Complexity Homework 3

Released: February 24, 2010

Due: March 10, 2010

Problem 1:

Show that **NP** is closed under complementation if and only if it is closed under Turing reductions: i.e., $\textbf{co-NP} \subseteq \textbf{NP} \iff \textbf{NP}^{\textbf{NP}} \subseteq \textbf{NP}$.

Problem 2:

Show that $\textbf{NP}^{\Sigma_k^p \cap \Pi_k^p} = \Sigma_k^p$ for all $k \geq 1$.

Problem 3:

In class, we related **ATIME** to **DSPACE**. In this problem you will relate it to **NSPACE**. Show that $\textbf{NSPACE}(S) \subseteq \textbf{ATIME}(S^2)$ for $S(n) \geq \log n$ (where S is space-constructible).

(Hint: Recall the algorithm in the proof of Savitch's theorem. Use existential quantifiers to guess the "middle node" in paths and universal quantifier to check both halfs of the path exist. To be clear, give the certificate version of the ATM, clearly stating how many certificate tapes are there, how long each one is, and how the deterministic verification works and in what time.)

Conclude that $\textbf{PSPACE} \subseteq \textbf{AP}$. (Of course, we have already seen in class that $\textbf{PSPACE} = \textbf{AP}$.)

Problem 4:

We often assume that a circuit has NOT gates only at the input level.

(a) Show how to convert a circuit *with fan-out one for all gates* (except possibly for the input gates) into an equivalent circuit of no larger size, but with NOT gates only at the inputs.

(b) Show how to convert a general circuit (with any fan-out for the gates) into an equivalent circuit of at most twice the size, with NOT gates only at the inputs.

Problem 5 (Extra Credit):

Show that the class ${\mathbf{\Sigma}_k^p}^{\mathbf{\Sigma}_\ell^p}$ is contained in the polynomial hierarchy. What is the lowest class in the hierarchy that can you can place it inside, for different values of k and ℓ? Prove your claim. (Remember that an oracle can be queried multiple times. Consider cases of k, ℓ being odd/even and 0, 1, 2 etc. To begin with, you may consider one or more of these special cases.)