CS 574: Randomized Algorithms

Lecture 7. More on Second Moment: Random SAT

September 15, 2015

Random k-SAT

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Conjecture

(Major Open Problem) For every $k \geq 2$ there exists a threshold value ${r_k}^* \in \mathbb{R}$ such that

$$r > {r_k}^* \Rightarrow Pr[\phi_k(n, rn) \quad satisfiable] \rightarrow 0, \quad as \quad r \Rightarrow \infty$$

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- Proved for 2-SAT, we will see shortly.

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Theorem

For every $k \geq 2$ *there exists a sequence* $\{r_k(n)\}$ *such that* $\forall \epsilon > 0$

$$r > (1-\epsilon) r_k(n) \Rightarrow Pr[\phi_k(n, rn) \quad satisfiable] \to 0, \quad as \quad r \Rightarrow \infty$$

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- Lower Bounds discussion: Achilioptas-Peres showed $r_k \geq 2^k \ln 2 - k$, while most other approaches yield $r_k \geq c\frac{2^k}{k}$.

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