Single-view Metrology and Camera Calibration

Computer Vision
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Last Class: Pinhole Camera

Camera Center \((t_x, t_y, t_z)\)

Principal Point \((u_0, v_0)\)

Projection Matrix

\[
P = \begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}
\]

Projection Point \((u, v)\)

Distance \(f\)

Distance \(Z\)
Last Class: Projection Matrix

\[
x = K[R \ t]X
\]

Where:
- \( K \) is the camera intrinsic matrix.
- \( R \) is the rotation matrix.
- \( t \) is the translation vector.
- \( X, Y, Z \) are the 3D world coordinates.
- \( u, v \) are the 2D image coordinates.
- \( f, s, u_0, v_0 \) are intrinsic parameters.
- \( r_{ij} \) are the elements of the rotation matrix.
- \( t_x, t_y, t_z \) are the translation components.
- \( \alpha, \beta, \gamma \) are scale factors.

The projection matrix \( P = K[R \ t] \) maps 3D world coordinates to 2D image coordinates.
Last class: Vanishing Points

Vertical vanishing point (at infinity)

Vanishing line

Vanishing point

Vanishing point

Slide from Efros, Photo from Criminisi
This class

• How can we calibrate the camera?

• How can we measure the size of objects in the world from an image?

• What about other camera properties: focal length, field of view, depth of field, aperture, f-number?
How to calibrate the camera?

\[ x = K[R \quad t] X \]

\[
\begin{bmatrix}
wu \\
wv \\
w
\end{bmatrix}
= \begin{bmatrix}
* & * & * & * & * \\
* & * & * & * & * \\
* & * & * & * & * \\
\end{bmatrix}
\begin{bmatrix}
X \\
Y \\
Z \\
1
\end{bmatrix}
\]
Calibrating the Camera

Method 1: Use an object (calibration grid) with known geometry
– Correspond image points to 3d points
– Get least squares solution (or non-linear solution)
Linear method

• Solve using linear least squares

\[
\begin{bmatrix}
w

\end{bmatrix} =
\begin{bmatrix}
m_{11} & m_{12} & m_{13} & m_{14} \\
m_{21} & m_{22} & m_{23} & m_{24} \\
m_{31} & m_{32} & m_{33} & m_{34}
\end{bmatrix}
\begin{bmatrix}
X \\
Y \\
Z \\
1
\end{bmatrix}
\]

\[
\begin{bmatrix}
X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -u_1X_1 & -u_1Y_1 & -u_1Z_1 & -u_1 \\
0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -v_1X_1 & -v_1Y_1 & -v_1Z_1 & -v_1 \\
0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & -u_nX_n & -u_nY_n & -u_nZ_n & -u_n \\
0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & -v_nX_n & -v_nY_n & -v_nZ_n & -v_n
\end{bmatrix}
\begin{bmatrix}
m_{11} \\
m_{12} \\
m_{13} \\
m_{14} \\
m_{21} \\
m_{22} \\
m_{23} \\
m_{24} \\
m_{31} \\
m_{32} \\
m_{33} \\
m_{34}
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

Ax=0 form
Calibration with linear method

• Advantages
  – Easy to formulate and solve
  – Provides initialization for non-linear methods

• Disadvantages
  – Doesn’t directly give you camera parameters
  – Doesn’t model radial distortion
  – Can’t impose constraints, such as known focal length
  – Doesn’t minimize projection error

• Non-linear methods are preferred
  – Define error as difference between projected points and measured points
  – Minimize error using Newton’s method or other non-linear optimization

Can solve for explicit camera parameters:
http://ksimek.github.io/2012/08/14/decompose/
Calibrating the Camera

Method 2: Use vanishing points

– Find vanishing points corresponding to orthogonal directions
Calibration by orthogonal vanishing points

• Intrinsic camera matrix
  – Use orthogonality as a constraint
  – Model K with only $f$, $u_0$, $v_0$

\[ \mathbf{p}_i = \mathbf{KRX}_i \]

For vanishing points

\[ \mathbf{X}_i^T \mathbf{X}_j = 0 \]

• What if you don’t have three finite vanishing points?
  – Two finite VP: solve $f$, get valid $u_0$, $v_0$ closest to image center
  – One finite VP: $u_0$, $v_0$ is at vanishing point; can’t solve for $f$
Calibration by vanishing points

• Intrinsic camera matrix

\[ p_i = KRX_i \]

• Rotation matrix
  – Set directions of vanishing points
    • e.g., \( X_1 = [1, 0, 0] \)
  – Each VP provides one column of \( R \)
  – Special properties of \( R \)
    • \( \text{inv}(R) = R^T \)
    • Each row and column of \( R \) has unit length
How can we measure the size of 3D objects from an image?
Perspective cues
Perspective cues
Perspective cues
Ames Room

Actual position of Person A

Apparent position of Person A

Actual and apparent position of person B

Apparent shape of room

Viewing peephole
Comparing heights

Vanishing Point
Measuring height

Camera height

5.3
3.3
2.8
Which is higher – the camera or the man in the parachute?
The cross ratio

A Projective Invariant

• Something that does not change under projective transformations (including perspective projection)

The cross-ratio of 4 collinear points

\[ \frac{\|P_3 - P_1\| \|P_4 - P_2\|}{\|P_3 - P_2\| \|P_4 - P_1\|} \]

Can permute the point ordering

• \(4! = 24\) different orders (but only 6 distinct values)

This is the fundamental invariant of projective geometry
Measuring height

scene cross ratio

\[
\frac{\|B - T\|}{\|B - R\|} \frac{\|\infty - R\|}{\|\infty - T\|} = \frac{H}{R}
\]

image cross ratio

\[
\frac{\|b - t\|}{\|b - r\|} \frac{\|v_z - r\|}{\|v_z - t\|} = \frac{H}{R}
\]

scene points represented as \( P = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \)

image points as \( p = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \)
Measuring height

\[ v \approx (b \times b_0) \times (v_x \times v_y) \]

vanishing line (horizon)

\[ \|t - b\| \|v_z - r\| \]
\[ \|r - b\| \|v_z - t\| \]

image cross ratio
What if the point on the ground plane $b_0$ is not known?

- Here the guy is standing on the box, height of box is known
- Use one side of the box to help find $b_0$ as shown above
What about focus, aperture, DOF, FOV, etc?
Adding a lens

• A lens focuses light onto the film
  – There is a specific distance at which objects are “in focus”
    • other points project to a “circle of confusion” in the image
  – Changing the shape of the lens changes this distance
Focal length, aperture, depth of field

A lens focuses parallel rays onto a single focal point
- focal point at a distance $f$ beyond the plane of the lens
- Aperture of diameter $D$ restricts the range of rays
The eye

- The human eye is a camera
  - **Iris** - colored annulus with radial muscles
  - **Pupil** (aperture) - the hole whose size is controlled by the iris
  - **Retina** (film): photoreceptor cells (rods and cones)

Slide source: Seitz
Depth of field

Changing the aperture size or focal length affects depth of field

Varying the aperture

Large aperture = small DOF

Small aperture = large DOF
Shrinking the aperture

- Why not make the aperture as small as possible?
  - Less light gets through
  - Diffraction effects
Shrinking the aperture

Slide by Steve Seitz
Relation between field of view and focal length

Field of view (angle width)

\[ \text{fov} = \tan^{-1} \left( \frac{d}{2f} \right) \]

Film/Sensor Width

Focal length
Dolly Zoom or “Vertigo Effect”

http://www.youtube.com/watch?v=NB4bikrNzMk

How is this done?

Zoom in while moving away

http://en.wikipedia.org/wiki/Focal_length
Review

How tall is this woman?

How high is the camera?

What is the camera rotation?

What is the focal length of the camera?

Which ball is closer?
Next class

- Image stitching