Cheat Sheet: Chernoff Type Inequalities

September 7, 2015

For more details, see here:

http://sarielhp.org/teach/13/b_574_rand_alg/lec/07_chernoff.pdf

Values	Probabilities	Inequality	Ref
$-1, +1$	$\mathbf{Pr}[X_i = -1] =$ $\mathbf{Pr}[X_i = 1] = 1/2$	$\mathbf{Pr}\left[Y \geq \Delta\right] \leq \exp\left(-\Delta^2/2n\right)$	Theorem 7.1.6$_{\mathrm{p6}}$
		$\mathbf{Pr}\left[Y \leq -\Delta\right] \leq \exp\left(-\Delta^2/2n\right)$	Theorem 7.1.6$_{\mathrm{p6}}$
		$\mathbf{Pr}\left[\lvert Y\rvert \geq \Delta\right] \leq 2\exp\left(-\Delta^2/2n\right)$	Corollary 7.1.7$_{\mathrm{p7}}$
$0, 1$	$\mathbf{Pr}[X_i = 0] =$ $\mathbf{Pr}[X_i = 1] = 1/2$	$\mathbf{Pr}\left[\left\lvert Y - \frac{n}{2}\right\rvert \geq \Delta\right] \leq 2\exp\left(-2\Delta^2/n\right)$	Corollary 7.1.8$_{\mathrm{p7}}$
0,1	$\mathbf{Pr}[X_i = 0] = 1 - p_i$ $\mathbf{Pr}[X_i = 1] = p_i$	$\mathbf{Pr}\left[Y > (1+\delta)\mu\right] < \left(\frac{e^\delta}{(1+\delta)^{1+\delta}}\right)^\mu$	Theorem 7.3.2$_{\mathrm{p12}}$
	For $\delta \leq 2e - 1$	$\mathbf{Pr}[Y > (1+\delta)\mu] < \exp\left(-\mu\delta^2/4\right)$	Theorem 7.3.2$_{\mathrm{p12}}$
	$\delta \geq 2e - 1$	$\mathbf{Pr}[Y > (1+\delta)\mu] < 2^{-\mu(1+\delta)}$	
	$\delta \geq e^2$	$\mathbf{Pr}[Y > (1+\delta)\mu] < \exp\left(-(\mu\delta/2)\ln\delta\right)$	
	For $\delta \geq 0$	$\mathbf{Pr}[Y < (1-\delta)\mu] < \exp\left(-\mu\delta^2/2\right)$	Theorem 7.3.5$_{\mathrm{p13}}$
$X_i \in [0, 1]$	X_is have arbitrary independent distributions.	$\mathbf{Pr}\left[Y - \mu \geq \varepsilon\mu\right] \leq \exp\left(-\varepsilon^2\mu/4\right)$	Theorem 7.4.5$_{\mathrm{p15}}$
		$\mathbf{Pr}\left[Y - \mu \leq -\varepsilon\mu\right] \leq \exp\left(-\varepsilon^2\mu/2\right)$.	
$X_i \in [a_i, b_i]$	X_is have arbitrary independent distributions.	$\mathbf{Pr}\left[\left\lvert Y - \mu\right\rvert \geq \eta\right] \leq 2\exp\left(-\frac{2\eta^2}{\sum_{i=1}^n (b_i - a_i)^2}\right)$	Theorem 7.5.3$_{\mathrm{p18}}$

Table 6.1: Summary of Chernoff type inequalities. Here we have n independent random variables $X_1, \ldots, X_n$, $Y = \sum_i X_i$ and $\mu = \mathbf{E}[Y]$.

Disclaimer: There are many more inequalities known, see for example:

D. Dubhashi and A. Panconesi.
Concentration of Measure for the Analysis of Randomized Algorithms.
Cambridge University Press, 2009.