1. (Sipser #8.17) Let $A$ be the language of properly nested parentheses. For example, $(())$ and (((()))) in $A$, but ) is not. Show that $A$ is in $L$.

2. (Sipser #9.19) Define the unique-sat problem to be $USAT = \{ \langle \varphi \rangle : \varphi$ is a boolean formula that has a unique satisfying assignment $\}$. Show that $USAT \in P$.

3. (Arora-Barak Problem 6.3) Describe a decidable language in $P/poly$ that is not in $P$.

4. (Sipser #9.13, #9.14): Consider the function $\text{pad} : \Sigma^* \times \mathbb{N} \rightarrow \Sigma^*\#^*$ that is defined as follows. Let $\text{pad}(s, \ell) = s\#^j$, where $j = \max(0, \ell - m)$ and $m$ is the length of $s$. Thus $\text{pad}(s, \ell)$ simply adds enough copies of the new symbol $\#$ to the end of $s$ so that the length of the result is at least $\ell$. For any language $A$ and function $f : \mathbb{N} \rightarrow \mathbb{N}$ define the language $\text{pad}(A, f(m))$ as $\text{pad}(A, f(m)) = \{ \text{pad}(s, f(m)) : s \in A, m = |s| \}$

(a) Prove that, if $A \in \text{TIME}(n^6)$, then $\text{pad}(A, n^2) \in \text{TIME}(n^3)$.

(b) Recall that $\text{EXP} = \text{TIME}(2^{\text{poly}(n)})$, and define $\text{NEXP} = \text{NTIME}(2^{\text{poly}(n)})$. Prove that, if $\text{EXP} \neq \text{NEXP}$, then $P \neq \text{NP}$.

5. (Sipser #9.16) Prove that $\text{TQBF} \notin \text{SPACE}(n^{1/3})$.

6. (Sipser #9.24, #9.25)

(a) Define the function $\text{majority}_n : \{0, 1\}^n \rightarrow \{0, 1\}$ as $\text{majority}_n(x_1, \ldots, x_n) = 1$ iff $\sum_i x_i \geq n/2$. Thus the $\text{majority}_n$ function returns the majority vote of the inputs. Directly show that the $\text{majority}_n$ function can be computed by size $O(n^2)$ size circuits.

(b) Recall that you may consider circuits that output strings over $\{0, 1\}$ by designating several output gates. Let $\text{add}_n : \{0, 1\}^{2n} \rightarrow \{0, 1\}^{n+1}$ take the sum of two $n$-bit binary integers and produce the $n + 1$ bit result. Show that you can compute the $\text{add}_n$ function with $O(n)$ size circuits.

(c) By recursively dividing the number of inputs in half, and using part (b), show that the $\text{majority}_n$ function can be computed by size $O(n \log n)$ size circuits.