3. **Image-Order Volume Visualization**

Consider the following ray through a pixel into a volume in which the scalar data are all in [0, 1]. One particular ray moves through the following 3 cells:

<table>
<thead>
<tr>
<th>0.75</th>
<th>0.25</th>
<th>0.5</th>
</tr>
</thead>
</table>

Suppose the transfer function we use is simply \( I(s) = (1 - s, 0, s, s) \)

Suppose the distance the ray traveled through each cell is 1

a. What is the color produced by a Maximum Intensity Projection?

\[ \text{max} = \frac{3}{4} \]

\[ I(\frac{3}{4}) = \left( \frac{1}{4}, 0, \frac{3}{4}, \frac{3}{4} \right) \]

b. What is the color produced by an Average Intensity Projection?

\[ \text{avg} = \frac{1}{2} \]

\[ I(\frac{1}{2}) = \left( \frac{1}{2}, 0, \frac{1}{2}, \frac{1}{2} \right) \]

c. How would the color be produced by compositing with the Over operator? Just write out an expression, don't do the computation.

\[ \left( \frac{1}{4}, 0, \frac{3}{4}, \frac{3}{4} \right) \text{ over } \left( \frac{3}{4}, 0, \frac{1}{4}, \frac{1}{4} \right) \text{ over } \left( \frac{1}{2}, 0, \frac{1}{2}, \frac{1}{2} \right) = \frac{3}{4} \left( \frac{1}{4}, 0, \frac{3}{4}, \frac{3}{4} \right) + \frac{1}{4} \left( \frac{3}{8}, \frac{3}{4}, 0, \frac{1}{4} \right) + \frac{1}{2} \left( \frac{1}{2}, 0, \frac{1}{2}, \frac{1}{2} \right) \]

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**Pre-multiplied alpha**

Suppose we use pre-multiplied alpha. A color \((r, g, b)\) and an alpha value \(\alpha\) is stored as \((\alpha r, \alpha g, \alpha b, \alpha)\). Compositing of two colors with alpha values can be accomplished using the over operator: \(C_A \text{ over } C_B = C_A + (1 - \alpha_A)C_B\)

4. Derive a set of expressions using pre-multiplied alpha that allows front to back volume rendering. What advantage in terms of optimizations does this expression have? Can this be done with post-multiplied alpha?

\[ \hat{C}_{i+1} = \hat{C}_i + (1 - \hat{A}_i) \hat{C}_i \]

\[ \hat{A}_{i+1} = \hat{A}_i + (1 - \hat{A}_i) A_i \]