Volume Visualization

The Over Operator

Suppose we store colors with an alpha value that indicates the level of transparency with 0 being transparent and 1 being opaque. We will use pre-multiplied alpha. A color (r,g,b) and an alpha value \( \alpha \) is stored as \( (ar, ag, ab) \).

Compositing of two colors with alpha values can be accomplished using the over operator: \( C_A \) over \( C_B = C_A + (1-\alpha_A)C_B \)

1. Blending

Suppose \( C_A = (0.5, 0.5, 0.75, 0.75) \)
\[ C_B = (0.0, 0.25, 0.25, 0.5) \]

a. Compute \( C_A \) over \( C_B \)
\[
\begin{pmatrix}
\frac{1}{2} & \frac{9}{16} & \frac{13}{16}
\end{pmatrix}
\]

b. Compute \( \alpha_{A\text{over}B} \)
\[
\frac{7}{8}
\]

2. Algebra for the over operator

a. Prove \( C_A \) over \( (C_B \) over \( C_C) = (C_A \) over \( C_B \) over \( C_C \)
(Associative Law)

See last page

b. Prove that the Over operator is not commutative
By counterexample: black over white and white over black both with alpha values of 1 produce different colors
3. Image-Order Volume Visualization

Consider the following ray through a volume in which the scalar data are all in \([0,1]\). One particular ray moves through the following 3 cells:

\[
\begin{array}{c|c|c}
0.75 & 0.25 & 0.5 \\
\hline
C_A & C_B & C_C
\end{array}
\]

Suppose the transfer function we use is simply \(I(s) = (1 - s, 0, s, s)\)

Suppose the distance the ray traveled through each cell is 1

a. What is the color produced by a Maximum Intensity Projection?

\[
\begin{align*}
I(3/4) &= (3/4, 3/4, 3/4) \\
\end{align*}
\]

b. What is the color produced by an Average Intensity Projection?

\[
\begin{align*}
I_{AV} &= 1/2 \\
I(1/2) &= (1/2, 0, 1/2, 1/2)
\end{align*}
\]

c. How would the color be produced by compositing with the Over operator? Just write out an expression, don’t do the computation.

\[
\begin{align*}
\frac{3/4(4,0,0) + 1/4(5/8, \frac{9}{8} \cdot 0, \frac{9}{8} \cdot 3/4, \frac{9}{8} \cdot 3/4)}{d_A C_A + (1 - d_A) C_B}
\end{align*}
\]

4. Suppose a ray enters two cells in succession and then leaves the volume:

1. enters a cell with scalar \(s_1\) at \((x_1, y_1)\)
2. enters a cell with scalar \(s_2\) at \((x_2, y_2)\)
3. leaves the volume at \((x_3, y_3)\)

Assume we are using a ray just to integrate the scalar function along a line. Derive a formula for the accumulated value along the ray.

\[
S = \frac{s_1 \text{ dist}(P_1, P_2) + s_2 \text{ dist}(P_2, P_3)}{\text{dist}(P_1, P_3)}
\]

For visualization we may normalize \(S/\text{dist}(P_1, P_3)\)
2. Algebra for the Over operator

a. Prove \( C_A \over (C_B \over C_C) = (C_A \over C_B) \over C_C \) 

(Associative Law)

Assume \( C_A \neq C_B \neq C_C \)

\[
C_A + (1 - x_A) \left( C_B + (1 - x_B) C_C \right) \\
C_A + (1 - x_A) C_B + (1 - x_B) (1 - x_D) C_C \\
C_{AB} + \left( (1 - x_A) - (1 - x_B)x_D \right) C_C \\
1 - (x_A + (1 - x_B)x_D) C_C \\
\]

For pre-multiplied \( \alpha \)