Last Time on CS 498

● Why are quaternions useful? What can they do that euler angles can’t?
  ○ Exam question detected.

● Give the matrix that would transform a 3D object by:
  ○ Rotating $\theta$ degrees around the Z axis
  ○ Then translating by $(x, y, z)$.

  ■ Would the answer change if the steps were reversed?
Homogeneous Transformation Matrices

- Translate by \( t = (3, 4, 5) \)
- Then rotate by \[
\begin{bmatrix}
1 & 0 & 0 \\
0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\
0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{bmatrix}
\]
- ... What matrix combines these two transformations?
Homogeneous Transformation Matrices

What is the geometric interpretation of the following matrices?
Homogeneous Transformation Matrices

Case #1
\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
= 
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Case #2
\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
= 
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Homogeneous matrix for R and \(t\)
Homogeneous Transformation Matrix Inverse
Switching Coordinate Frames

Flower: $F = (2, 0, 1)$

Pupil: $P = (1, 0, 3)$

Steve’s local coordinate system coincides with the global system. Then Steve rotates by $\pi/2$ yaw and translates by $(-10, 10, 0)$.

- Write Steve’s homogeneous transformation matrix.
- Find the coordinates of the flower from Steve’s (LCF) perspective after the transformation.
Switching Coordinate Frames

Flower: $F = (2, 0, 1)$

Pupil: $P = (1, 0, 3)$

Steve’s local coordinate system coincides with the global system. Then Steve rotates by $\pi/2$ yaw and translates by $(-10, 10, 0)$.

Steve’s transform matrix:

$$T = \begin{bmatrix}
& & & \\
& & & \\
& & & \\
0 & 0 & 0 & 1
\end{bmatrix}$$
Switching Coordinate Frames

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Steve’s local coordinate system coincides with the global system. Then Steve rotates by $\pi/2$ yaw and translates by $(-10, 10, 0)$.
Switching Coordinate Frames

Flower: $F = (2, 0, 1)$

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Steve’s local coordinate system coincides with the global system. Then Steve rotates by $\pi/2$ yaw and translates by $(-10, 10, 0)$.

The inverse of $T$ represents the switch of the “view point” from global coordinate frame to local coordinate frame.
Viewing Transformations
From World Coordinate Frame to Pixels on Screen

Goal:

Ignore for now:

What is different from previous geometric transformations?
Object Frame to World Frame

The chain of transformations starts with:

- for moving objects.
- for stationary objects.
World Frame to Eye Frame

The eye is a rigid body, too.
Eye Frame to Canonical Frame

In canonical frame

$x, y$ coordinates:

$z$ coordinate:
Canonical Frame to Viewport Frame
Algebraic Representation
Algebraic Representation: Cyclopean Eye Transformation

Consider a “look at”:

1. Coordinate axis for the eye in the world:

\[ \hat{x} = \hat{y} = \hat{z} = \]

In Graphics:

In VR:

Rotation matrix:

\[ R = \left[ \begin{array}{ccc} \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \end{array} \right] \]
Algebraic Representation: Cyclopean Eye Transformation

To place the eye in the world:

To convert from world frame to eye frame:
Algebraic Representation: Left Eye Transformation

To place the left eye in the world:

To convert from world frame to left eye frame:
From Alternate World Generator to GPU
Canonical Transformation
Canonical Transformation

Incorrect perspective

Correct perspective
Canonical Transformation: 2D Analogy
Canonical Transformation: 2D Analogy

\[
\begin{align*}
&\text{if } z_p = n \Rightarrow z_{\text{near}} = z_p + f - \frac{nf}{z_p} = n \\
&\text{if } z_p = f \Rightarrow z_{\text{far}} = z_p + f - \frac{nf}{z_p} = f \\
&z_p = \frac{n + f}{2} \Rightarrow . \\
\end{align*}
\]

\[
\begin{bmatrix}
  n & y_p \\
  (n+f)z_p - nf & z_p \\
\end{bmatrix}
= \begin{bmatrix}
  n & 0 & 0 \\
  0 & n+f & -nf \\
  0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  y_p \\
  z_p \\
  1
\end{bmatrix}
\]

\[
y = \frac{n}{z_p} y \\
z = n+f - \frac{nf}{z_p}
\]

Depth order is present.
Canonical Transformation

\[ T_p = \begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -nf \\ 0 & 0 & 1 & 0 \end{bmatrix} \]
Canonical Transformation

\[
T_{st} = \begin{bmatrix}
\frac{2}{s-e} & 0 & 0 & \frac{-s}{s-e} \\
0 & \frac{2}{t-b} & 0 & \frac{-t}{t-b} \\
0 & 0 & \frac{2}{n-f} & \frac{-n}{n-f} \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
Viewport Transformation

$T_{vp}$ converts -1 .. 1 range to pixel coordinates:

$n_x = \#$ horizontal pixels

$n_y = \#$ vertical pixels
Next Time on CS 498: Light and Optical Systems

Alternate World Generator: proper lighting and shadows.

Lens: proper correction for the lens distortion.
Review

● How could a matrix that is the product of several homogeneous transformations be inverted?
  ○ What would the inverted matrix “mean”?

● Does Unity store global object coordinates or local object coordinates?
  ○ Why is Unity’s choice the “natural” choice? Does it make your life easier?
Announcements

- MP 2.1 & Team Formation was due!
  - But you already did them so it’s fine.

- MP 2.2-2.4 is due **next Monday (02/19)**.

- Read Ch. 3.4 & 3.5