CS 574: Randomized Algorithms

Lecture 20. Random Walks and Electrical Networks, contd.
Introduction to Markov Chains.

October 29, 2015

A better bound on the Cover Time

Theorem

If $G = (V, E)$ is a connected graph and $R = \max_{x,y \in V} R_{eff}(x, y)$ is the maximum effective resistance in G, then

$$|E|R \leq cov(G) \leq O(\log n)|E|R$$

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Compare the bounds from this to the complete graph and the lollipop.

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Some Examples

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- Card Shuffling examples.