New CS 473: Algorithms, Spring 2015

Homework 11

This homework will not be graded.

1. In Homework 10, we considered several different problems that can be solved by reducing them to a linear programming problem:

   - Finding a line that fits a given set of \( n \) points in the plane with minimum \( L_1 \) error.
   - Finding a line that fits a given set of \( n \) points in the plane with minimum \( L_\infty \) error.
   - Finding the largest matching in a bipartite graph.
   - Finding the smallest vertex cover in a bipartite graph.

   The specific linear programs are described in the homework solutions. For each of these linear programs, answer the following questions in the language of the original problem:

   (a) What is a basis?
   (b) (For the line-fitting problems only:) How many different bases are there?
   (c) What is a feasible basis?
   (d) What is a locally optimal basis?
   (e) What is a pivot?

2. Let \( G = (V,E) \) be an arbitrary directed graph with weighted vertices; vertex weights may be positive, negative, or zero. A prefix of \( G \) is a subset \( P \subseteq V \), such that there is no edge \( u \to v \) where \( u \notin P \) but \( v \in P \). A suffix of \( G \) is the complement of a prefix. Finally, an interval of \( G \) is the intersection of a prefix of \( G \) and a suffix of \( G \). The weight of a prefix, suffix, or interval is the sum of the weights of its vertices.

   (a) Describe a linear program that characterizes the maximum-weight prefix of \( G \). Your linear program should have one variable per vertex, indicating whether that vertex is or is not in the chosen prefix.

   (b) Describe a linear program that characterizes the maximum-weight interval of \( G \).

   [Hint: Don't worry about the solutions to your linear programs being integral; they will be. If all vertex weights are negative, the maximum-weight interval is empty; if all vertex weights are positive, the maximum-weight interval contains every vertex.]