

1. Prove that the recursive languages are closed under the following operations:
  - union
  - intersection
  - complement
  - concatenation

Drawing a diagram is sufficient. For example to show recursive languages are closed under union, you simply need to draw a diagram showing how you'd use (always halting) TMs  $M_1$  and  $M_2$  for languages  $L_1$  and  $L_2$  to create a TM  $M_{L_1 \cup L_2}$  that decides membership in  $L_1 \cup L_2$ . Pseudocode would also be okay.

2. Prove that if  $L_1$  and  $L_2$  are recursive, then so is  $\text{SHUFFLE}(L_1, L_2) = \{w \mid w = \alpha_1\beta_1\alpha_2\beta_2\dots\alpha_k\beta_k \text{ for some } k \geq 0 \text{ and strings } \alpha_1, \dots, \alpha_k \text{ and } \beta_1, \dots, \beta_k, \text{ such that } \alpha_1\alpha_2\dots\alpha_k \in L_1 \text{ and } \beta_1\beta_2\dots\beta_k \in L_2\}$ .

3. Show that if  $L_1$  and  $L_2$  are recursively enumerable, then so is  $\text{SHUFFLE}(L_1, L_2)$ .