# Interactive Proofs

Lecture 19
And Beyond

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- Perfect completeness: One-sided-error-AM = AM
  - Similar to BPP ⊆ $\Sigma_2^P$ (yields MAM protocol; MAM=AM)

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- Similarly, MA $\subseteq \Sigma_2^P$

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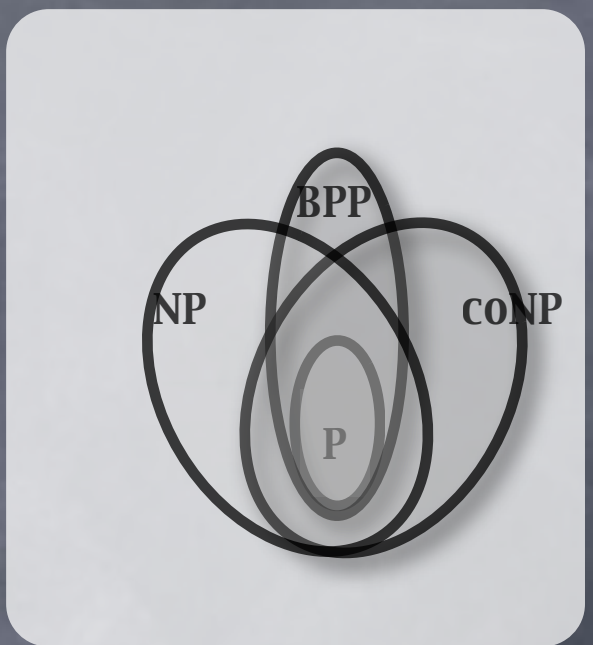


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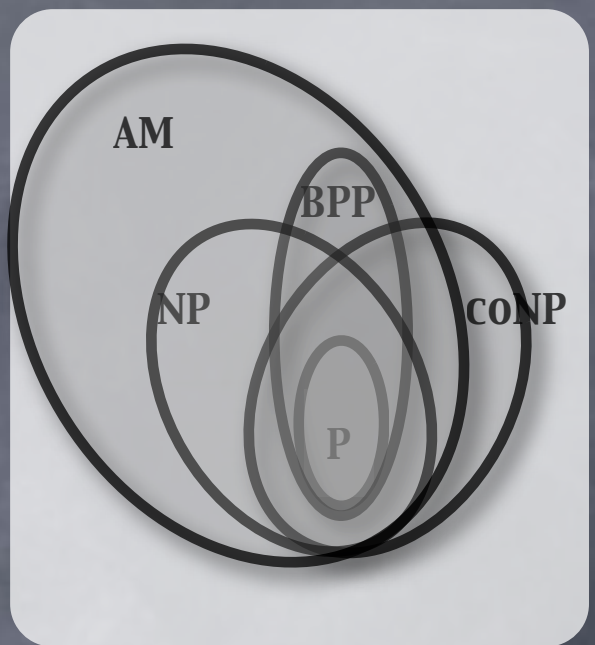


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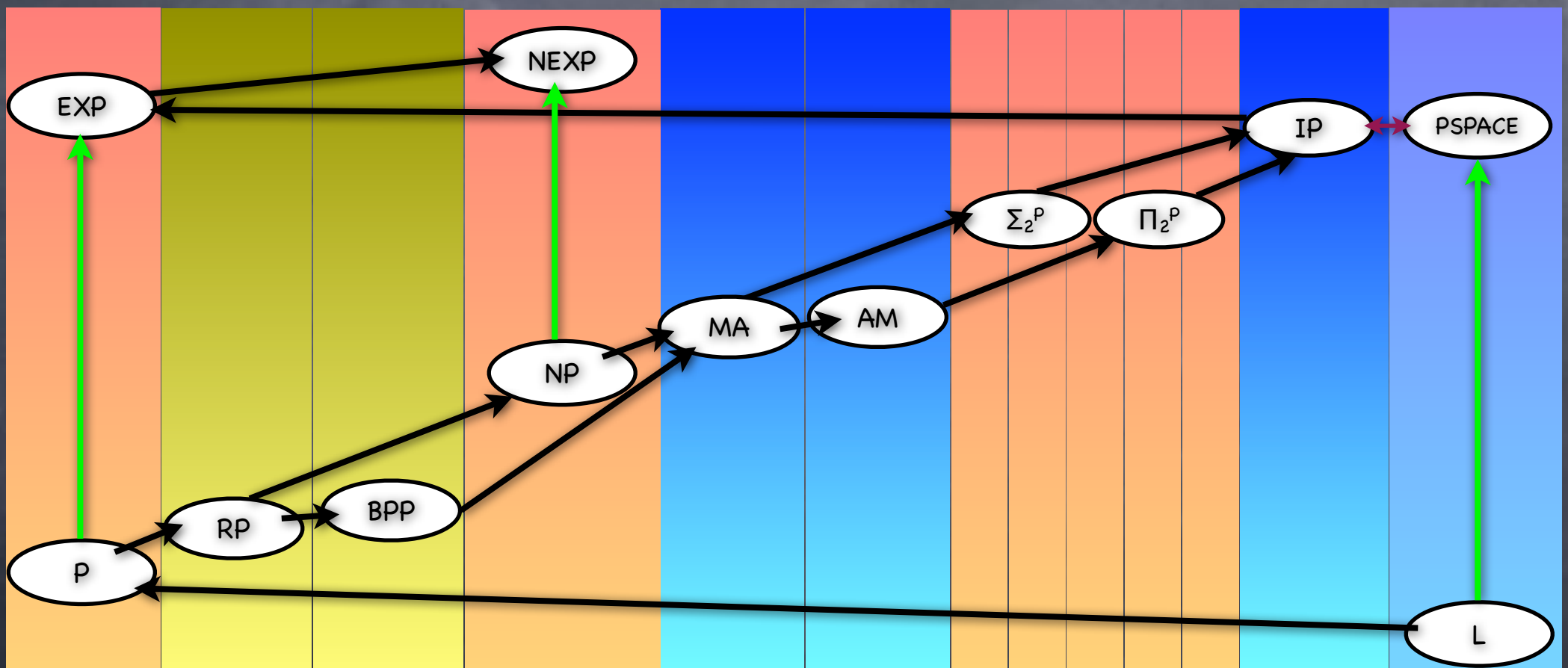
# Zoo



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- User: In fact I just care if it works correctly on the inputs I want to solve. Maybe for each input I have, your machine could prove correctness using an IP protocol?
- Vendor: But I don't have a (nano-bio-quantum) implementation of the prover's program...

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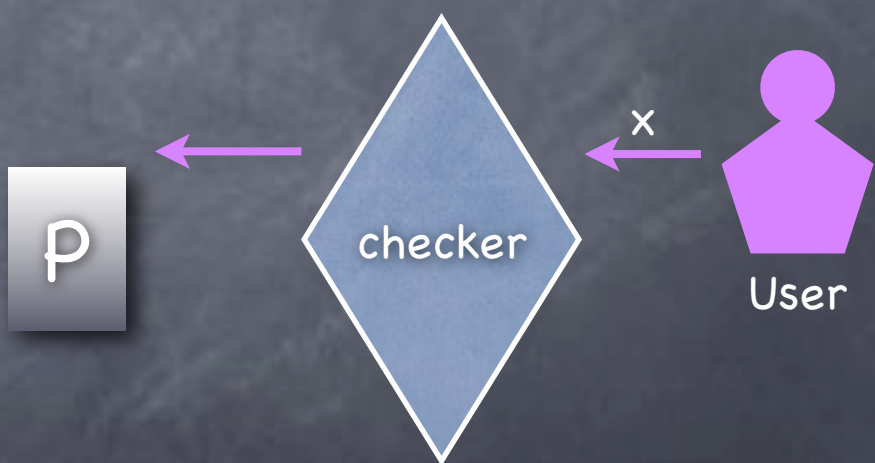

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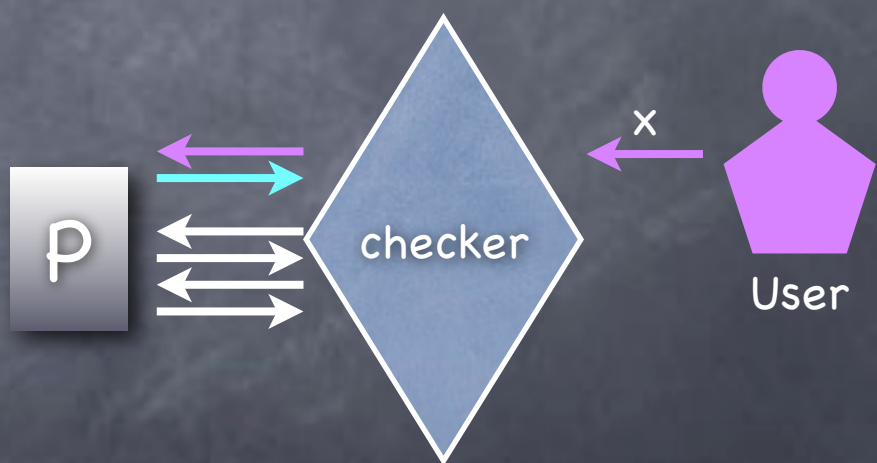

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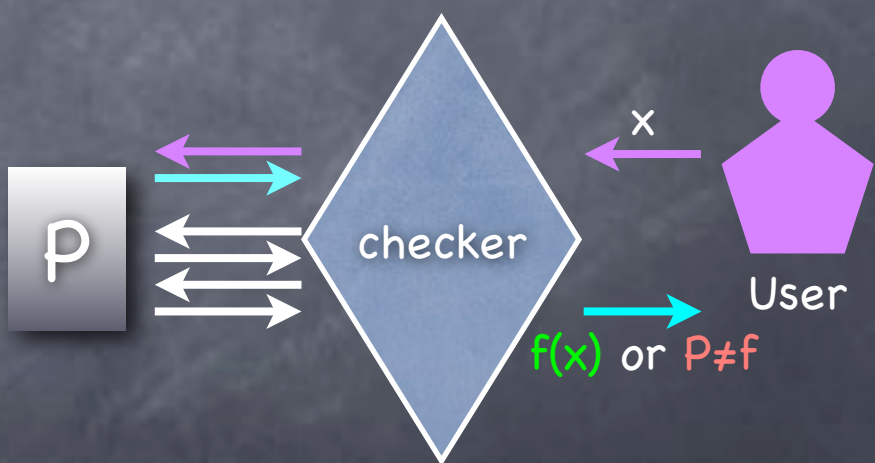

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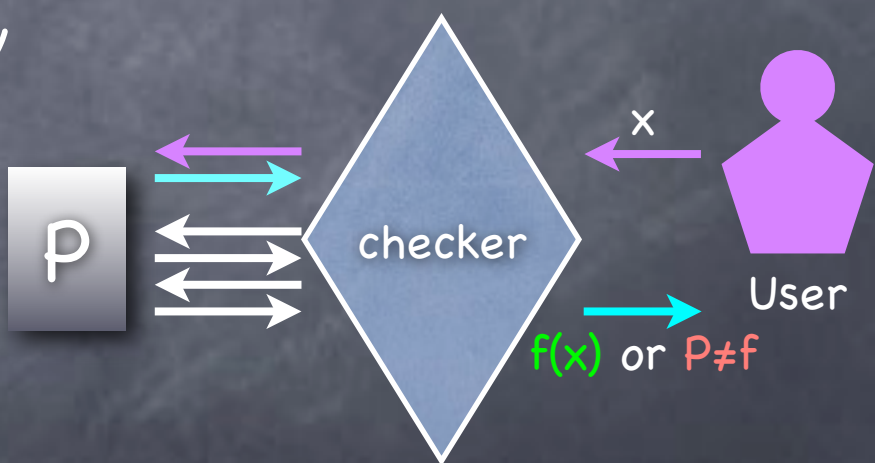

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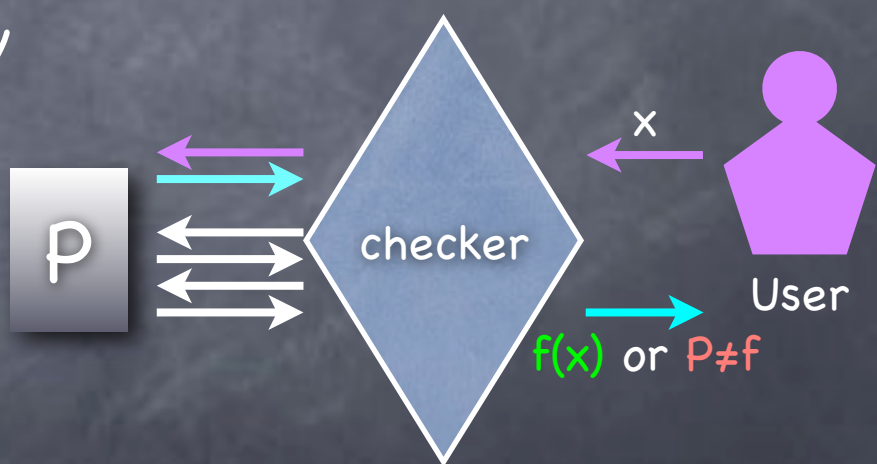

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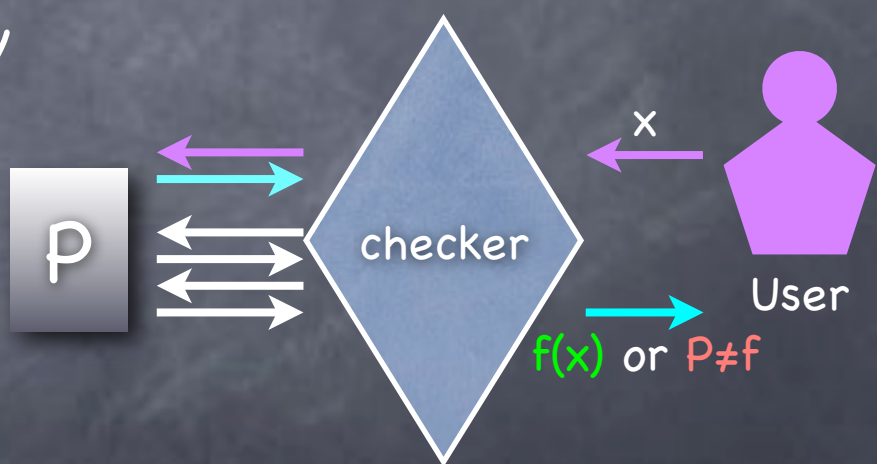

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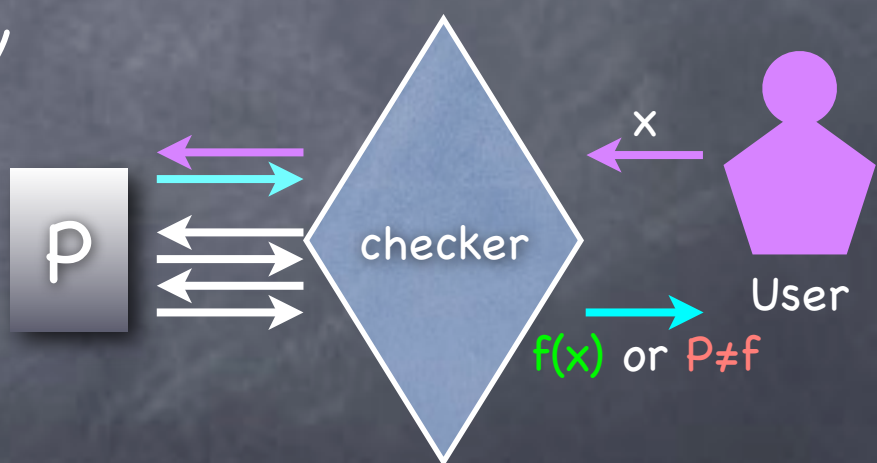
- Will consider boolean f (i.e., a language L)



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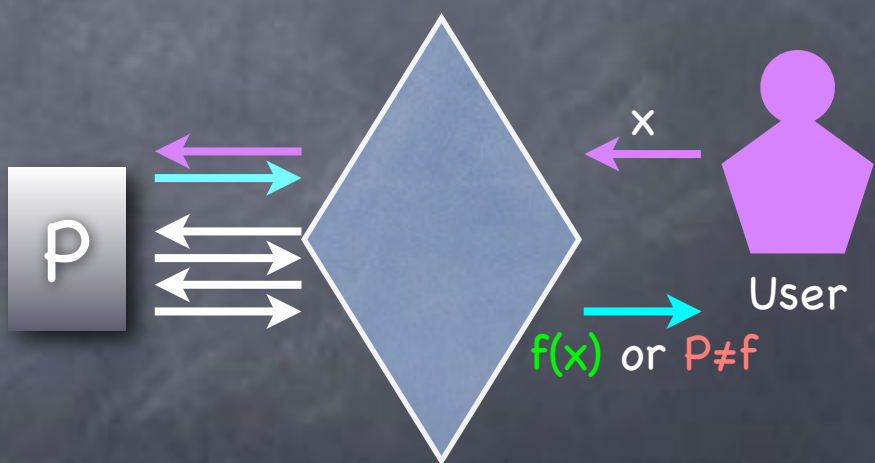

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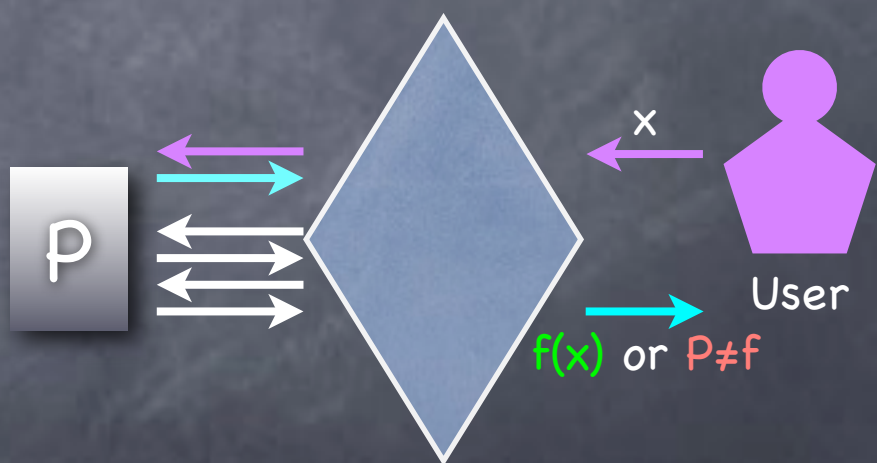

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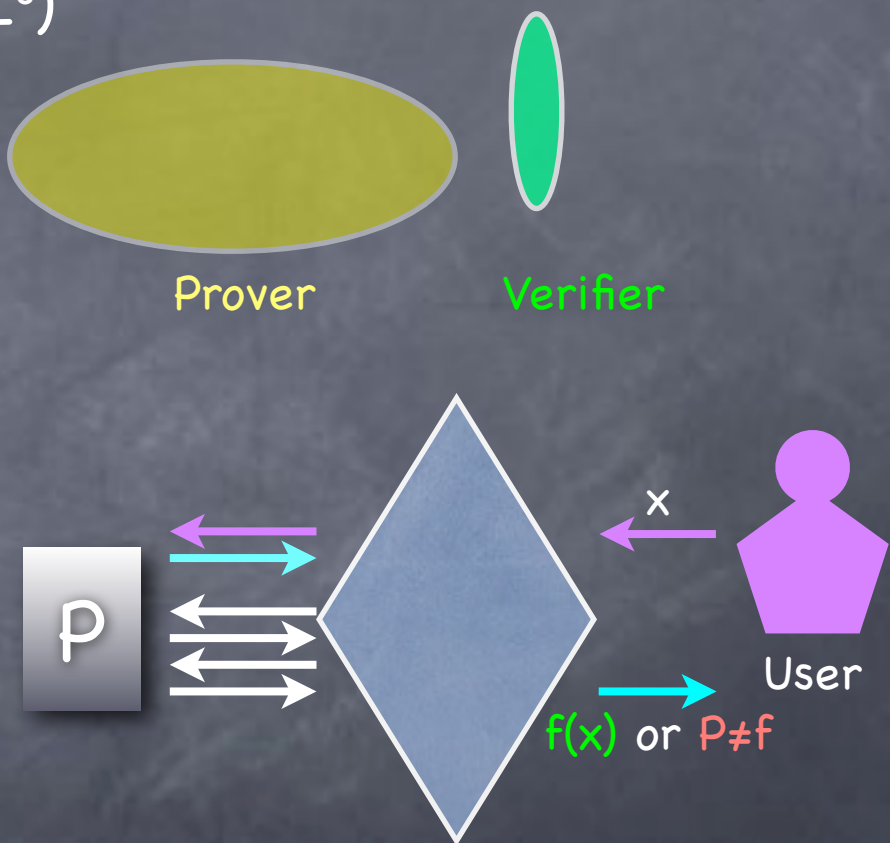

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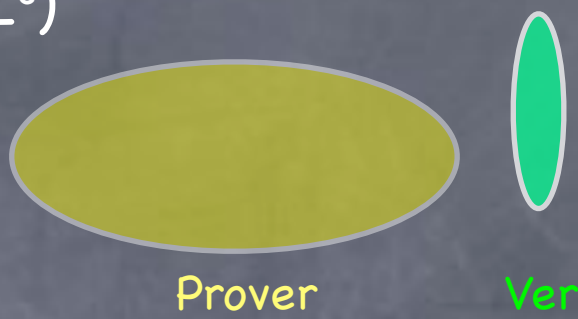


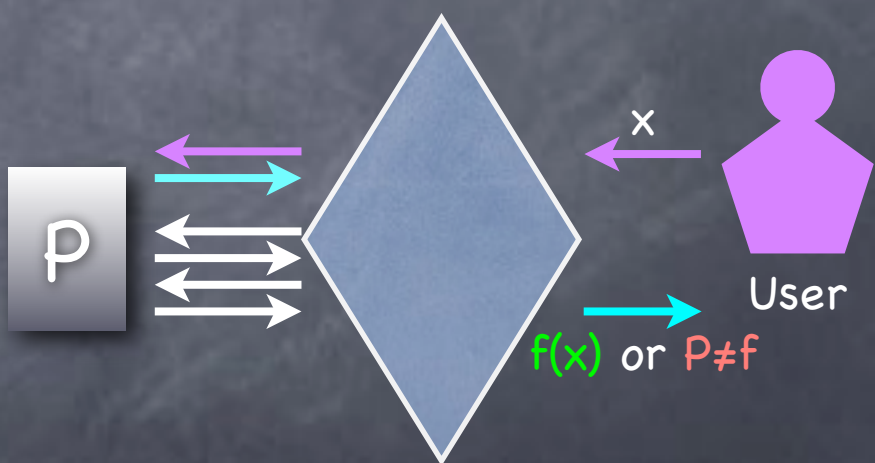

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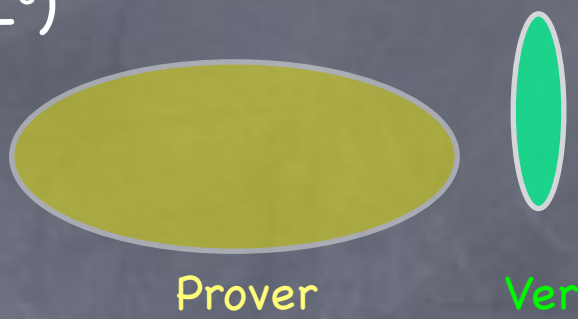


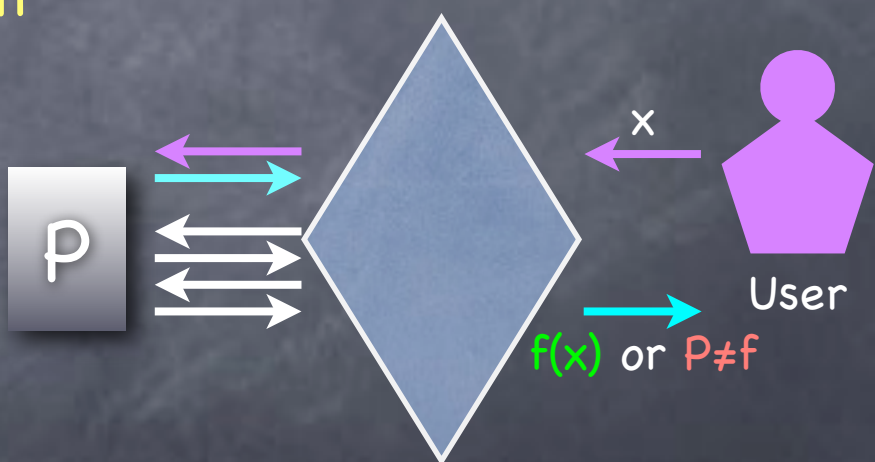

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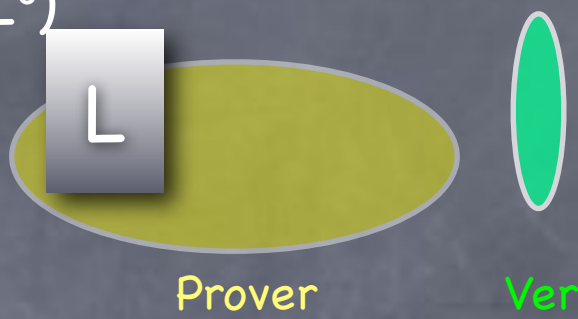


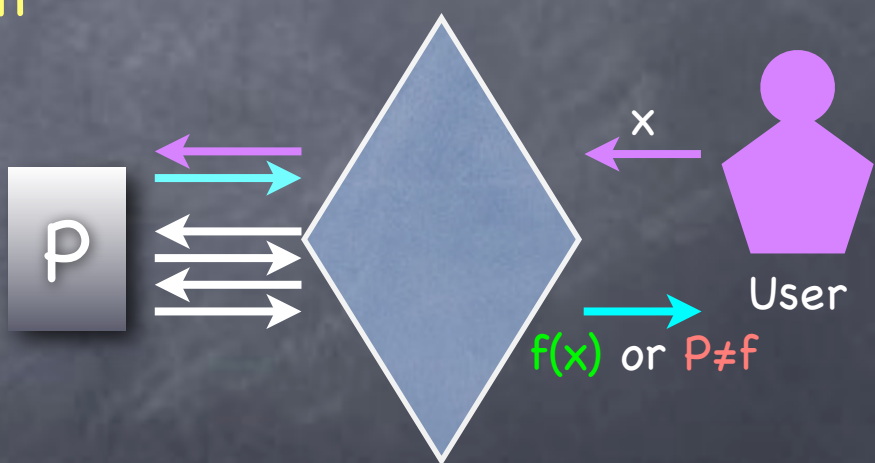

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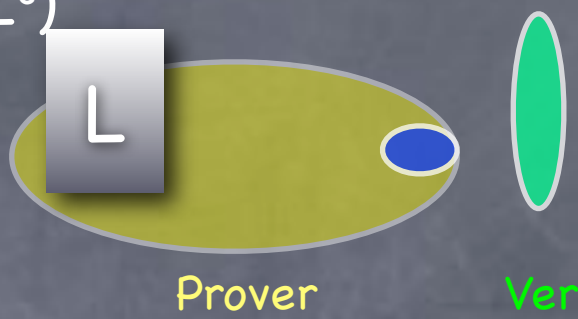


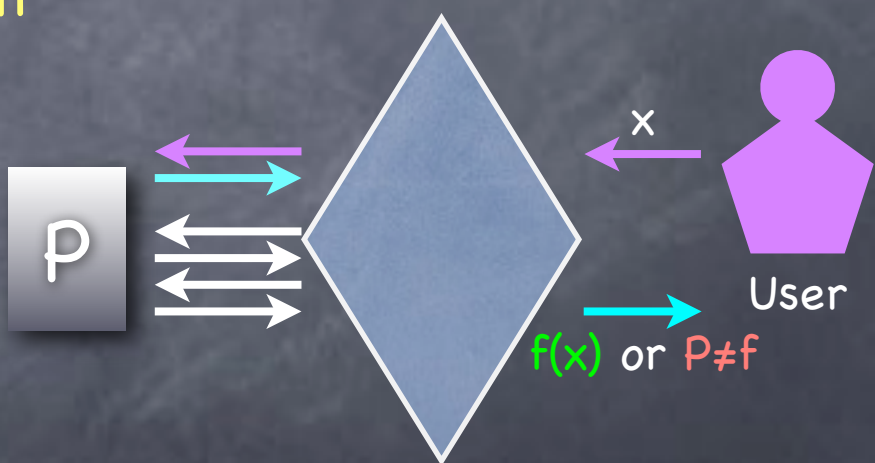

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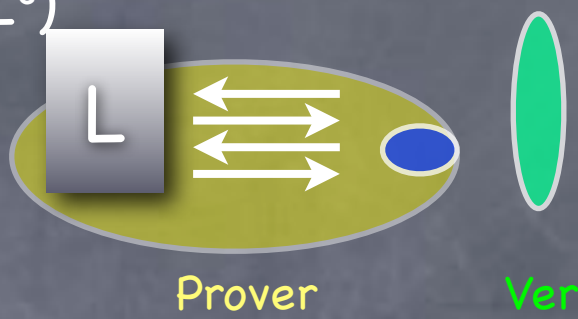


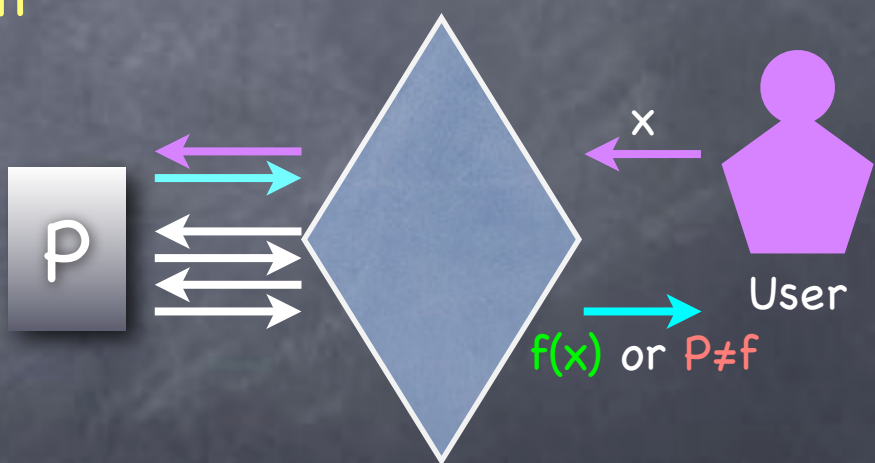

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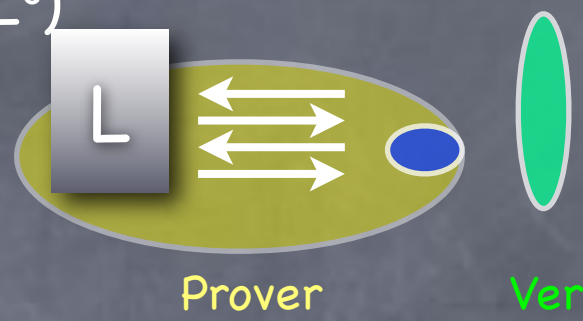


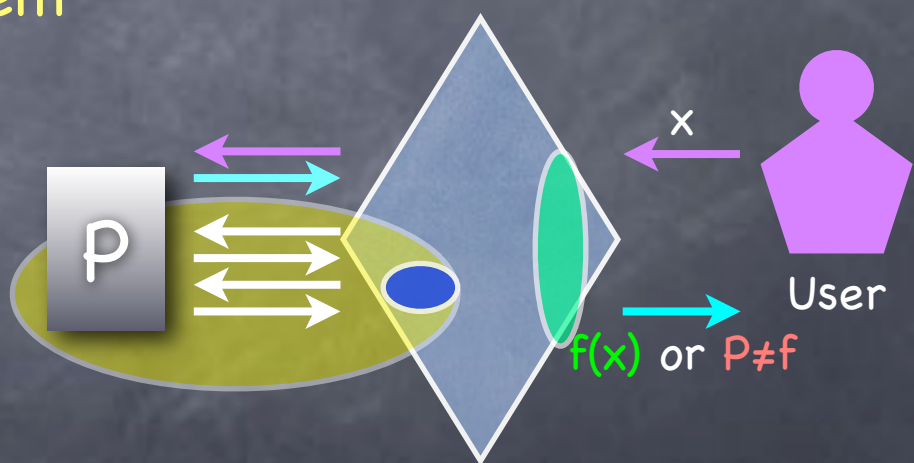

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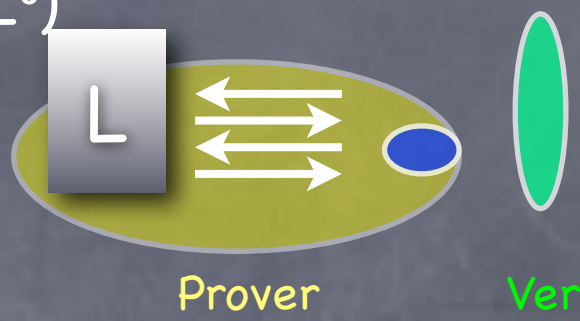


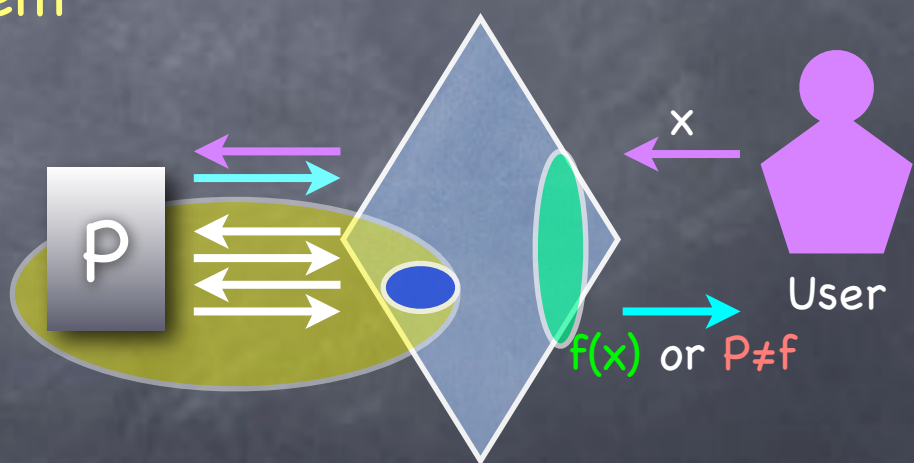

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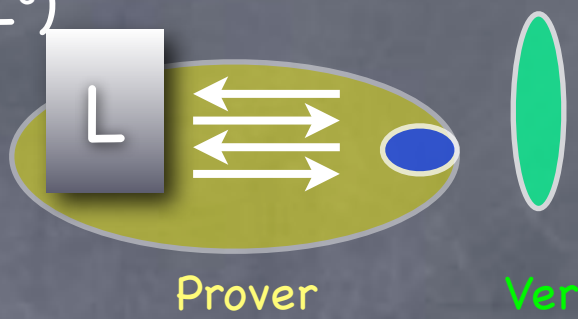


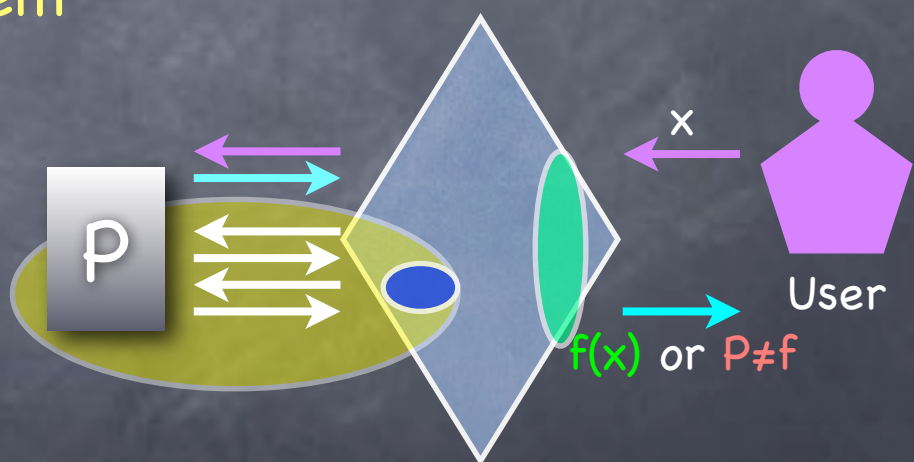

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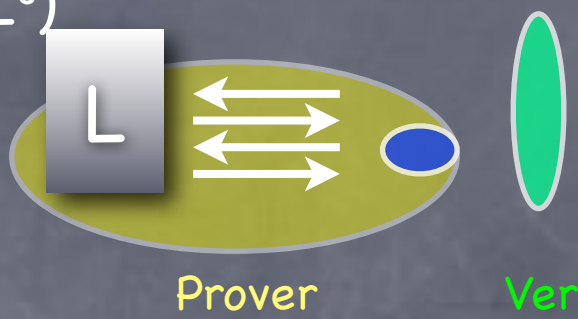
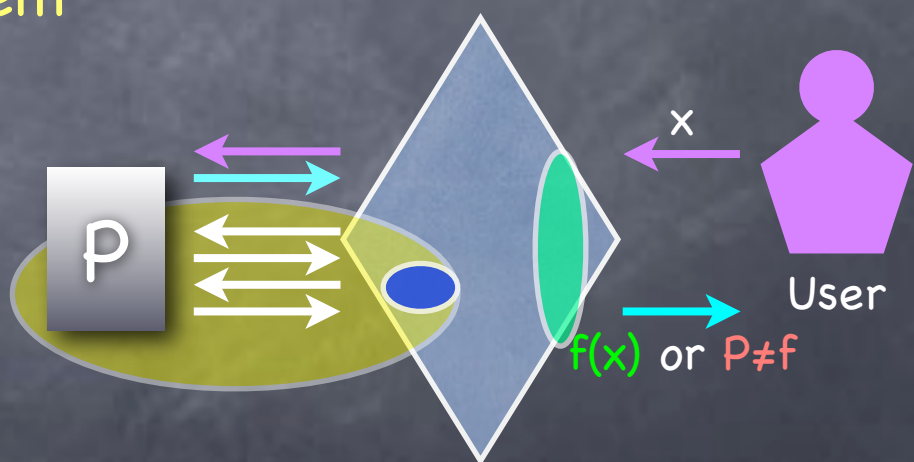
- How about Graph Isomorphism?



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  - Else remember $v_1 \mapsto u$, and continue with $v_2$; keep old markers and use new larger markers to get $G_0''$ and $G_1''$

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    - Also useful in certain cryptographic protocols

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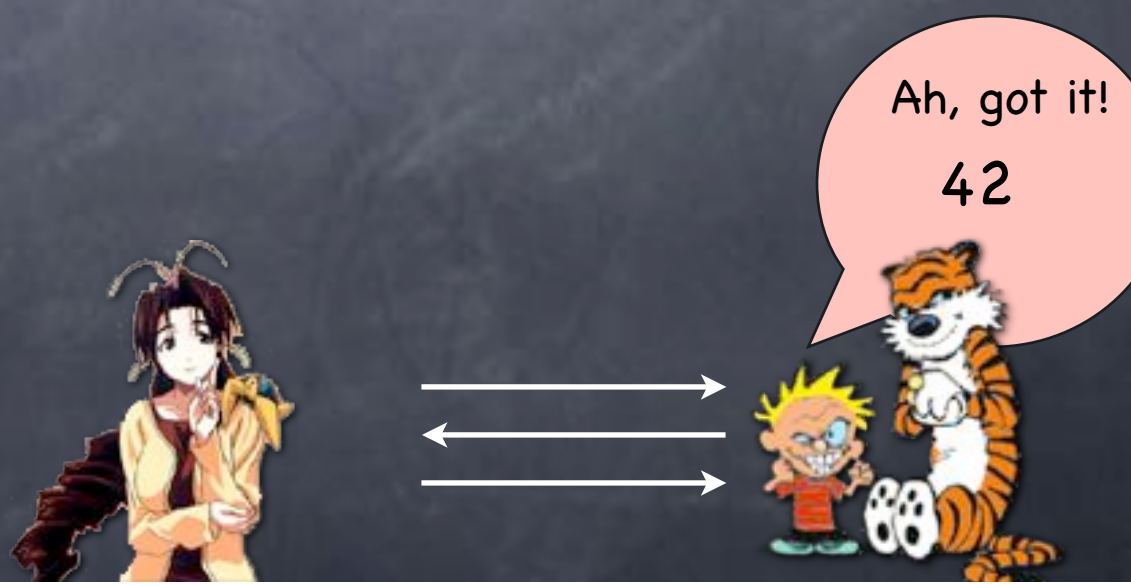

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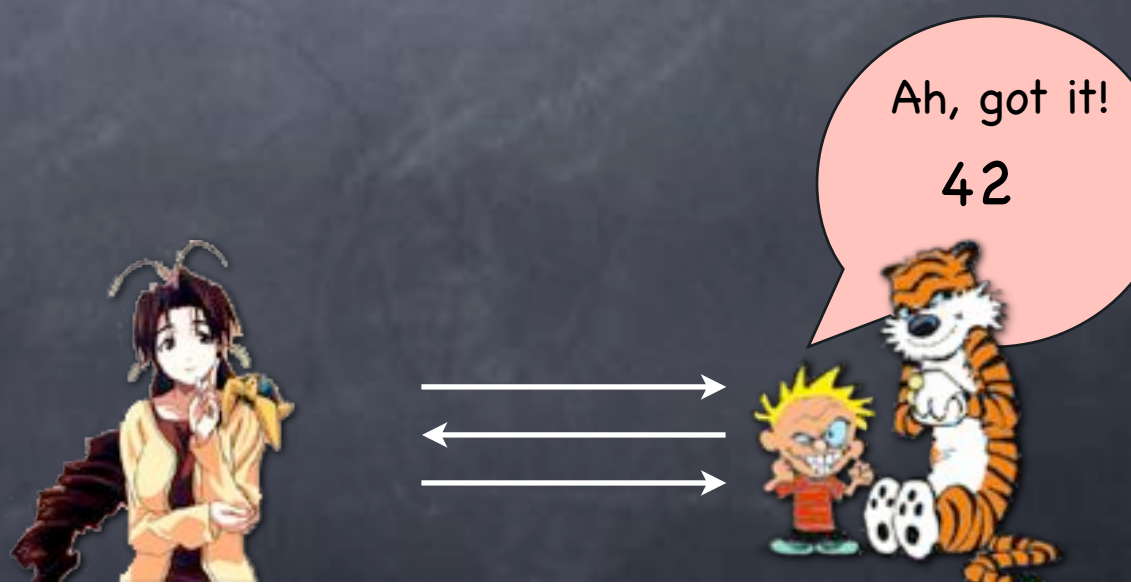

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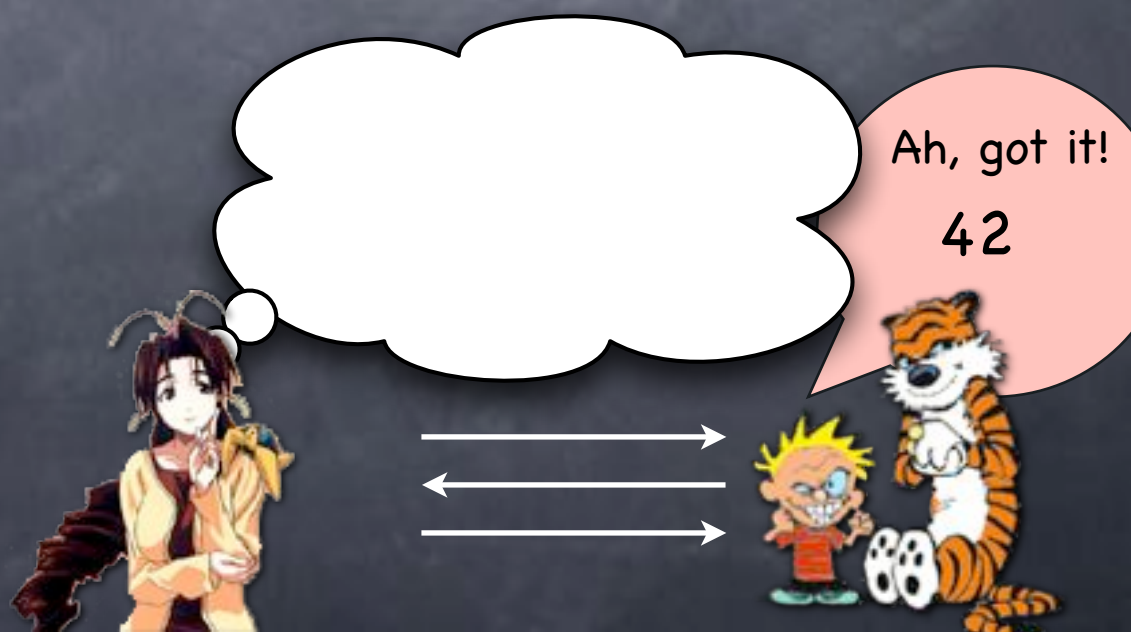

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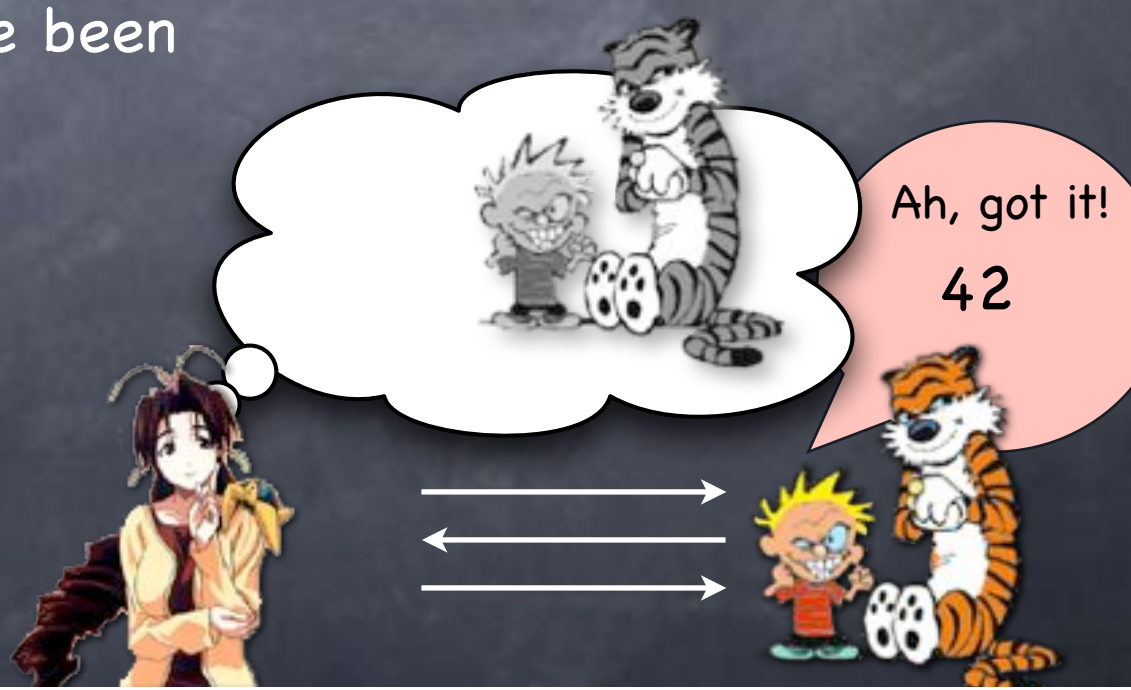

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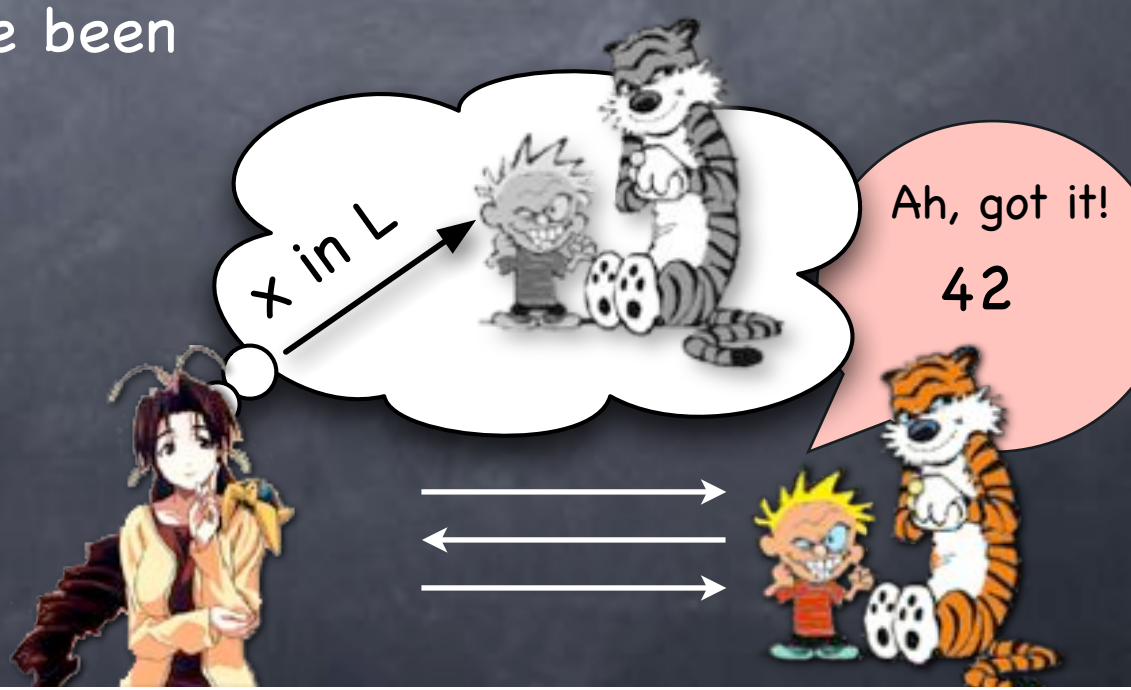

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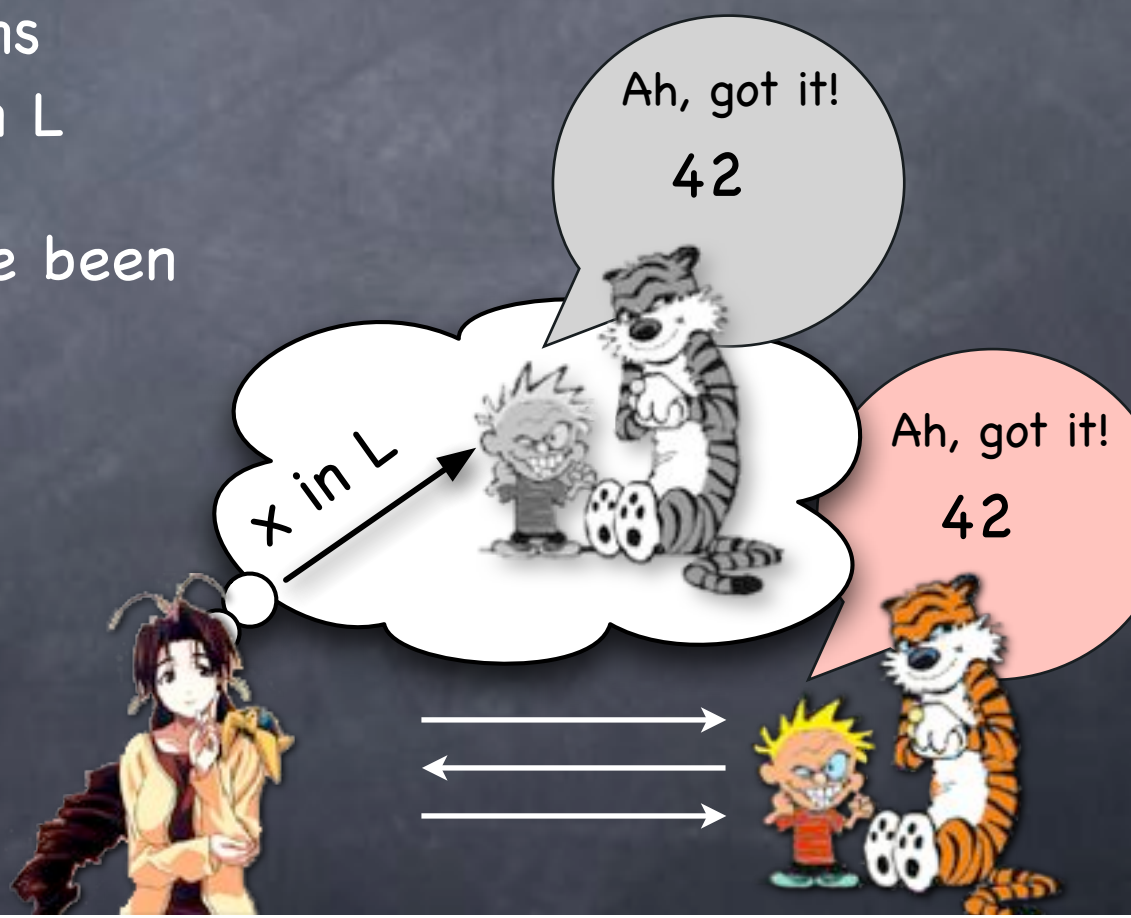

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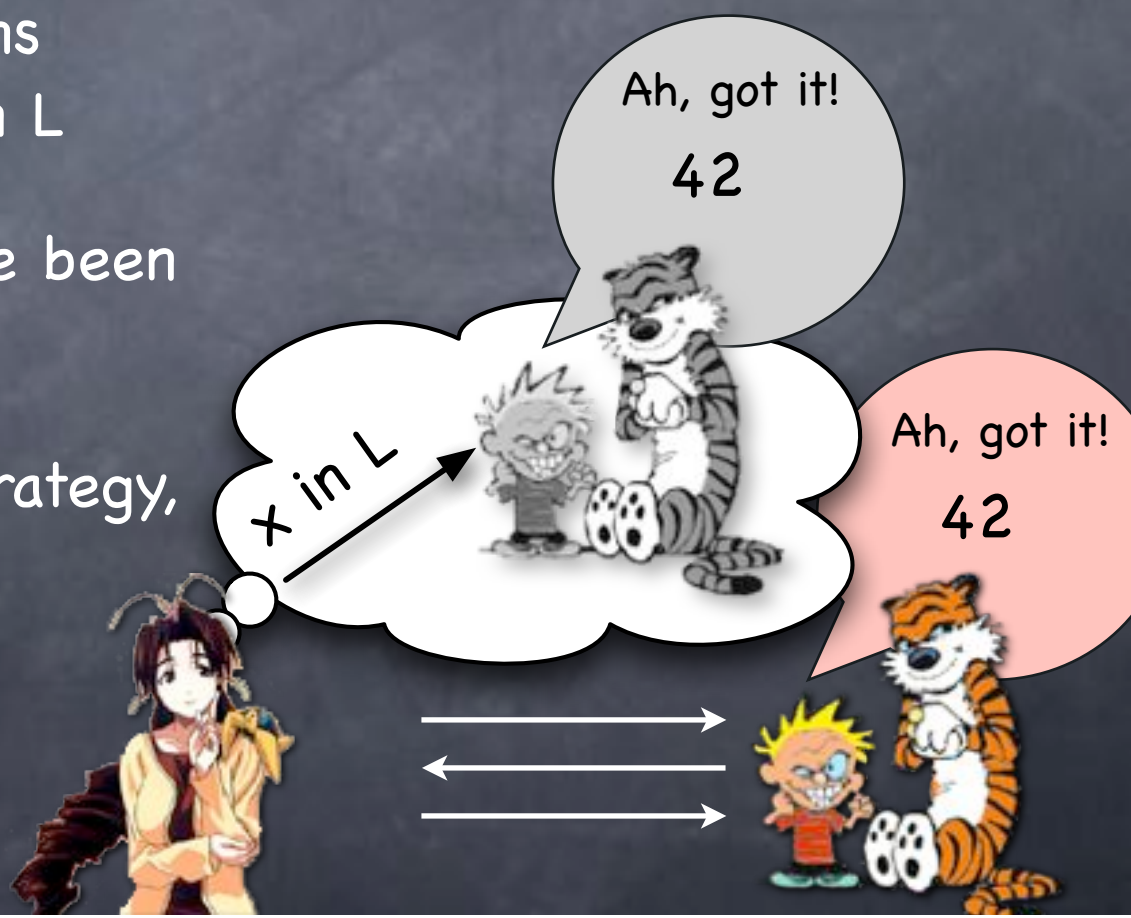

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- Understanding power of interaction/non-determinism and randomness
  - Useful in "hardness of approximation", in cryptography, ...