Sorting networks

0-1 principle

n inputs → n outputs

Bitonic sort

shift

Bitonic sequence

HC[n]

BS[n]

$D_{BS}(n) = 1 + D_{BS}(\frac{n}{2}) = \log n$

$\#_{BS}(n) = |HC(n)| + 2\#_{BS}(\frac{n}{2}) = O(n \log n)$

Lemma: Bitonic sorter sorts bitonic sequences with a circuit

gates $= O(n \log n)$

depth $= \log n$

MS[n]

BS[n]

iterative / recursive

$D_{MS}(n) = D_{MS}(\frac{n}{2}) + D_{BS}(n) = D_{MS}(\frac{n}{2}) + O(\log n) = O(\log^2 n)$

$= \log n + \log \frac{n}{2} + \log \frac{n}{4} \cdots$

$\leq \log n$

$G(n) = 2G(\frac{n}{2}) + \#_{BS}(n)$

$= 2G(\frac{n}{2}) + O(n \log n) = O(n \log^2 n)$

Bitonic sorting network

Parallel algorithms

PRAM — Parallel RAM

n cores, n^k cores

Assumption: memory access is $O(1)$

memory

L2

L3 cache

memory

PRAM is real

Sorting: $O(\log n)$ time, $O(n \log n)$ ... $O(n \log n)$

Prefix sum

$b_i := \sum_{k=1}^{i} a_k$

$O(\log n)$ — $O(n)$

B

n processors

$O(\log n)$

binary tree

$n + \frac{n}{2} + \frac{n}{4} \cdots = O(n)$ — n cpus, $O(\log n)$ time

Parallel quicksort

< p | p | > p

last

$O(\log n)$

active prefix sum

$O(\log n)$

$O(\log n)$ parallel time partition

$T(n) = O(\log n) + T(\frac{3}{4}n)$

$= O(\log n) + \log(\frac{4}{3}n) + \log((\frac{3}{4})^2 n) + \cdots + \log((\frac{3}{4})^{\log n} n) \cdot 1$

$n \to \frac{3}{4}n \to (\frac{3}{4})^2 n$

$(\frac{3}{4})^{\log_{\frac{4}{3}} n} n$

$O(\log n)$ ← | $O(\log n)$