Computational Complexity

Lecture 1
in which we talk about
Time Complexity, P, NP and coNP

Evolution of Computation

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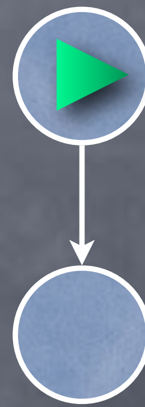

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  - output explicitly encoded in the final configuration (say, in the control-state)

Time Complexity

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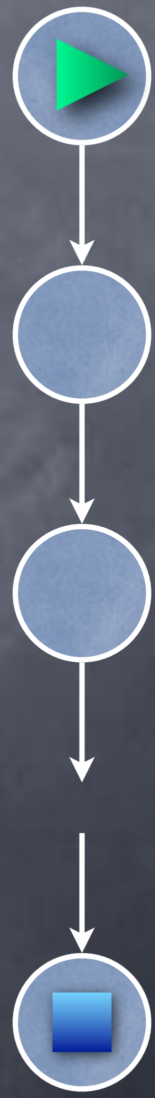

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  - (Note: complexity T is a <u>function</u> of n)

worst-case complexity

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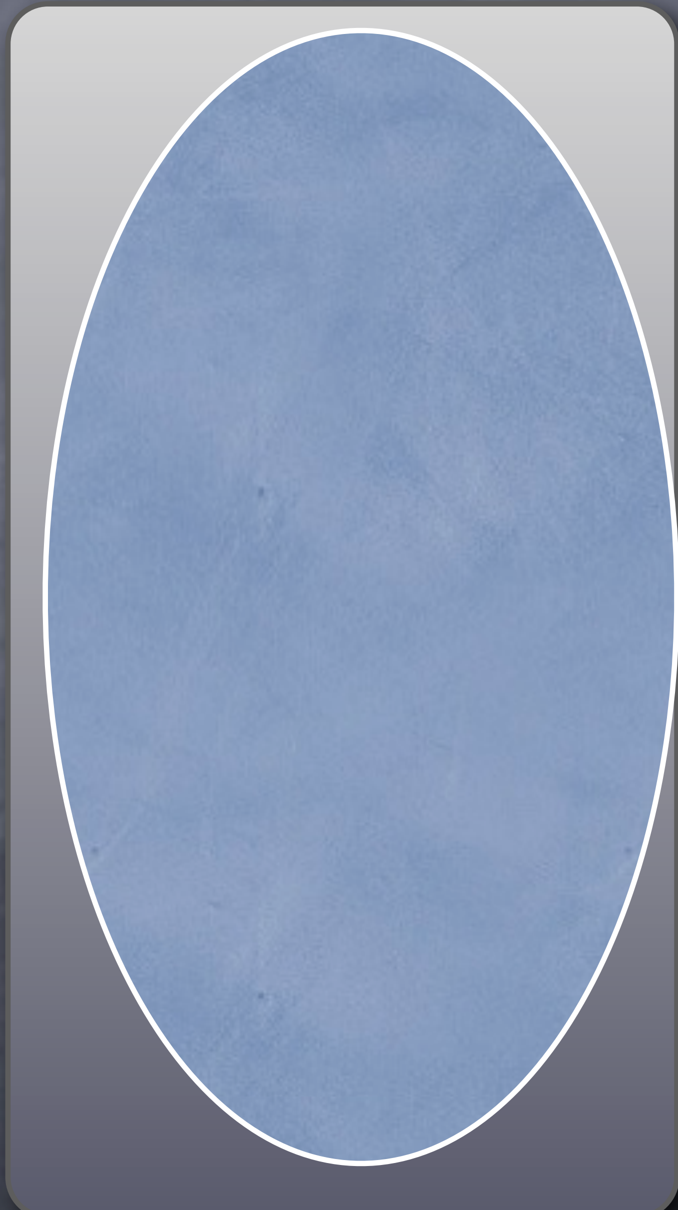

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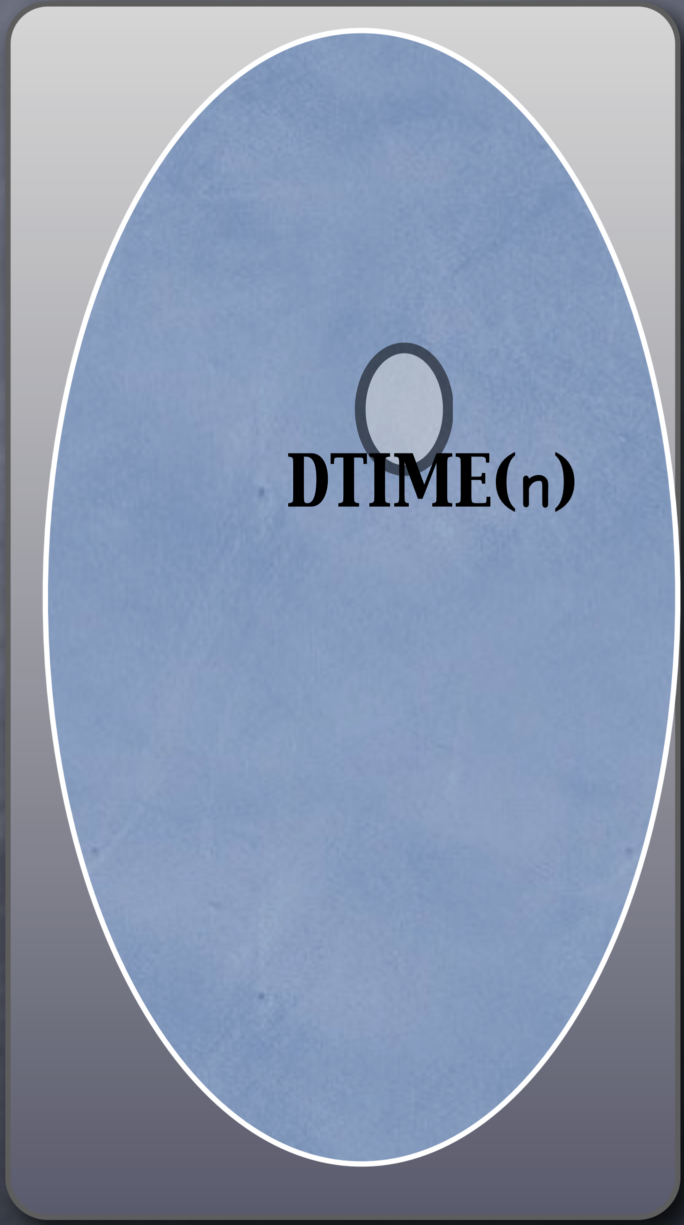

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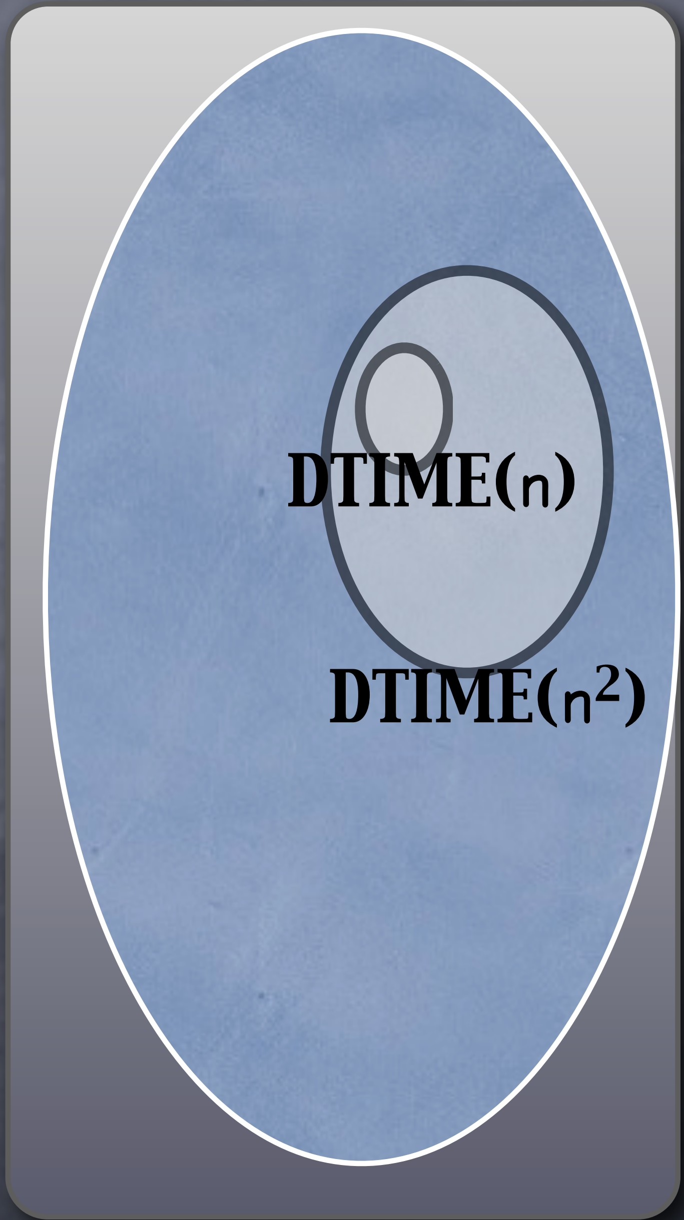

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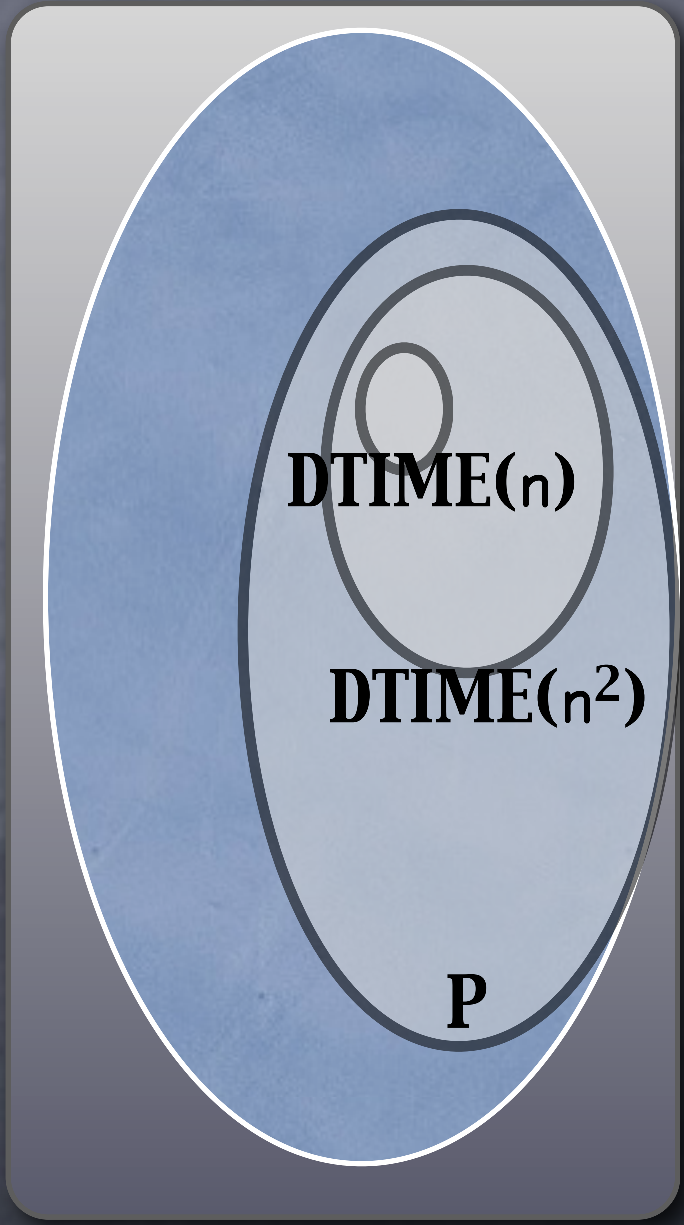

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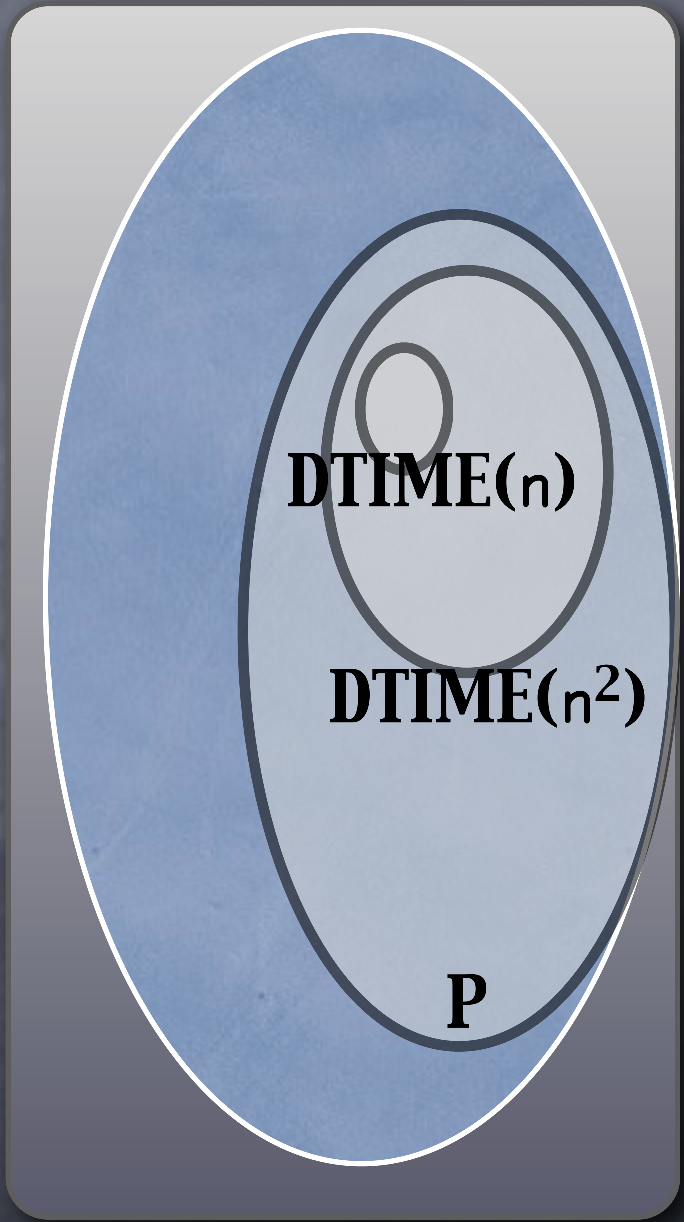

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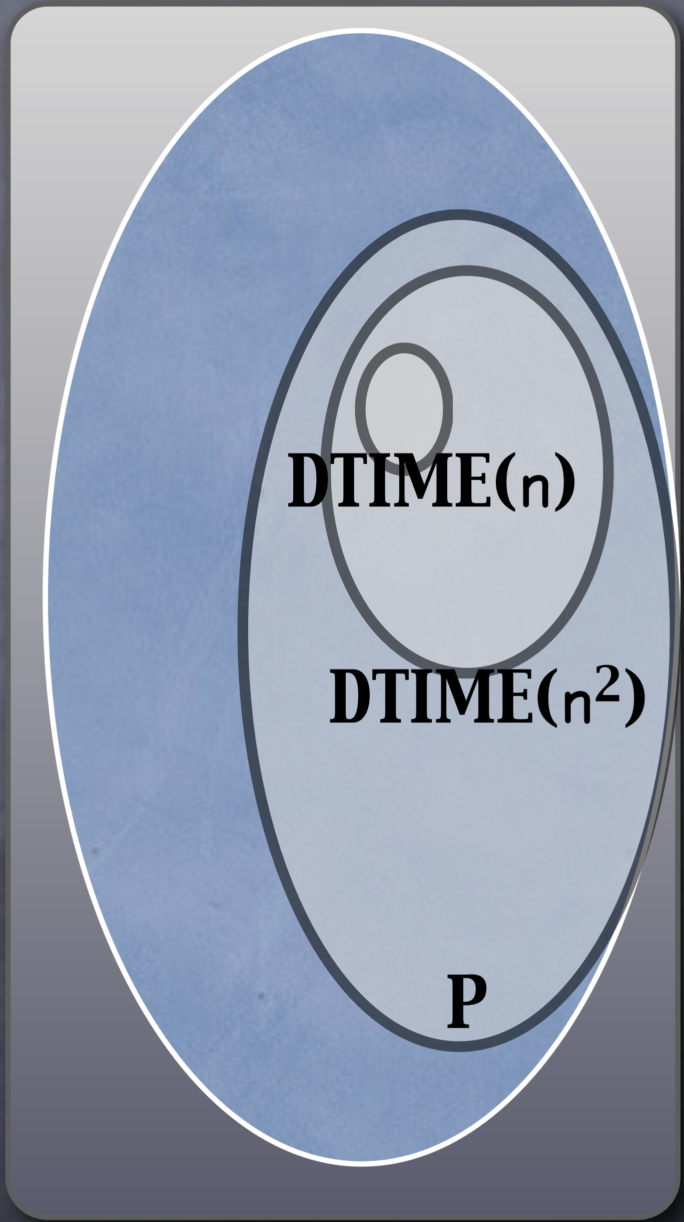
- But P is robust: Models can simulate each other with only "polynomial slow down"



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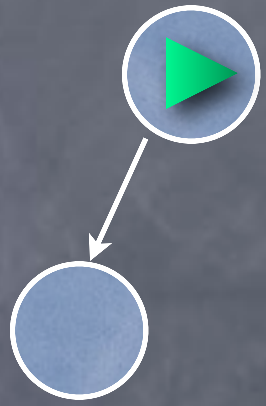

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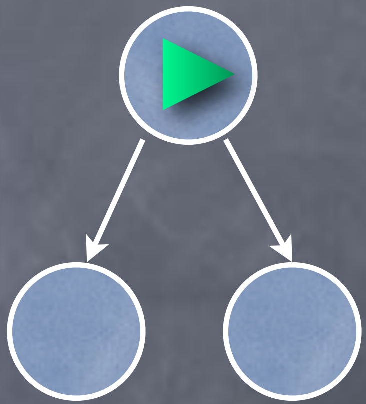

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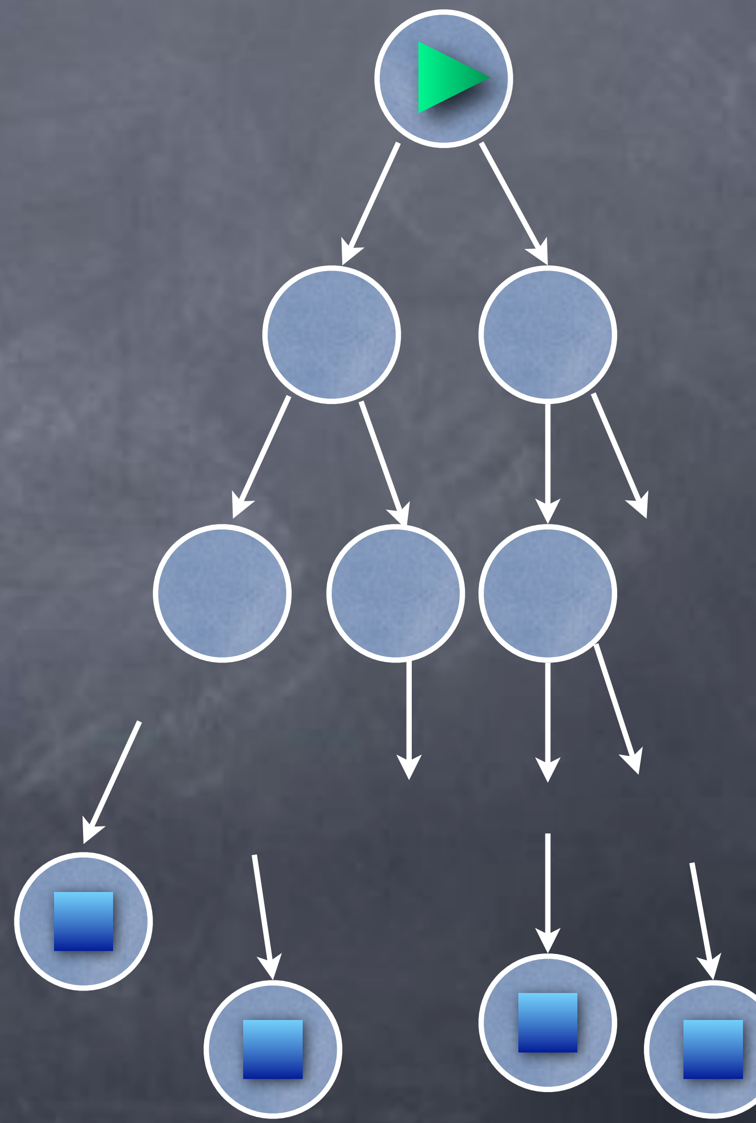

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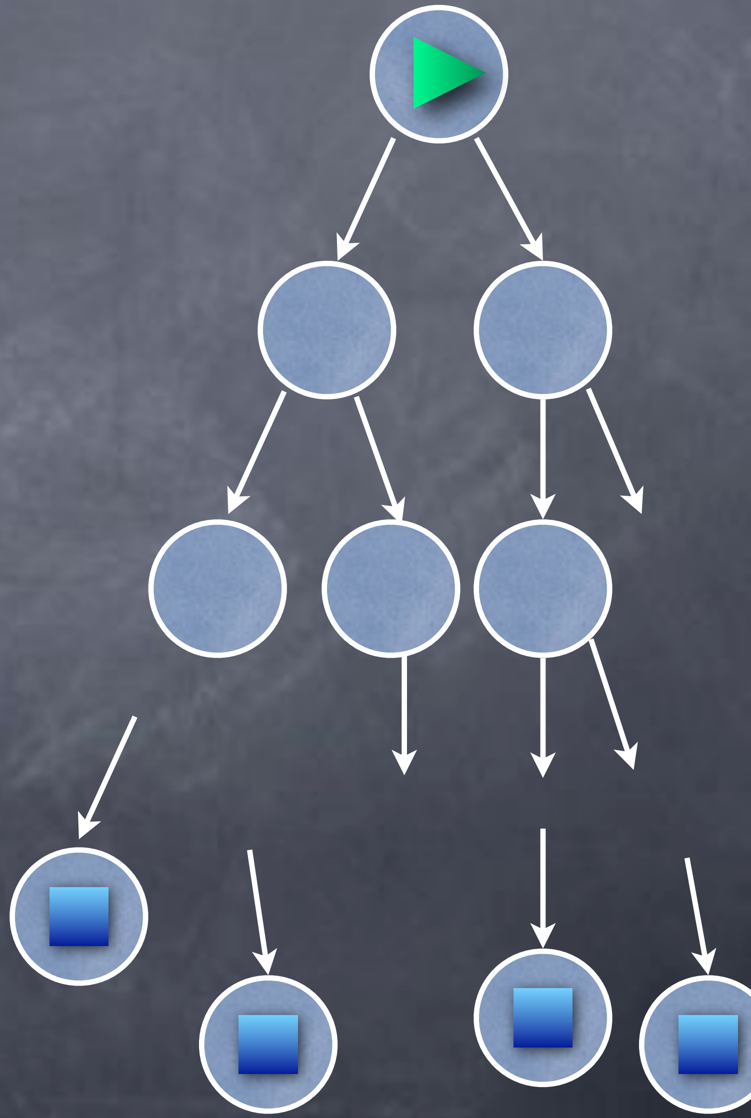

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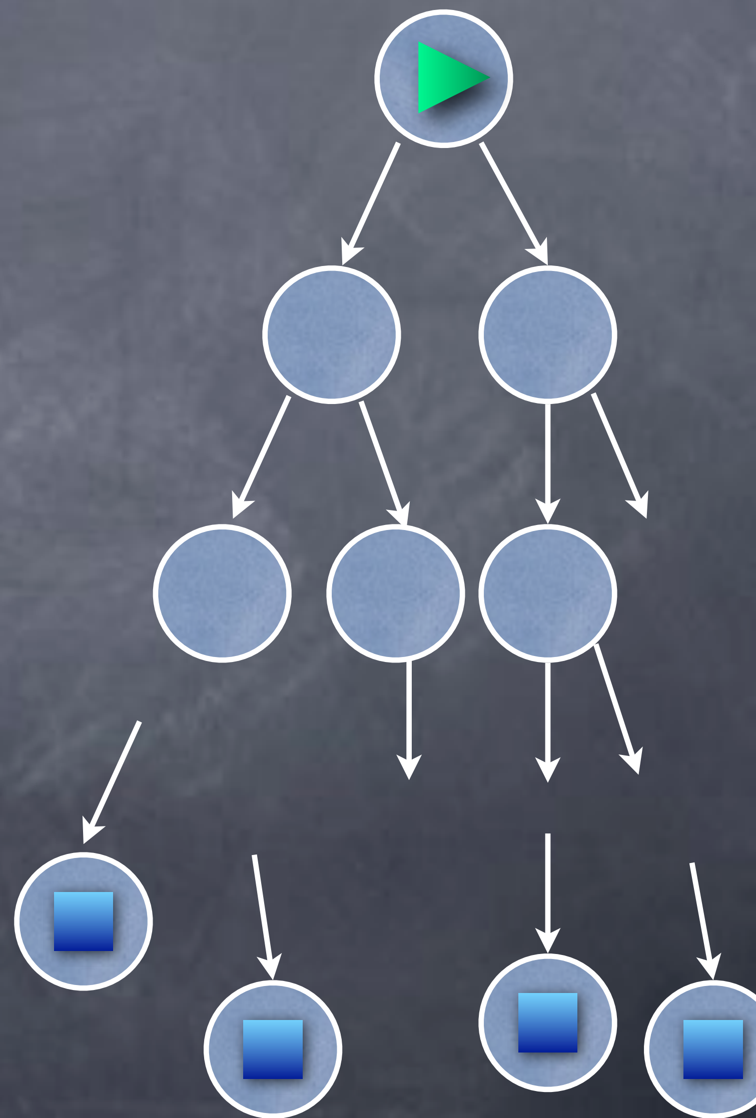

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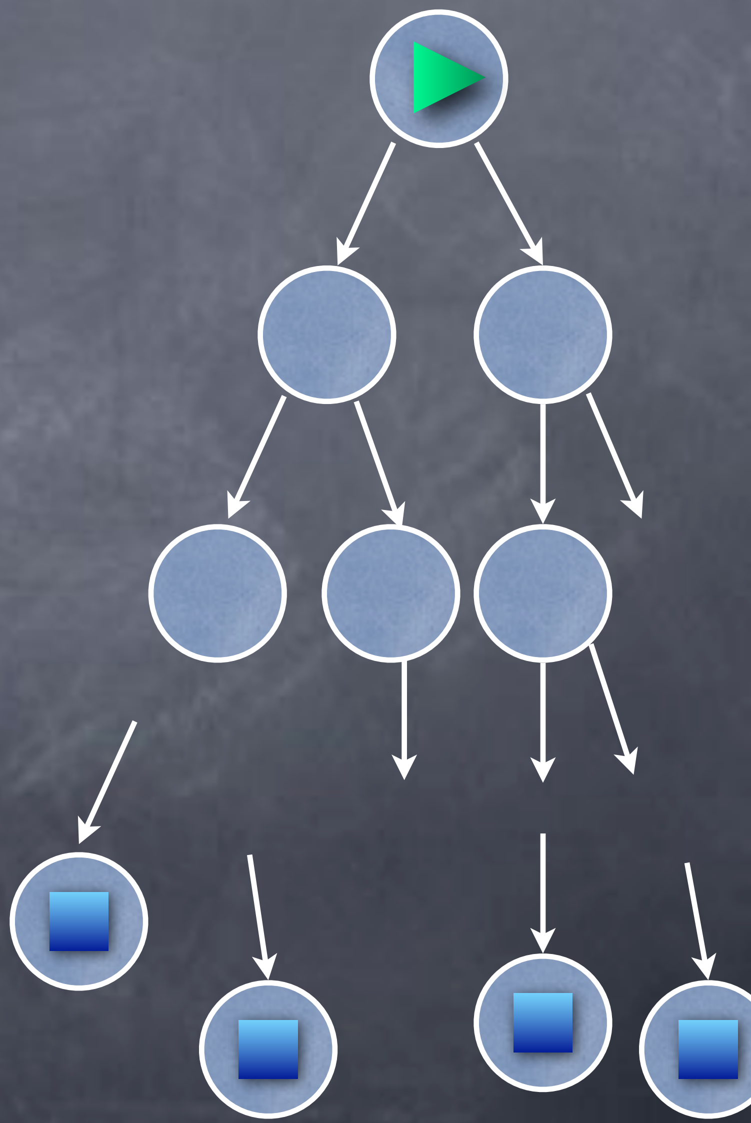
- L ∈ NTIME(T): an NTM decides L in time at most T



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  - Finding a certificate (or even finding if there exists a certificate) may take longer

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- Note: Completeness and soundness

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- All problems in P (empty certificate)

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  - For next bit query L_1-oracle with (x,00) and (x,01)

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- Solve all sorts of optimization problems efficiently!

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  - $M_{L^c} \leftrightarrow M_L$?
    - flip accept/reject states <u>and</u> flip "there exists" and "for all" in the acceptance criterion (NTM ↔ "co-NTM")

co-Class

- co-X = { L | L^c is in X } (where L^c = { x | $x \notin L$ })
- co-DTIME(T) = DTIME(T)
  - L^c in DTIME(T) iff L in DTIME(T)
  - $M_{L^c} \leftrightarrow M_L$: flip accept/reject states
- co-NTIME(T): all L s.t. L^c is in NTIME(T)
  - $M_{L^c} \leftrightarrow M_L$?
    - flip accept/reject states and flip "there exists" and "for all" in the acceptance criterion (NTM ↔ "co-NTM")
  - L^c = { x | $\nexists w$ s.t. $(x,w) \in L'$ } = { x | $\forall w$ $(x,w) \in L'^c$ }

co-Class

- co-X = { L | L^c is in X } (where L^c = { x | x∉L })
- co-DTIME(T) = DTIME(T)
  - L^c in DTIME(T) iff L in DTIME(T)
  - $M_{L^c} \leftrightarrow M_L$: flip accept/reject states
- co-NTIME(T): all L s.t. L^c is in NTIME(T)
  - $M_{L^c} \leftrightarrow M_L$?
  - flip accept/reject states and flip "there exists" and "for all" in the acceptance criterion (NTM ↔ "co-NTM")
  - L^c = { x | ∄w s.t. (x,w) ∈ L' } = { x | ∀w (x,w) ∈ L'^c }

no counter-example

P, NP and co-NP

P, NP and co-NP

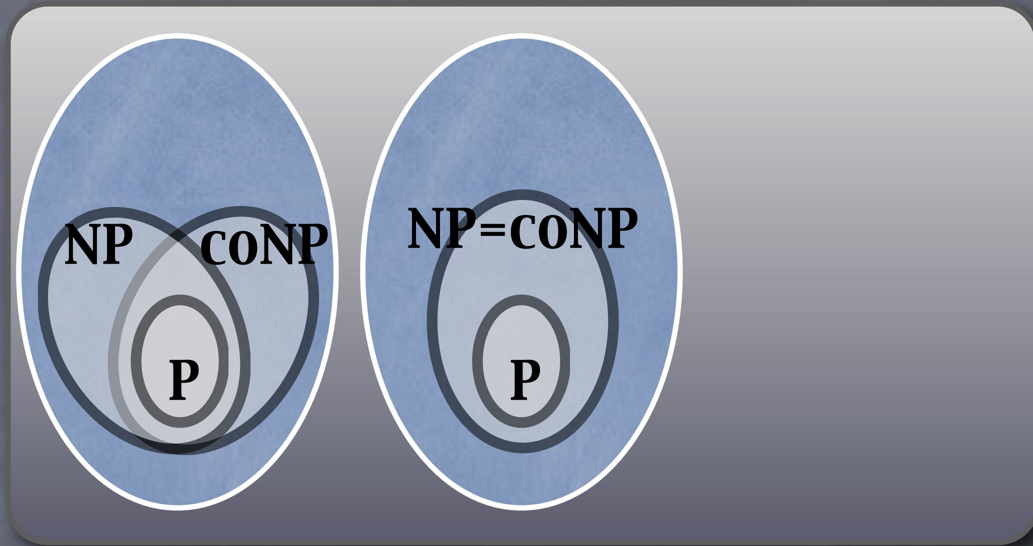
- Different possibilities

P, NP and co-NP

- Different possibilities

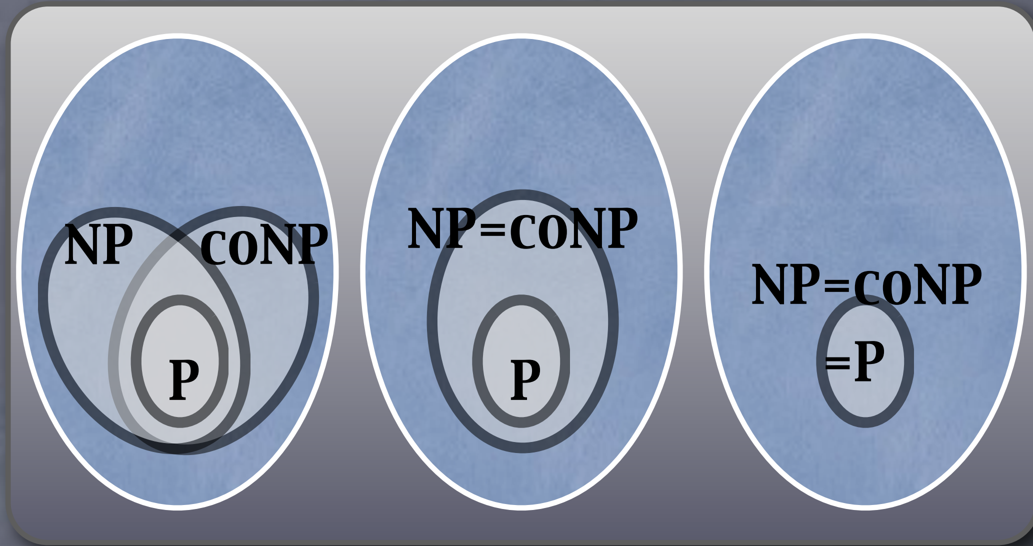
P, NP and co-NP

- Different possibilities



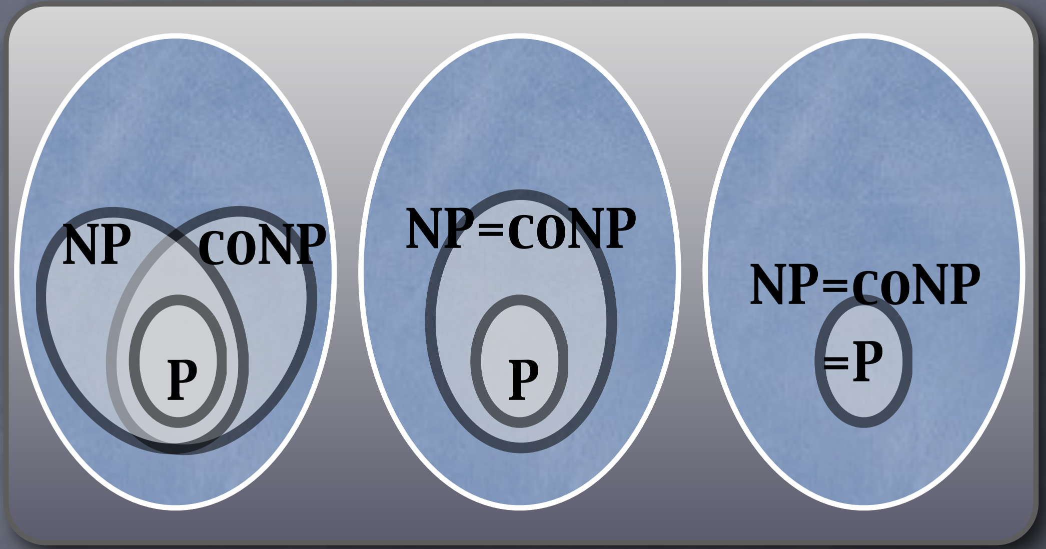
P, NP and co-NP

- Different possibilities

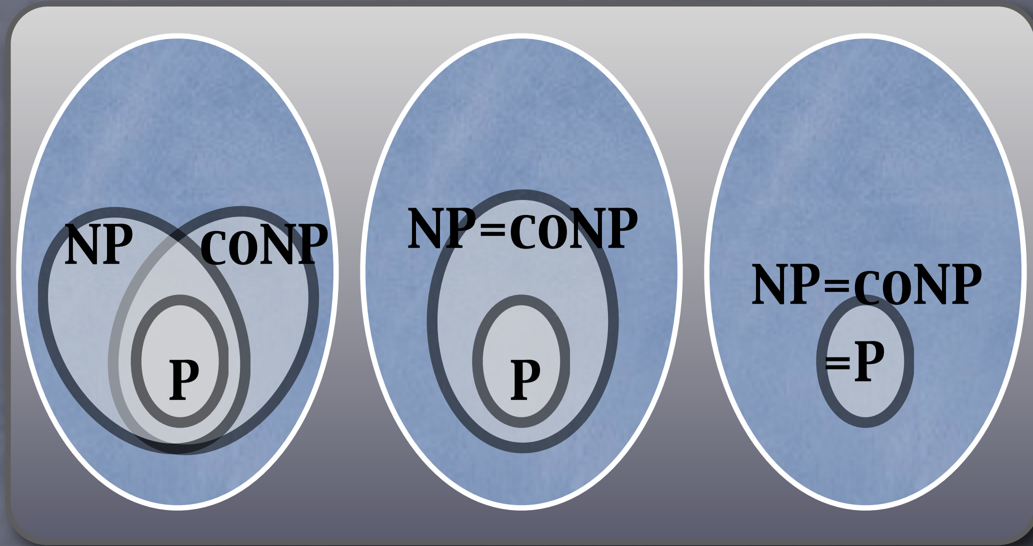


P, NP and co-NP

- Different possibilities
- If P=NP, then



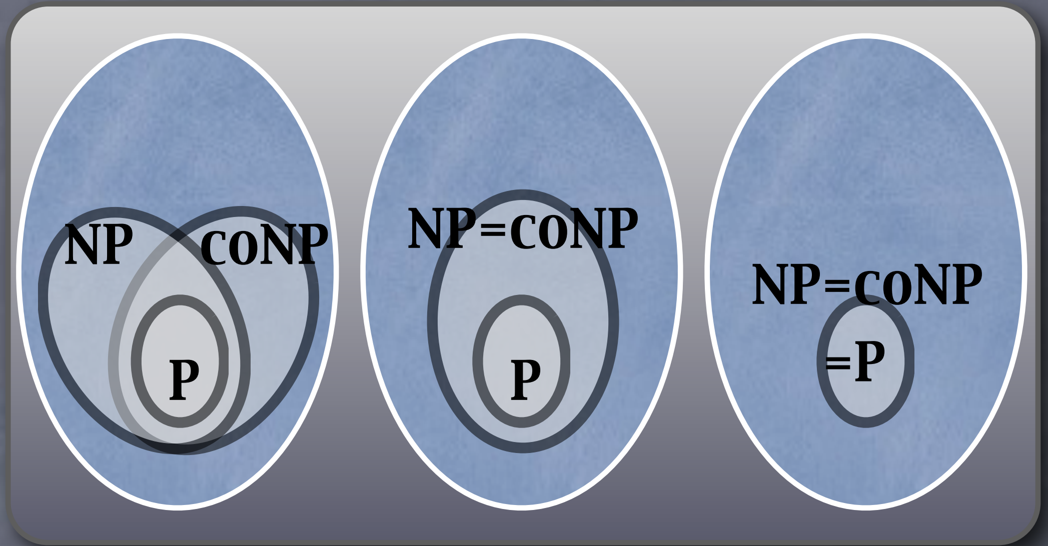
P, NP and co-NP



- Different possibilities
- If P=NP, then
  - coNP = coP = P = NP

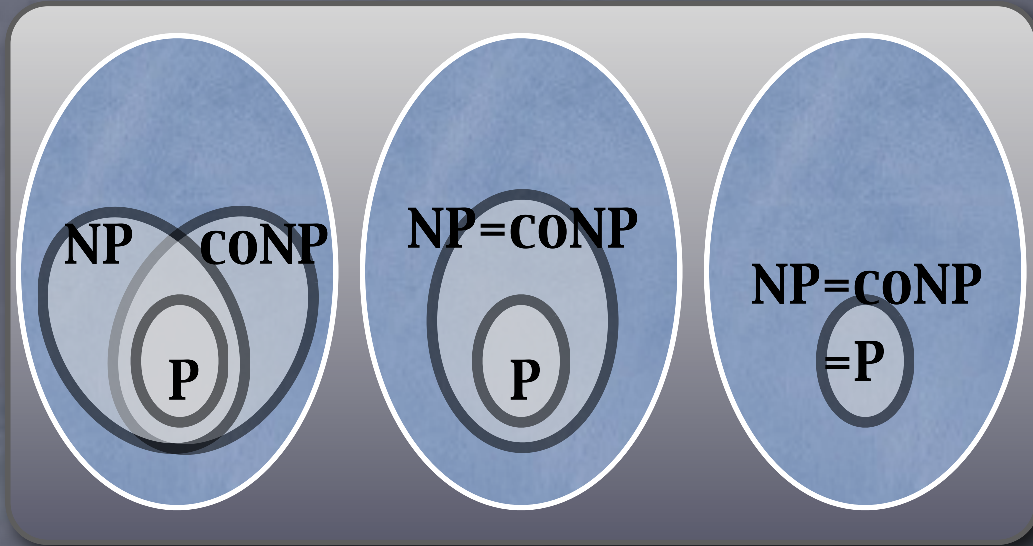
P, NP and co-NP

- Different possibilities
- If P=NP, then
  - coNP = coP = P = NP
  - Also, EXP = NEXP [Exercise]

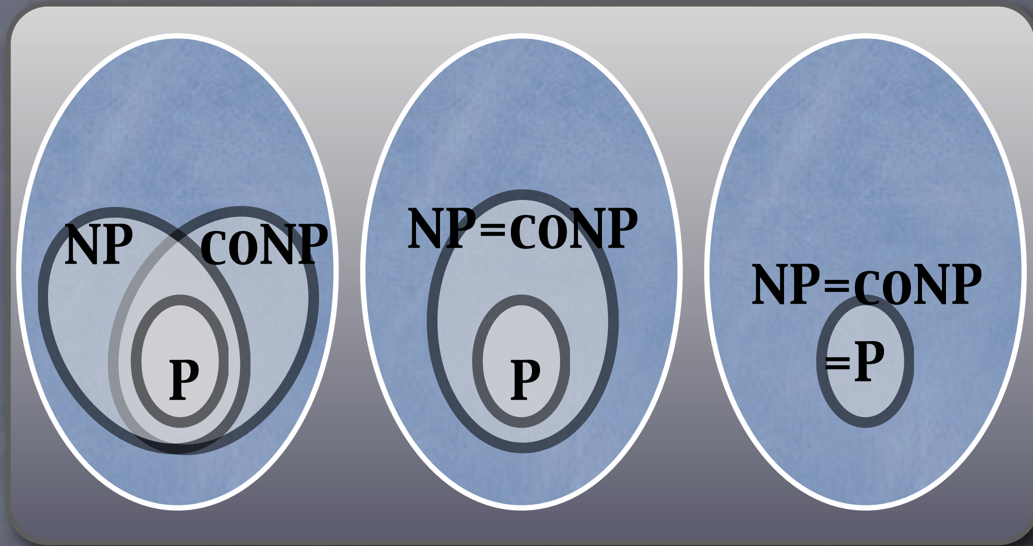


P, NP and co-NP

- Different possibilities
- If P=NP, then
  - coNP = coP = P = NP
  - Also, EXP = NEXP [Exercise]
    - padding to scale up both classes

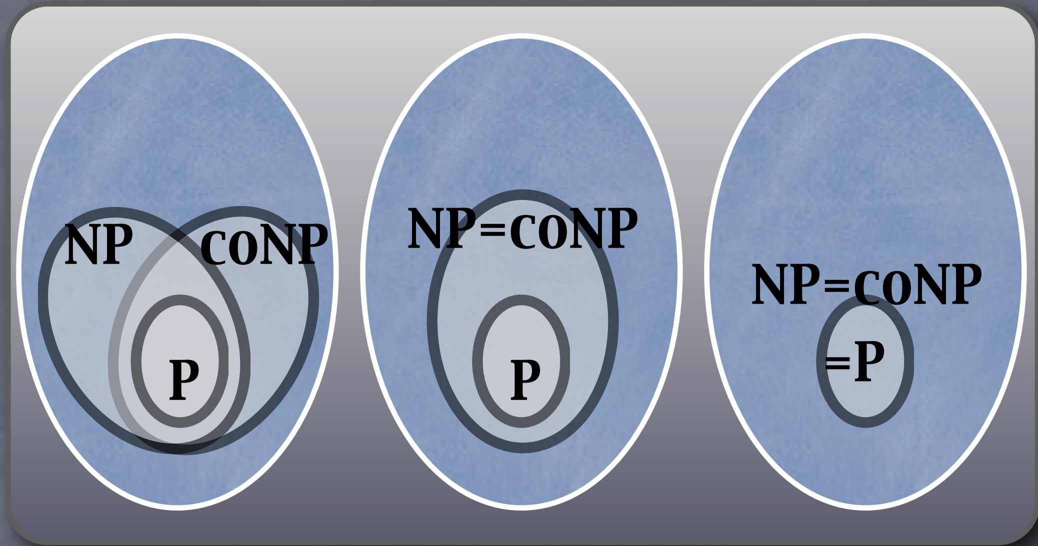


P, NP and co-NP



- Different possibilities
- If P=NP, then
  - coNP = coP = P = NP
  - Also, EXP = NEXP [Exercise]
    - padding to scale up both classes
    - x → (x,pad), so that Exp(|x|) = Poly(|x,pad|)

P, NP and co-NP



- Different possibilities
- If P=NP, then
  - coNP = coP = P = NP
  - Also, EXP = NEXP [Exercise]
    - padding to scale up both classes
    - x → (x,pad), so that Exp(|x|) = Poly(|x,pad|)
- If P=NP, then the complexity landscape would get greatly simplified than believed (more later)

Today

Today

- DTIME

Today

- DTIME
  - P, EXP

Today

- DTIME
  - P, EXP
- NTIME

Today

- DTIME
  - P, EXP
- NTIME
  - Two views: non-determinism and certificate

Today

- DTIME
  - P, EXP
- NTIME
  - Two views: non-determinism and certificate
  - NP, NEXP

Today

- DTIME
  - P, EXP
- NTIME
  - Two views: non-determinism and certificate
  - NP, NEXP
- co-NTIME

Today

- DTIME
  - P, EXP
- NTIME
  - Two views: non-determinism and certificate
  - NP, NEXP
- co-NTIME
  - Two views: co-NTM and “no counter-example”

Next ~~Class~~ Lecture

Next ~~Class~~ Lecture

- NP completeness

Next ~~Class~~ Lecture

- NP completeness
  - As hard as it gets inside NP

Next ~~Class~~ Lecture

- NP completeness
  - As hard as it gets inside NP
    - a la reductions (of course)