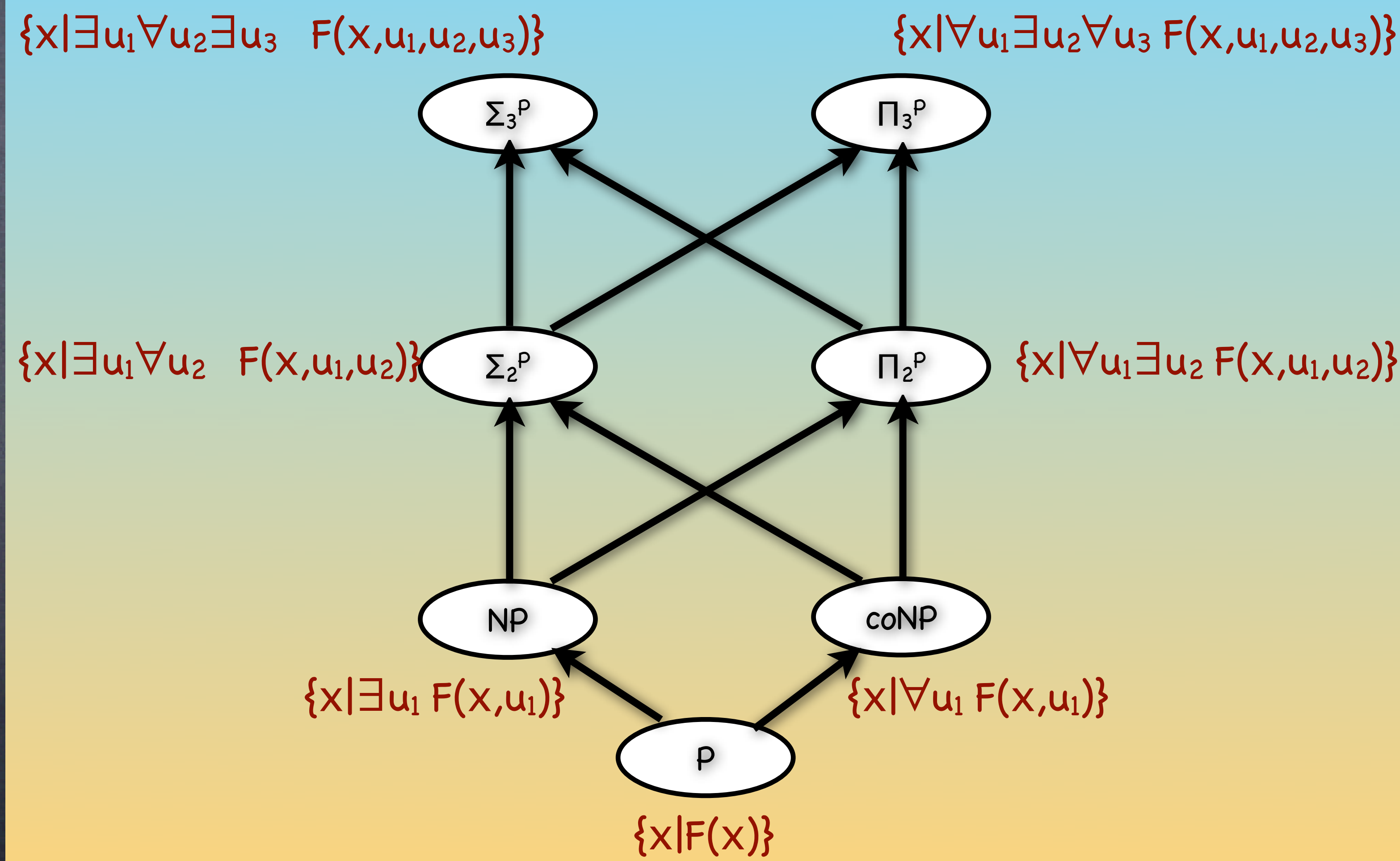
Computational Complexity

Lecture 8

More of the Polynomial Hierarchy

Oracle-based Definition

Recall PH



{x|∃u₁∀u₂∃u₃ F(x,u₁,u₂,u₃)}
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Σ₃ᴾ
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Σ₂ᴾ
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{x|∀u₁∃u₂ F(x,u₁,u₂)}
NP
coNP
{x|∃u₁ F(x,u₁)}
{x|∀u₁ F(x,u₁)}
P
{x|F(x)}

Oracle Machines

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  - Can run an oracle machine with any oracle
  - Oracle fully specified by the input-output behavior
  - Language oracle: answer is a single bit
    - This is what we consider

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  - $NP \cup co\text{-}NP \subseteq NP^{SAT}$
    - Can we better characterize NP^{SAT}?

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    - $L = \{ x | \exists w\ (x,w) \in L' \}$, where L' in $\Pi_k{}^P$

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    - But $NP^{L'} \subseteq NP^{\Pi_k} = NP^{\Sigma_k}$

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  - So L in $NP^{L'}$ where L' in Π_k^P
    - But $NP^{L'} \subseteq NP^{\Pi_k} = NP^{\Sigma_k}$
- So $\Sigma_{k+1}^P \subseteq NP^{\Sigma_k}$
- Now to show $NP^{\Sigma_k} \subseteq \Sigma_{k+1}^P$

$$NP^{\Sigma_k} \subseteq \Sigma_{k+1}$$

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  - To show $C = \{(x,w,ans) \mid M^{<ans>}(x,w)=1$ and "ans correct"$\}$ in $\Sigma_{k+1}{}^P$

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    - Then B also in Σ_{k+1}^P

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  - Changes for 2 queries: $z=Z(x,w) \rightarrow (z^{(1)},z^{(2)}) = Z(x,w,ans)$, $u_i \rightarrow u_i^{(1)},u_i^{(2)}$, $v_i \rightarrow v_i^{(1)},v_i^{(2)}$, and use conjunction of two checks (for j=1 and j=2) of the form $[\ (ans^{(j)}=1 \wedge F(z^{(j)},u_1^{(j)},\ldots)=1)$ or $(ans^{(j)}=0 \wedge F(z^{(j)},v_1^{(j)},\ldots)=0)\ ]$

Oracle Version

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- **Σ_{k+1}^P = NP^{Σ_k}** (with Σ_0^P = P)

Oracle Version

- $\Sigma_{k+1}{}^{P} = NP^{\Sigma_k}$ (with $\Sigma_0{}^{P} = P$)
- $\Pi_{k+1}{}^{P} = \text{co-}NP^{\Pi_k}$ (with $\Pi_0{}^{P} = P$)

Oracle Version

- **$\Sigma_{k+1}^P = NP^{\Sigma_k}$** (with $\Sigma_0^P = P$)
- **$\Pi_{k+1}^P = co\text{-}NP^{\Pi_k}$** (with $\Pi_0^P = P$)
  - $\Pi_{k+1}^P = co\text{-}(NP^{\Sigma_k}) = co\text{-}NP^{\Sigma_k} = co\text{-}NP^{\Pi_k}$

Δ_k^P

$\Delta_k{}^P$

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- Note that Δ_2^P = co-Δ_2^P
- $\Delta_{k+1}^P \supseteq \Sigma_k^P \cup \Pi_k^P$
- $\Delta_{k+1}^P \subseteq \Sigma_{k+1}^P \cap \Pi_{k+1}^P$ (why?)

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  - $P^{\Sigma_k} \subseteq NP^{\Sigma_k} \cap coNP^{\Sigma_k}$

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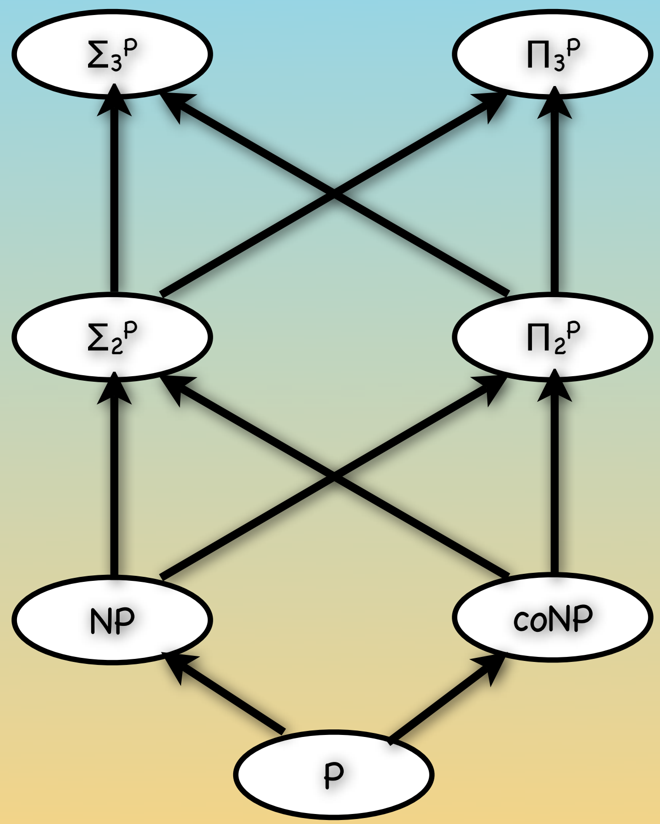

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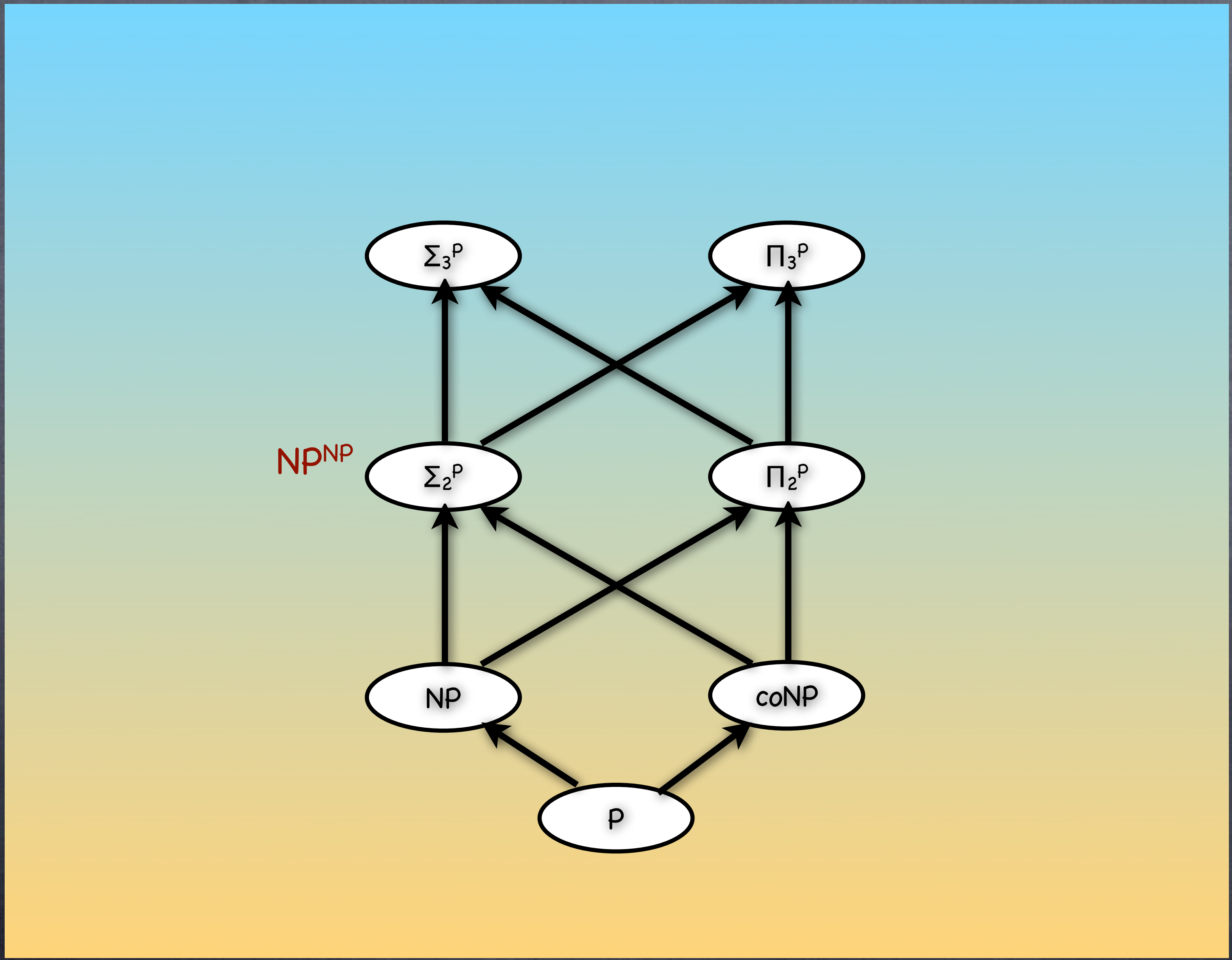

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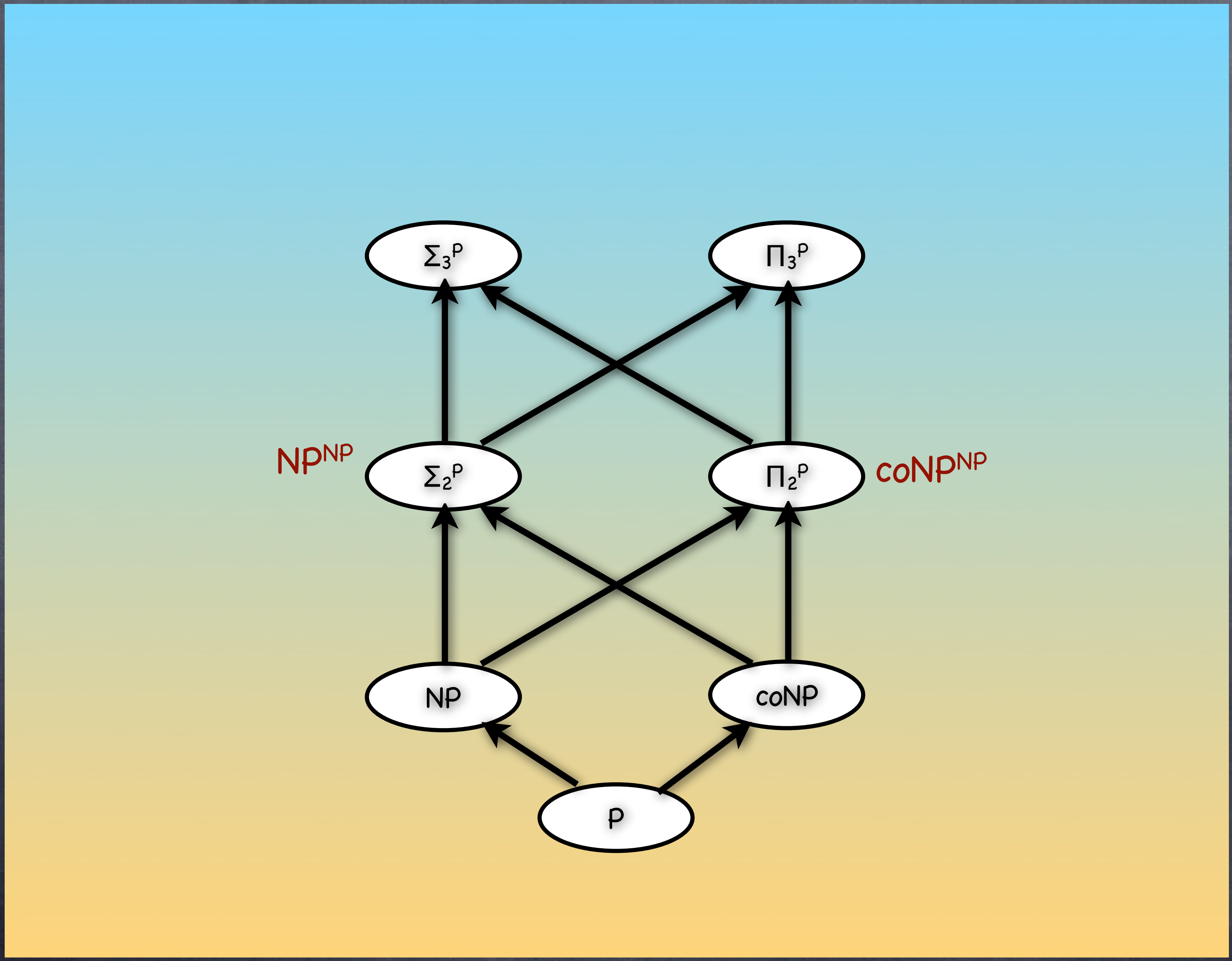

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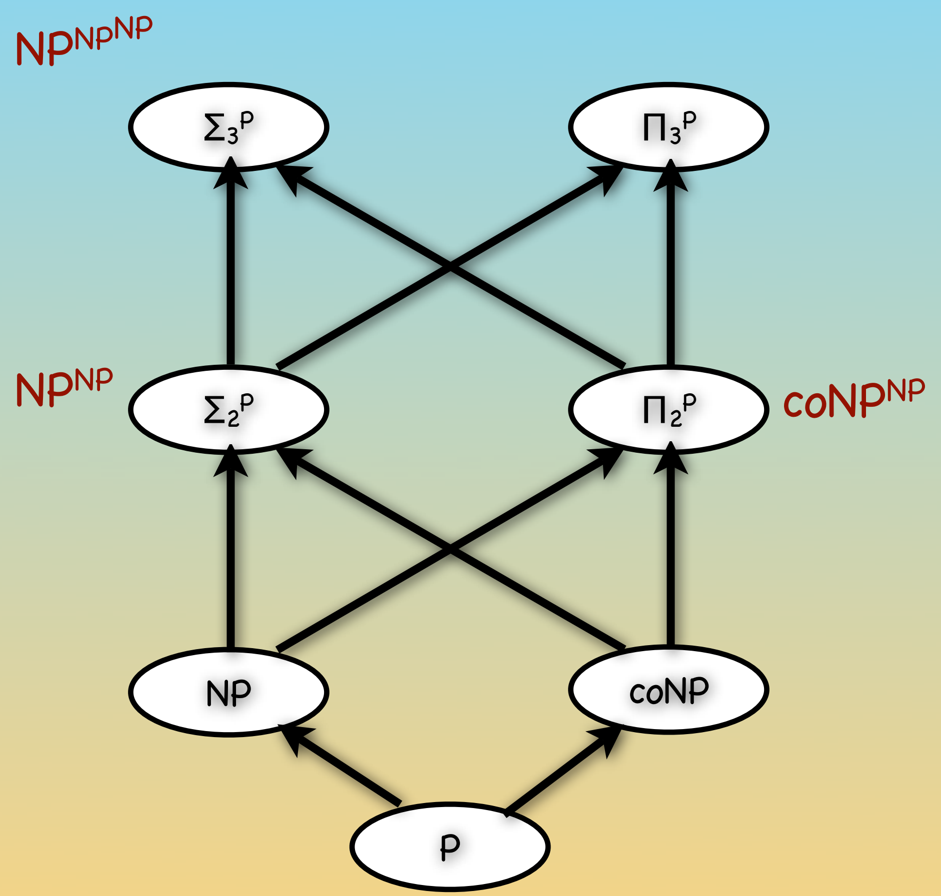

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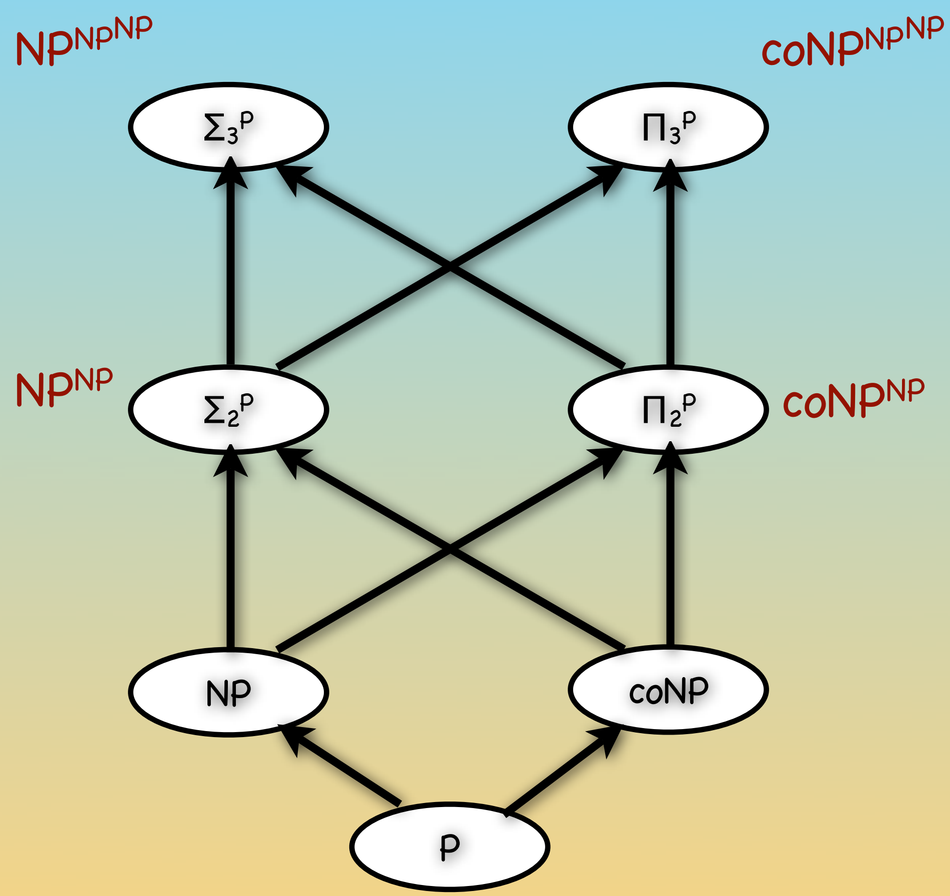

$NP^{NP^{NP}}$
$coNP^{NP^{NP}}$
Σ_3^P
Π_3^P
NP^{NP}
Σ_2^P
Π_2^P
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NP
coNP
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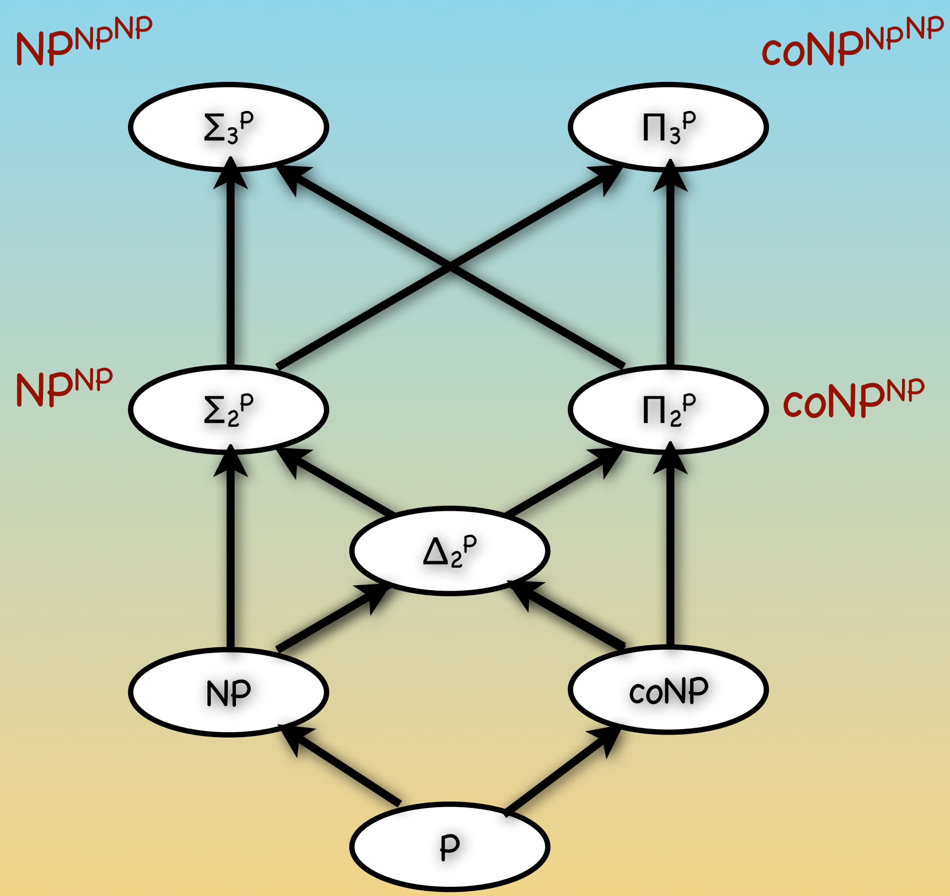
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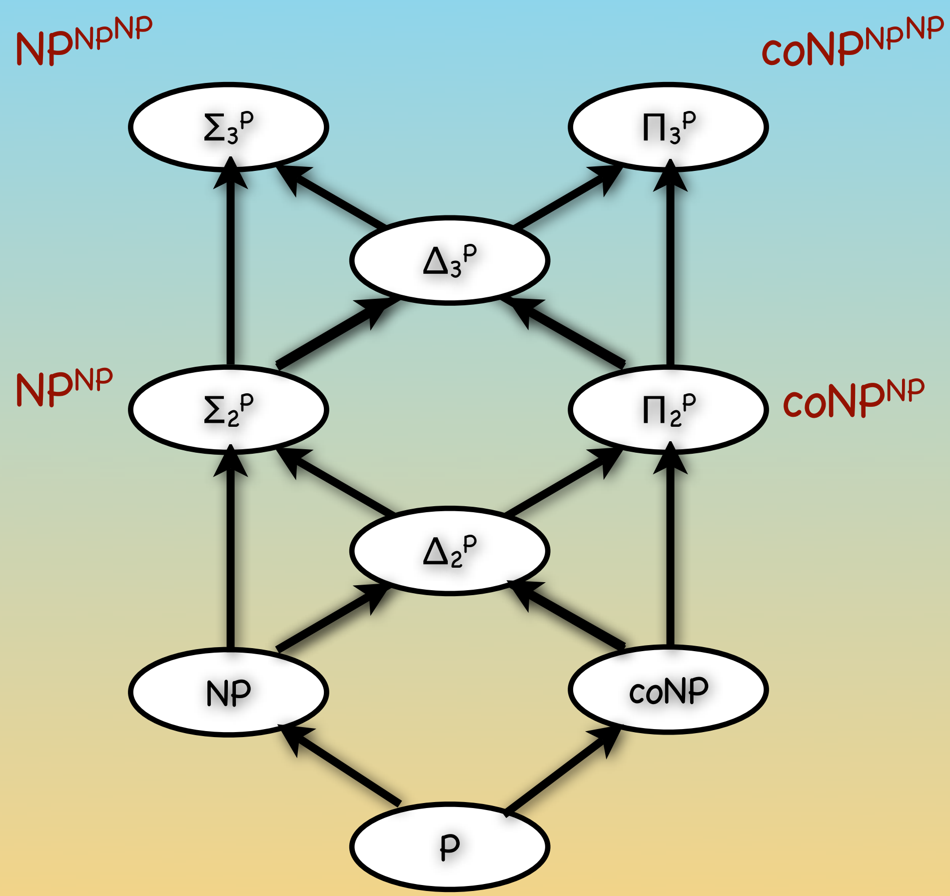

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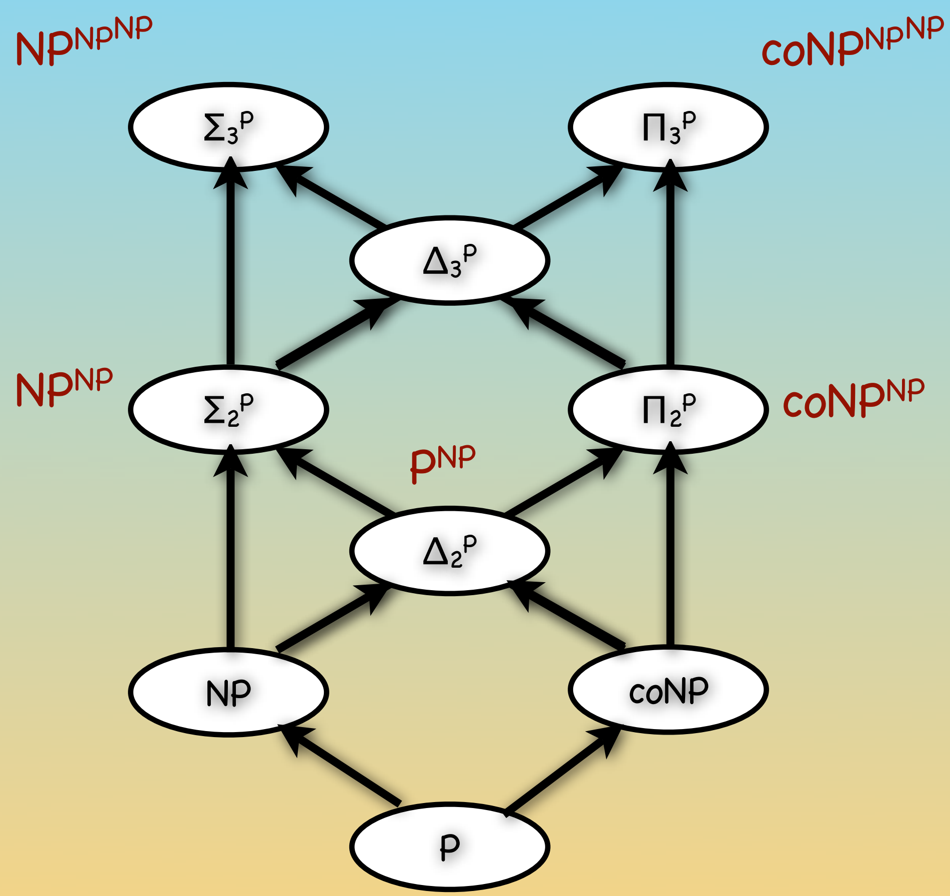

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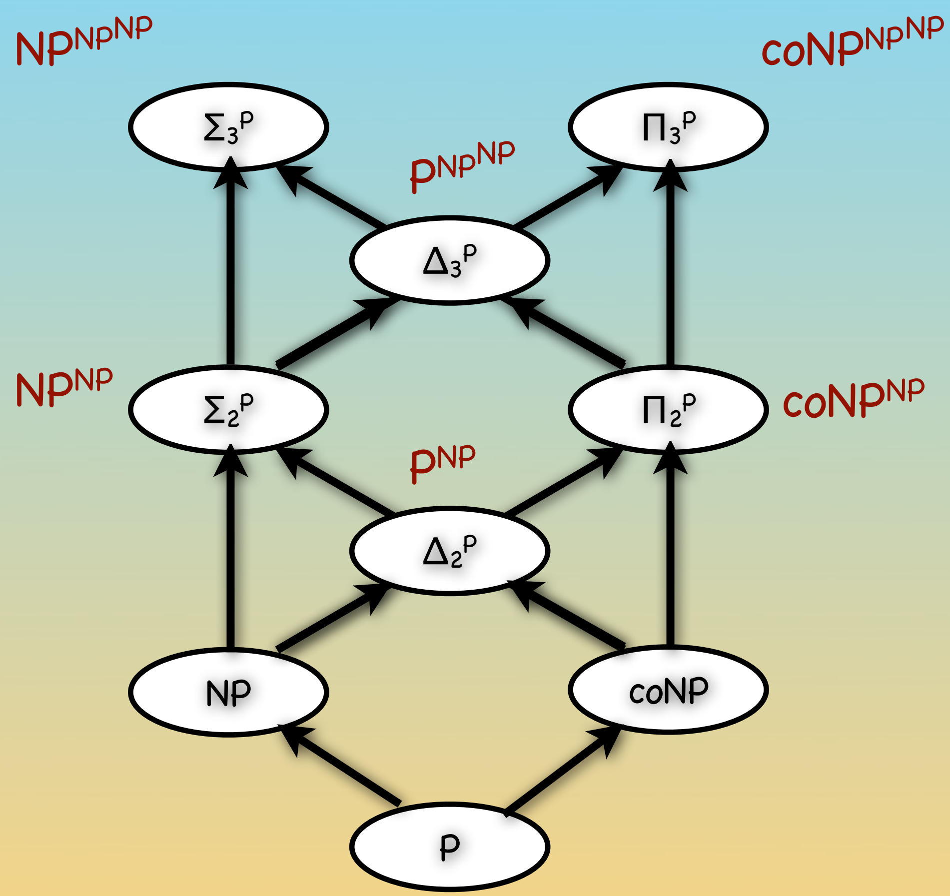

NP^NP^NP
coNP^NP^NP
Σ3P
Π3P
P^NP^NP
Δ3P
NP^NP
Σ2P
Π2P
coNP^NP
P^NP
Δ2P
NP
coNP
P

Today

Today

- Today, more PH

Today

- Today, more PH
  - Oracle-based definitions (in particular $NP^{NP} = \Sigma_2^P$)

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- Next lecture, more PH

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  - Alternating TM-based definitions

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  - Alternating TM-based definitions
  - Time-Space tradeoffs