1. **Classifying Critical Points**

Suppose we have a 2D vector field defined as \( \mathbf{v}(x) = (y^2 + y, x^2 + x) \)

- **a.** What is a critical point in the vector field? The point \((0,0)\) since the magnitude of the field is 0 at that point \( \|0^2 + 0, 0^2 + 0\| = 0 \)

- **b.** What is the Jacobian of the vector field? Recall that in 2D it takes the form of the following matrix:

\[
J = \begin{vmatrix}
\frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} \\
\frac{\partial v_y}{\partial x} & \frac{\partial v_y}{\partial y}
\end{vmatrix}
\]

\[
= \begin{bmatrix}
0 & 2y + 1 \\
2x + 1 & 0
\end{bmatrix}
\]

- **c.** Evaluate the Jacobian at the critical point.

\[
\begin{bmatrix}
0 & 1 \\
1 & 0
\end{bmatrix}
\]

- **d.** What are the eigenvalues of the Jacobian at the critical point?

\[
\begin{vmatrix}
0 - \lambda & 1 \\
1 & 0 - \lambda
\end{vmatrix} = \lambda^2 - 1, \text{ so the roots are } +1 \text{ and } -1
\]

- **e.** Classify the critical point based on the eigenvalues. Is it a source, sink, saddle, center, or focus? **Saddle**
2. **Displacement Surfaces**

Suppose we place a sphere of radius 1 centered at the origin within a vector field defined by \( \mathbf{v}(x,y,z) = (x^2 + x, y^2 + x, z^2 + x) \)

1. If the surface of the sphere is displaced by the vector field using the formula

\[
S'_{\text{displ}} = \{ x + \mathbf{v}(x) \Delta t, \ \forall x \in S \}
\]

where is the sphere surface point \((1,0,0)\) moved to by the field if we use a timestep of \(\frac{1}{2}\)?

\[
(1,0,0) + \frac{1}{2} <2,1,1> = (2, \frac{1}{2}, \frac{1}{2})
\]

2. We can limit displacement to non-tangential motion by calculating the following displacement

\[
S'_{\text{displ}} = \{ x + (\mathbf{v}(x) \mathbf{n}(x)) \mathbf{n}(x) \Delta t, \ \forall x \in S \}
\]

Using that formula and the same values as part 1, to where is \((1,0,0)\) displaced?

The normal is \(<1,0,0>\) since the tangent plane to the sphere is the \(x=1\) plane.

\[
(1,0,0) + \left( (1,0,0) \cdot (2,1,1) \frac{1}{2} \right) = (2,0,0)
\]