1. Recall that $L_u = \{\langle M\rangle \# w \mid M(w) \text{ accepts}\}$ is not decidable. In class we showed reductions from L_u to various languages L to show that L was undecidable. One of the languages shown undecidable was the "halting language" $L_{halt} = \{\langle M\rangle \mid M \text{ halts on blank input}\}$.

   In order to show a language L is undecidable, it is often just as easy, or even easier, to show a reduction from L_{halt} to L.

   Example: We show that $L_{1^*} = \{\langle M\rangle \mid L(M) = 1^*\}$ is not decidable by showing $L_{halt} \leq L_{1^*}$

   Reduction: We show how a decider for L_{1^*} could be used to decide L_{halt}. The reduction takes an instance of L_{halt} (i.e., a TM M that we'd like to know if it halts on blank input) and outputs an instance of M' for L_{1^*}.

   We need the following to be true: M halts on blank input if and only if $L(M') = 1^*$.

   Here is the (partial) code for M'. Fill in the blank, and then answer parts (a), (b), and (c).

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   M'(x:  string)
       run M until, if ever, it halts
       if M halted, then accept x iff x__________
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   (a) If M doesn't halt when run on blank input, what is $L(M')$? _______________

   (b) If M halts when run on blank input, what is $L(M')$? _______________

   (c) Briefly argue that no decider for L_{1^*} can exist.

2. Let $L_{even} = \{\langle M\rangle \mid L(M) = \{w : |w| \text{ is even}\}\ \}$.
   Prove that L_{even} is not decidable by showing that $L_{halt} \leq L_{even}$.

3. Let $L_h = \{\langle M\rangle \# w \mid M(w) \text{ halts}\}$. Show how to use a decider for L_h to build a decider for L_u.