

Prove that the following languages are undecidable *using Rice’s Theorem*:

**Rice’s Theorem.** The language  $\{\langle M \rangle \mid L(M) \text{ satisfies property } P\}$  is undecidable for any property  $P$  that is satisfied by at least one, and not all, recursively enumerable languages.

1.  $\text{ACCEPTREGULAR} := \{\langle M \rangle \mid L(M) \text{ is regular}\}$

**Example solution:** We need to show that the property of being regular is satisfied by at least one, but not all, r.e. languages, and then Rice’s theorem will be applicable. Clearly for any regular language  $R$  there is a TM  $M$  such that  $L(M) = R$ , so the property of being regular holds for at least one r.e. language (namely,  $L(M)$ ). Just as clearly, there is a TM  $M'$  that accepts  $\{0^n 1^n \mid n \geq 0\}$ , a nonregular language, so  $L(M')$  is an r.e. language that is not regular. Since at least one, but not all, r.e. languages satisfy the property of being regular, we can apply Rice’s theorem and conclude that  $\text{ACCEPTREGULAR}$  is not decidable.

2.  $\text{ACCEPTILLINI} := \{\langle M \rangle \mid \text{ILLINI} \in L(M)\}$
3.  $\text{ACCEPTPALINDROME} := \{\langle M \rangle \mid M \text{ accepts at least one palindrome}\}$
4.  $\text{ACCEPTTHREE} := \{\langle M \rangle \mid M \text{ accepts exactly three strings}\}$
5.  $\text{ACCEPTUNDECIDABLE} := \{\langle M \rangle \mid L(M) \text{ is undecidable}\}$

**Recursive, recursively enumerable (but not recursive), or not recursively enumerable?**

Give the best answer

1.  $\{\langle M \rangle \mid M \text{ has more than 374 states}\}$
2.  $\{\langle M \rangle \mid M \text{ on blank input never moves its head}\}$
3.  $\{\langle M \rangle \mid L(M) \text{ contains a string with 374 characters}\}$
4.  $\{\langle M \rangle \mid L(M) \text{ contains only strings with 374 characters}\}$
5.  $\{\langle M \rangle \mid L(M) \text{ contains all of the strings of 374 characters}\}$
6.  $\{\langle M \rangle \mid L(M) \text{ contains none of the strings of 374 characters}\}$