1	2	3
v_i	1200	200	200
	$\boxed{900}$	170	200

Truthful

Pay the highest bid = FP Bid$_i$

Pay the highest bid = FP Bid_i

pay the second highest bid = SP Bid_i

$\boxed{900}$ 170 200

Truthful

$\boxed{1200}$ 200 200

☆ First price : Highest bidder wins, pays her bid.

$n = 2$. Suppose $b_2 = \frac{V_2}{2}$ $\quad V_1, V_2 \sim U[0, 1]$

if 1 bids b_1, then $U_1(b_1) = (V_1 - b_1) \Pr[b_1 \geq b_2] + 0 \Pr[b_1 < b_2]$

$= (V_1 - b_1) \Pr\left[\frac{V_2}{2} \leq b_1\right]$

$= (V_1 - b_1) \Pr[V_2 \leq 2b_1]$

$= (V_1 - b_1) \cdot 2b_1$

$$\underset{b_1 \in [0,1]}{\arg\max} \ (V_1 - b_1) \cdot 2b_1$$

$$\frac{dU_1}{db_1} = 2V_1 - 4b_1 = 0$$

$$b_1 = \frac{V_1}{2}$$

Similarly, if $b_1 = \frac{V_1}{2}$ then $b_2 = \frac{V_2}{2}$ is best for player 2.

$\left(\frac{V_1}{2}, \frac{V_2}{2}\right)$ is a NE.

☆ In general : n bidders $\quad V_1, \ldots, V_n \sim U[0, 1]$

Suppose fix $b_i = \frac{(n-1) V_i}{n}$ $\forall i > 1$.

then we can show that $b_1 = \frac{(n-1) V_1}{n}$ is the

Then we can show that $b_i = \frac{(n-1)\, v_i}{n}$ is the best strategy for player 1.

$\left(\frac{n-1}{n} v_1, \ldots, \frac{n-1}{n} v_n\right)$ is a NE.

- $v_1, \ldots, v_n$ may come from a complex distribution
  " from different " "

★ **Second Price**: Highest bidder wins & pays the second highest bid.

Thm (Vickrey '61): For each agent i, $b_i = v_i$ is the best strategy no matter what others bid.

$$\Updownarrow$$

$b_i = v_i$ is the dominant strategy &

$(v_1, \ldots, v_n)$ is the DSE.

$v_i - (\lambda b_i + (1-\lambda) B_i)$

Pf: Fix an agent i & bids b_K $\forall K \neq i$.

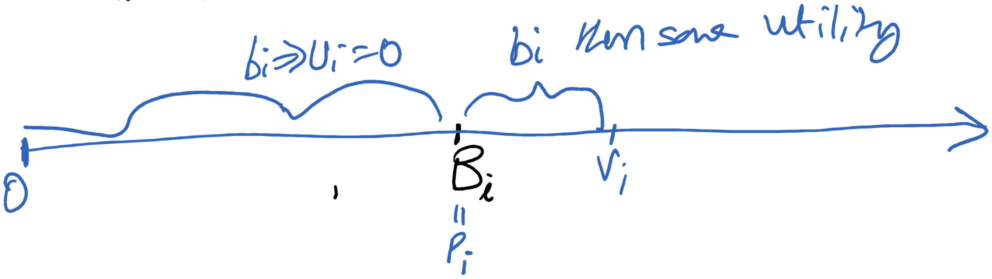
Agent i bids b_i

Case I: i wins $\Rightarrow$ $U_i = v_i - \max_{K \neq i} b_K$ $\rightarrow$ B_i

payment is independent of your bid

Case II: i looses $\Rightarrow$ $U_i = 0$

$b_i = v_i$ is best $\Leftarrow$



$b_i = v_i$ is best $\Leftarrow$

v_i $\quad$ B_i

$b_i \Rightarrow U_i = 0$ same as $(b_i = v_i)$ $\qquad$ $b_i \Rightarrow U_i = -ve$

☆ EBay = English auction

→ start with very low price where probably everyone wants to buy. And keep increasing until only one remains

$v_1 \ge v_2 \ge \dots \ge v_n$ $\qquad$ Price

second price auction

all n agents $\ge p_0$

n_1 agents p_1

$\vdots$

1 agent p_k

☆ Dutch auction $\rightsquigarrow$ essentially first price auction.

$v_1 \ge \dots \ge v_n$

$p_0 \ge$ →
