Announcements

- Work in groups - groups of 2 for MPs, groups of 2-4 for the final project.
- MP1 is due tonight, 11:59pm.
- MP2 is out today, due Sep 29 11:59pm.
- Read Chapter 6 and 7 of Shirley for background on transformations.
Transformations

Why math?
Representation of Rotations: Unit Quaternions

\[ q = (a, b, c, d) \in \mathbb{R}^4, \]

The set of all unit \( q \) is a hypersphere \( S^3 \)

\( S^2 \) lives in \( \mathbb{R}^3 \)
\( S^1 \) lives in \( \mathbb{R}^2 \)
\( S^0 \) lives in \( \mathbb{R}^1 \)

In Unity 3D:

In math:
From Axis-Angle to Unit Quaternions

Axis-angle: \((\theta, \vec{v})\)

Corresponding unit quaternion:

Sanity check:
Unit Quaternions: Examples

\[ q = \left( \cos \frac{\theta}{2}, \mathbf{v}_1 \cdot \sin \frac{\theta}{2}, \mathbf{v}_2 \cdot \sin \frac{\theta}{2}, \mathbf{v}_3 \cdot \sin \frac{\theta}{2} \right) \]

\[(1, 0, 0, 0) \quad (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0)\]

\[(0, 1, 0, 0) \quad (\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}, 0)\]

\[(0, 0, 1, 0) \quad (\frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}})\]

\[(0, 0, 0, 1) \quad (\frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}})\]

\[(1, 1, 1, 1, 1) ? \]

https://www.wolframalpha.com/input/?i=quaternion%3A+0%2B2i-j-3k&lk=3
Unit Quaternions: Inverses and Duplicates

\[ q = \left( \cos \frac{\theta}{2}, \ v_1 \cdot \sin \frac{\theta}{2}, \ v_2 \cdot \sin \frac{\theta}{2}, \ v_3 \cdot \sin \frac{\theta}{2} \right) \]

\[(a, b, c, d)\]
Unit Quaternions: Multiplication

\[ q = (a, b, c, d) \]

Order of operations?

Inverses?

Efficiency?
Conversions

Special Orthogonal Matrix $\mathbf{R}$

Quaternions $\mathbf{q}$

Yaw, Pitch, Roll $R_y(\phi) R_x(\theta) R_z(\psi)$

Axis-angle $(\mathbf{v}, \theta)$
Sample Problem

Steve is a Minecraft character. His head is a cube. The center of his head is the origin of Steve's coordinate frame, in which his left pupil has coordinates (1, 0, 3). Initially, Steve was placed in a position (x, y, z) = (10, 10, 10) and orientation (α, β, γ) = (0, 0, 0) in the global coordinate frame. After that, Steve turned his head by a yaw of 90 degrees and walked along a vector (10, 0, 10). Calculate the coordinates of his left pupil in the global coordinate frame after Steve's walk.

http://www.shamusyoung.com/twentyssidedtale/?p=11241
Sample Problem

Steve is a Minecraft character. His head is a cube. The center of his head is the origin of Steve's coordinate frame, in which his left pupil has coordinates $(1, 0, 3)$. Initially, Steve was placed in a position $(x, y, z) = (10, 10, 10)$ and orientation $(1, 0, 0, 0)$ in the global coordinate frame. After that, Steve turned his head by a quaternion $(0.5, 0.5, 0.5, 0.5)$ and walked along a vector $(10, 0, 10)$. Calculate the coordinates of Steve's left pupil in the global coordinate frame after his walk.
Applying Quaternion Rotation to a Vector

Vector \( (x, y, z) \in \mathbb{R}^3 \)

Rotate by quaternion \( q \)

\[ p = (0, x, y, z) \]

\[ p' = q \circ p \circ q^{-1} \]

To read the result, take the last three components of \( p' \) only.
Using Quaternions for Head-Tracking