Computational Complexity

Lecture 1
in which we talk about
Time Complexity, P, NP and coNP
Evolution of Computation
Evolution of Computation

- The program (Turing Machine) starts in an initial configuration (tape-contents, control-state, head-position)
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Evolution of Computation

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- Input explicitly encoded in the initial configuration.

At every step the configuration evolves.

Until computation terminates: final configuration.

- Output explicitly encoded in the final configuration (say, in the control-state).
Time Complexity
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- Deterministic TM computation model
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- Program (deterministic TM) succinctly specifies the "next configuration" function
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- **Deterministic TM computation model**
- Program (deterministic TM) succinctly specifies the "next configuration" function
- Time Complexity of language $L$ (worst case): if there is a **TM that decides** $L$ (correct on all instances), and for any input instance of size $n$, it **takes at most** $T(n)$ **steps** then $L$ in class **DTIME(T)**
Time Complexity

- Deterministic TM computation model

- Program (deterministic TM) succinctly specifies the “next configuration” function

- Time Complexity of language L (worst case): if there is a TM that decides L (correct on all instances), and for any input instance of size n, it takes at most $T(n)$ steps then L in class $\text{DTIME}(T)$

  (Note: complexity $T$ is a function of $n$)
P for Polynomial Time
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If a problem is in $\text{DTIME}(T)$ and $T(n) = O(n^c)$ for some $c$, then the problem is in $\text{P}$. 

$\Box$
P for Polynomial Time

If a problem is in DTIME(T) and $T(n)=O(n^c)$ for some $c$, then the problem is in $P$

$P = \bigcup_{a,b,c > 0} DTIME(a.n^c+b)$
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$P = \bigcup_{a,b,c > 0} \text{DTIME}(a.n^c + b)$

DTIME(T) depends on the specifics of the TM model (no. of tapes, alphabet size).

But $P$ is robust: Models can simulate each other with only “polynomial slow down”.
Non-deterministic Computation
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Not “realistic” as a computation model, but has realistic interpretations (coming up)
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- An NTM is said to accept an input if any of the threads of execution accepts it
- Time: longest execution thread
- $L \in \text{NTIME}(T)$: an NTM decides $L$ in time at most $T$
NTIME(T): alt view
L is in \text{NTIME}(T) iff it can be defined in the following way:
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\[ L = \{ x \mid \exists w \text{ s.t. } (x, w) \in L' \} \]
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Where $L'$ is in DTIME($T(|x|)$) (with an extra read-once input tape for $w$)
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i.e., in time \( T \), deterministic TM for \( L' \) can verify a certificate of membership for \( L \)
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i.e., in time T, deterministic TM for \( L' \) can verify a certificate of membership for L.

Non-deterministic computation: essentially guess w and verify.
$L \in \text{NTIME}(T)$: 
Equivalent views
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- Non-deterministic $M$
L ∈ NTIME(T):
Equivalent views

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- input: x
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\( L \in \text{NTIME}(T): \) Equivalent views

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$L \in \text{NTIME}(T)$: Equivalent views

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- in at most $T(|x|)$ steps
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- Deterministic $M'$
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- Deterministic \( M' \)
  - input: \( x \) and cert. \( w \)
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  - Input: $x$ and cert. $w$
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L ∈ NTIME(T):
Equivalent views

- Non-deterministic M
  - input: x
  - makes non-det choices
  - x ∈ L iff some thread of M accepts
  - in at most T(|x|) steps

- Deterministic M’
  - input: x and cert. w
  - reads bits from the cert.
  - x ∈ L iff for some cert. w, M’ accepts
  - in at most T(|x|) steps
NP
$\text{NP} = \bigcup_{a,b,c > 0} \text{NTIME}(a.n^c + b)$
NP

\[ NP = \bigcup_{a,b,c > 0} \text{NTIME}(a \cdot n^c + b) \]

There's an NTM that decides \( L \) in polynomial time
(some fixed polynomial)
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- There's an NTM that decides \( L \) in polynomial time (some fixed polynomial)
- There's a TM that verifies certificates for membership in \( L \), in polynomial time
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- Or, \( L = \{ x \mid \exists w \text{ s.t. } (x,w) \in L', \ |w| = O(poly(|x|)) \}, \) and \( L' \) in \( P \).
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Or, \( L = \{ x \mid \exists w \text{ s.t. } (x,w) \in L' \}, |w| = O(\text{poly}(|x|)) \), and
\( L' \) in \( P \)

Note: Completeness and soundness
Some Problems in NP
Some Problems in NP

- Graph properties: has a clique of size $n/2$, has a "Hamiltonian cycle", graph has an "Eulerian tour", two graphs are isomorphic.
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Numerical properties: is a composite number, is a prime number (not obvious)
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Constraint satisfaction: equation has solution, Linear Program (LP) is feasible, Integer LP is feasible, has a short Traveling Salesperson tour
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- Numerical properties: is a composite number, is a prime number (not obvious)

- Constraint satisfaction: equation has solution, Linear Program (LP) is feasible, Integer LP is feasible, has a short Traveling Salesperson tour

- All problems in P (empty certificate)
Search using Decision
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Say, given x, need to find w s.t. \((x,w) \in L'\) (if such w exists)

Consider \(L_1\) in NP: \((x,y) \in L_1\) iff \(\exists z\) s.t. \((x,yz) \in L'\). (i.e., can y be a prefix of a certificate for x).
Search using Decision

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- consider L₁ in NP: (x,y) ∈ L₁ iff ∃z s.t. (x,yz) ∈ L’. (i.e., can y be a prefix of a certificate for x).

- Query L₁-oracle with (x,0) and (x,1). If ∃w, one of the two must be positive: say (x,0) ∈ L₁; then first bit of w be 0.
Search using Decision

Suppose given “oracles” for deciding all NP languages, can we easily find certificates?

Yes! So, if decision easy (decision-oracles realizable), then search is easy too!

Say, given $x$, need to find $w$ s.t. $(x,w) \in L'$ (if such $w$ exists)

consider $L_1$ in NP: $(x,y) \in L_1$ iff $\exists z$ s.t. $(x,yz) \in L'$. (i.e., can $y$ be a prefix of a certificate for $x$).

Query $L_1$-oracle with $(x,0)$ and $(x,1)$. If $\exists w$, one of the two must be positive: say $(x,0) \in L_1$; then first bit of $w$ be 0.

For next bit query oracle with $(x,00)$ and $(x,01)$
What if NP = P
What if $\text{NP} = \text{P}$

“Can find as efficiently as can verify” (broadly speaking)
What if \( NP = P \)

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Mathematics: Proofs are easy to verify efficiently (if written in full). So we can generate them too efficiently?! Prove/discover theorems mechanically!
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Cryptography: If someone’s private key (well, key generation info) given, can verify that it corresponds to a public key. So we can find the private key efficiently?! No public-key crypto!
What if NP = P

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Cryptography: If someone’s private key (well, key generation info) given, can verify that it corresponds to a public key. So we can find the private key efficiently?! No public-key crypto!

Solve all sorts of optimization problems efficiently!
EXP and NEXP
EXP and NEXP

EXP is $\text{DTIME}(2^{\text{poly}(n)})$: 

EXP and NEXP

EXP is $\text{DTIME}(2^{\text{poly}(n)})$:

$$\text{EXP} = \bigcup_{a,b,c > 0} \text{DTIME}(2^{an^c+b})$$
EXP and NEXP

- **EXP** is DTIME($2^{\text{poly}(n)}$):
  - $\exp = \bigcup_{a,b,c > 0} \text{DTIME}(2^{an^c+b})$

- **NEXP** is NTIME($2^{\text{poly}(n)}$):
EXP and NEXP

- EXP is $\text{DTIME}(2^{\text{poly}(n)})$:
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$\text{NEXP} = \text{all } L \text{ of the form:}$
EXP and NEXP

- EXP is DTIME($2^{\text{poly}(n)}$):
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  \text{EXP} = \bigcup_{a,b,c > 0} \text{DTIME}(2^{an^c+b})
  \]

- NEXP is NTIME($2^{\text{poly}(n)}$):
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  \[
  L = \{x \mid \exists w \text{ s.t. } (x,w) \in L', \; |w| = O(2^{\text{poly}(|x|)})\}, \text{ and } L' \text{ in EXP?}
  \]
EXP and NEXP

EXP is $\text{DTIME}(2^{\text{poly}(n)})$:

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$L = \{x \mid \exists w \text{ s.t. } (x,w) \in L', |w| = O(2^{\text{poly}(|x|)}) \}, \text{ and } L' \text{ in EXP?}$

No! $L'$ in $\text{DTIME}(2^{\text{poly}(|x|)})$
EXP and NEXP

EXP is \(\text{DTIME}(2^{\text{poly}(n)})\):

\[ \text{EXP} = \bigcup_{a,b,c>0} \text{DTIME}(2^{an^c+b}) \]

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NEXP = all \(L\) of the form:

\[ L = \{x \mid \exists w \text{ s.t. } (x,w) \in L', \ |w| = O(2^{\text{poly}(|x|)}) \}, \text{ and } L' \text{ in EXP?} \]

No! \(L'\) in \(\text{DTIME}(2^{\text{poly}(|x|)})\)

i.e., \(L'\) in \(P\)
co-Class
co-Class

\[ co-X = \{ L \mid L^c \text{ is in } X \} \text{ (where } L^c = \{ x \mid x \notin L \} \) \]
co-Class

\[ \text{co-}X = \{ L \mid L^c \text{ is in } X \} \text{ (where } L^c = \{ x \mid x \notin L \} \text{) } \]

\[ \text{co-DTIME}(T) = \text{DTIME}(T) \]
co-Class

\[ \text{co-}X = \{ L \mid L^c \text{ is in } X \} \text{ (where } L^c = \{ x \mid x \notin L \} \) \]

\[ \text{co-DTIME}(T) = \text{DTIME}(T) \]

\[ L^c \text{ in } \text{DTIME}(T) \text{ iff } L \text{ in } \text{DTIME}(T) \]
co-Class

co-\(X = \{ L \mid L^c \text{ is in } X \} \) (where \(L^c = \{ x \mid x \notin L \} \))

co-DTIME(T) = DTIME(T)

\(L^c\) in DTIME(T) iff \(L\) in DTIME(T)

\(M_{L^c} \leftrightarrow M_L: \text{flip accept/reject states}\)
co-Class

\[ \text{co-}X = \{ L \mid L^c \text{ is in } X \} \quad \text{(where } L^c = \{ x \mid x \notin L \} \text{)} \]

\[ \text{co-DTIME}(T) = \text{DTIME}(T) \]

\[ L^c \text{ in DTIME}(T) \text{ iff } L \text{ in DTIME}(T) \]

\[ M_{L^c} \leftrightarrow M_L: \text{flip accept/reject states} \]

\[ \text{co-NTIME}(T): \quad \text{all } L \text{ s.t. } L^c \text{ is in NTIME}(T) \]
co-Class

co-X = \{ L \mid L^c \text{ is in } X \} \text{ (where } L^c = \{ x \mid x \notin L \} \)

co-DTIME(T) = DTIME(T)

L^c \text{ in } DTIME(T) \iff L \text{ in } DTIME(T)

M_{L^c} \leftrightarrow M_L: \text{ flip accept/reject states}

co-NTIME(T): \text{ all } L \text{ s.t. } L^c \text{ is in } NTIME(T)

M_{L^c} \leftrightarrow M_L? \text{ flip accept/reject states and flip } \text{“there exists” and “for all” (NTM } \leftrightarrow \text{“co-NTM”)}
co-Class

\[ \text{co-}X = \{ L \mid L^c \text{ is in } X \} \] (where \( L^c = \{ x \mid x \notin L \} \))

\[ \text{co-DTIME}(T) = \text{DTIME}(T) \]

\[ L^c \text{ in DTIME}(T) \iff L \text{ in DTIME}(T) \]

\[ M_{L^c} \leftrightarrow M_L: \text{flip accept/reject states} \]

\[ \text{co-NTIME}(T): \text{ all } L \text{ s.t. } L^c \text{ is in NTIME}(T) \]

\[ M_{L^c} \leftrightarrow M_L? \text{ flip accept/reject states and flip “there exists” and “for all” (NTM} \leftrightarrow \text{“co-NTM”}) \]

\[ L^c = \{ x \mid \nexists w \text{ s.t. } (x,w) \in L' \} = \{ x \mid \forall w \ (x,w) \in L'^c \} \]
co-Class

co-X = \{ L \mid L^c \text{ is in } X \} \text{ (where } L^c = \{ x \mid x \notin L \})

c-o-DTIME(T) = DTIME(T)

L^c \text{ in DTIME(T)} \iff L \text{ in DTIME(T)}

M_{L^c} \leftrightarrow M_L: \text{ flip accept/reject states}

c-o-NTIME(T): \text{ all } L \text{ s.t. } L^c \text{ is in NTIME(T)}

M_{L^c} \leftrightarrow M_L? \text{ flip accept/reject states and flip “there exists” and “for all” (NTM} \leftrightarrow \text{“co-NTM”)}}

L^c = \{ x \mid \not\exists w \text{ s.t. } (x, w) \in L' \} = \{ x \mid \forall w (x, w) \in L^c \}
P, NP and co-NP
P, NP and co-NP

Different possibilities
P, NP and co-NP

- Different possibilities
P, NP and co-NP

Different possibilities
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Different possibilities
P, NP and co-NP

- Different possibilities
- If $P=NP$, then $NP=coNP$
P, NP and co-NP

- Different possibilities
- If P=NP, then
  - coNP = coP = P = NP
P, NP and co-NP

- Different possibilities
- If P=NP, then
  - coNP = coP = P = NP
- Also, EXP = NEXP [Exercise]
Different possibilities

If $P=NP$, then

- $\text{coNP} = \text{coP} = P = NP$

Also, $\text{EXP} = \text{NEXP}$ [Exercise]

padding to scale up both classes
P, NP and co-NP

- Different possibilities

- If \( P = NP \), then
  - \( \text{coNP} = \text{coP} = P = NP \)

- Also, \( \text{EXP} = \text{NEXP} \) \([\text{Exercise}]\)
  - padding to scale up both classes
  - \( x \rightarrow (x, \text{pad}) \), so that \( \text{Exp}(|x|) = \text{Poly}(|x, \text{pad}|) \)
Different possibilities

If $P=NP$, then

- $coNP = coP = P = NP$

Also, $EXP = NEXP$ [Exercise]

- **padding** to scale up both classes

  - $x \rightarrow (x, \text{pad})$, so that $\text{Exp}(|x|) = \text{Poly}(|x, \text{pad}|)$

If $P=NP$, then the complexity landscape would get greatly simplified than believed (more later)
Today
Today

DTIME
Today

- DTIME
- P, EXP
Today

- DTIME
- \( P, \text{ EXP} \)
- NTIME
Today

- **DTIME**

- **P, EXP**

- **NTIME**

- Two views: non-determinism and certificate
Today

- DTIME
- \( P, \ EXP \)
- NTIME
- Two views: non-determinism and certificate
- NP, NEXP
Today

- DTIME
- P, EXP
- NTIME
- Two views: non-determinism and certificate
- NP, NEXP
- co-NTIME
Today

- DTIME
  - P, EXP

- NTIME
  - Two views: non-determinism and certificate

- NP, NEXP

- co-NTIME
  - Two views: co-NTM and “no counter-example”
Next Class Lecture
Next Class Lecture

- NP completeness
Next Class Lecture

- NP completeness
- As hard as it gets inside NP
Next Class Lecture

NP completeness

As hard as it gets inside NP

a la reductions (of course)