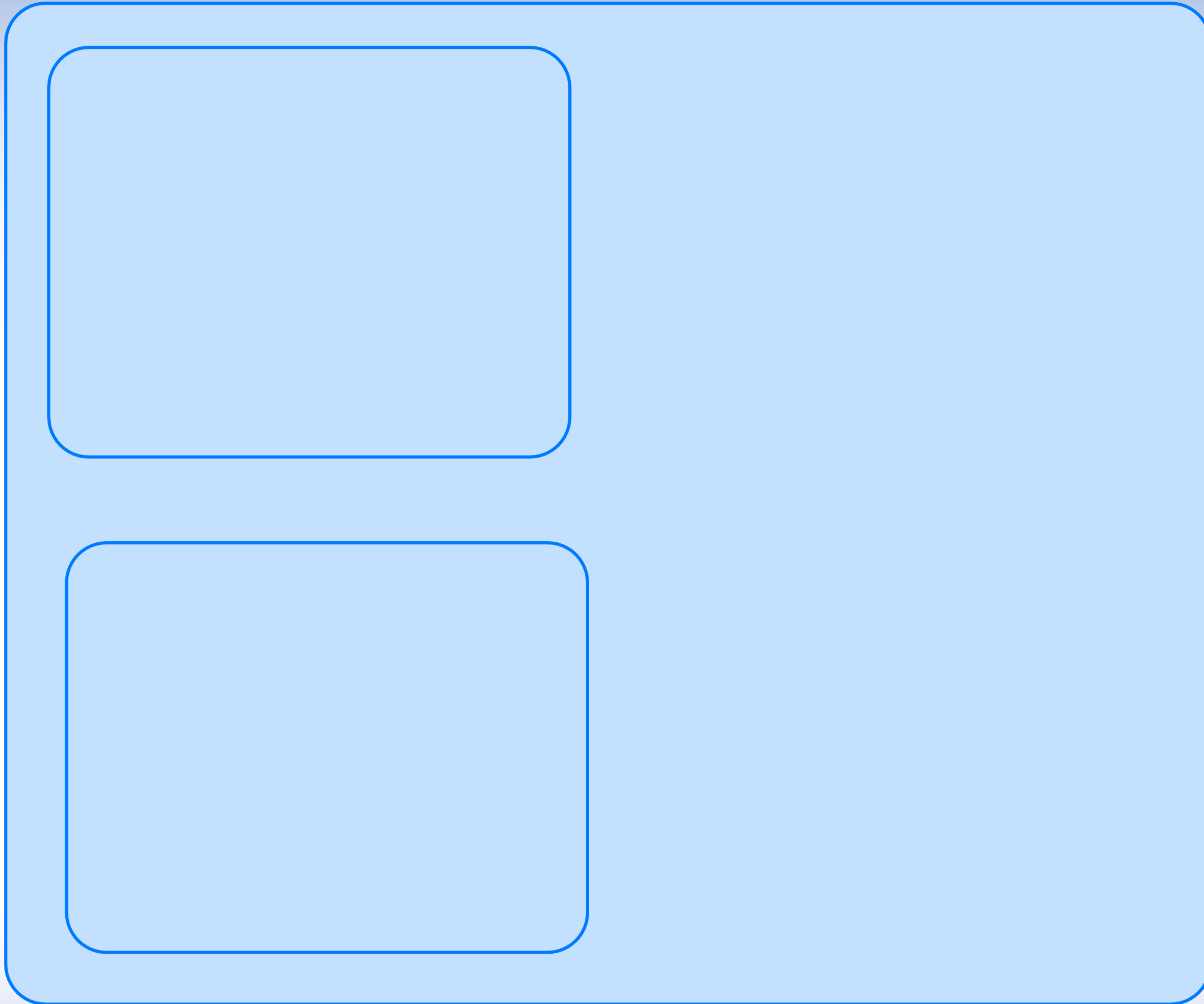


Announcements

- Work in groups - groups of 2 for MPs, groups of 2-4 for the final project.
- MP1 is due tonight, 11:59pm.
- MP2 is out today, due Sep 29 11:59pm.
- Read Chapter 6 and 7 of Shirley for background on transformations.

Transformations

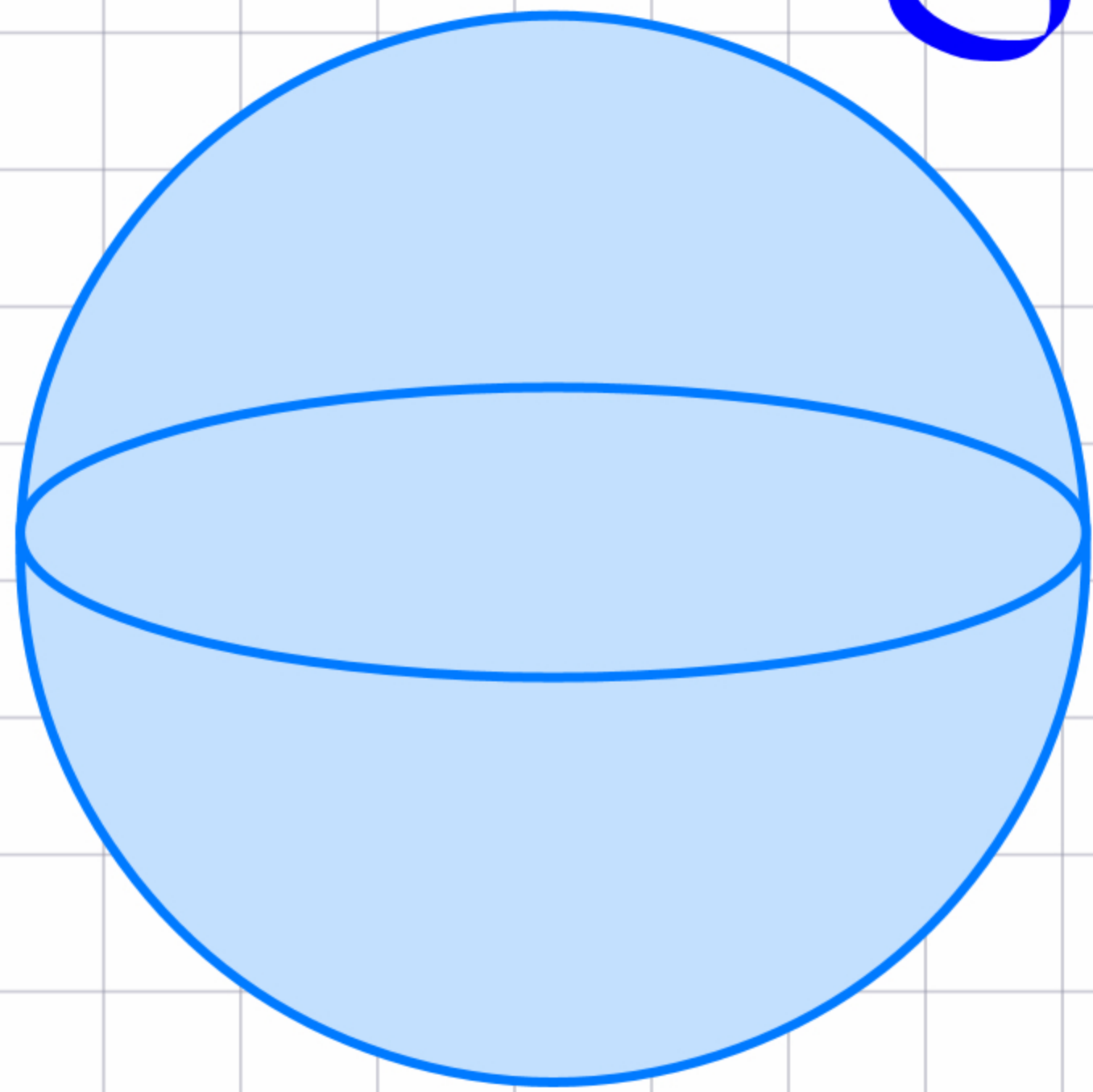


Why math?

Better Representation of Rotations: Unit Quaternions

$$q = (a, b, c, d) \in \mathbb{R}^4,$$

The set of all unit q is a hypersphere (S^3)



S^2 lives in \mathbb{R}^3

S^1 lives in \mathbb{R}^2

S^h lives in \mathbb{R}^{h+1}

In Unity 3D:

In math:

From Axis-Angle to Unit Quaternions

Axis-angle: (θ, \vec{v})

Corresponding unit quaternion:

Sanity check:

Unit Quaternions: Examples

$$q = \left(\cos \frac{\theta}{2}, v_1 \cdot \sin \frac{\theta}{2}, v_2 \cdot \sin \frac{\theta}{2}, v_3 \cdot \sin \frac{\theta}{2} \right)$$

$$(1, 0, 0, 0)$$

$$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0 \right)$$

$$(0, 1, 0, 0)$$

$$\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}, 0 \right)$$

$$(0, 0, 1, 0)$$

$$\left(\frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}} \right)$$

$$(0, 0, 0, 1)$$

$$(1, 1, 1, 1) ?$$

Unit Quaternions: Inverses and Duplicates

$$q = \left(\cos \frac{\theta}{2}, v_1 \cdot \sin \frac{\theta}{2}, v_2 \cdot \sin \frac{\theta}{2}, v_3 \cdot \sin \frac{\theta}{2} \right)$$

(a, b, c, d)

Unit Quaternions: Multiplication

$$q = (a, b, c, d)$$

Order of operations?

Inverses?

Efficiency?

Conversions

Special
Orthogonal
Matrix R

Quaternions
 q

Yaw, Pitch, Roll
 $R_y(\alpha) R_x(\beta) R_z(\delta)$

Axis-angle
 (\vec{v}, θ)

Sample Problem

Steve is a Minecraft character. His head is a cube. The center of his head is the origin of Steve's coordinate frame, in which his left pupil has coordinates $(1, 0, 3)$. Initially, Steve was placed in a position $(x, y, z) = (10, 10, 10)$ and orientation $(\alpha, \beta, \gamma) = (0, 0, 0)$ in the global coordinate frame. After that, Steve turned his head by a yaw of 90 degrees and walked along a vector $(10, 0, 10)$. Calculate the coordinates of his left pupil in the global coordinate frame after Steve's walk.



Sample Problem

Steve is a Minecraft character. His head is a cube. The center of his head is the origin of Steve's coordinate frame, in which his left pupil has coordinates $(1, 0, 3)$. Initially, Steve was placed in a position $(x, y, z) = (10, 10, 10)$ and orientation $(1, 0, 0, 0)$ in the global coordinate frame. After that, Steve turned his head by a quaternion $(0.5, 0.5, 0.5, 0.5)$ and walked along a vector $(10, 0, 10)$. Calculate the coordinates of Steve's left pupil in the global coordinate frame after his walk.



Applying Quaternion Rotation to a Vector

Vector $(x, y, z) \in \mathbb{R}^3$

Rotate by quaternion q

$$p = (0, x, y, z)$$

$$p' = q \circ p \circ q^{-1}$$

To read the result take the
last three components of p' only.

Using Quaternions for Head-Tracking

