

1. Design a one-tape TM that computes the function $f(x) = 2x$. More specifically, when started in the initial state scanning the first 0 in a block of x consecutive 0s (i.e., the “Instantaneous Description” (or “ID”) at the beginning is just $q_0 0^x$) your TM should halt scanning the first 0 in a block of $2x$ consecutive 0s (i.e. the ID at the end is just $q_{\text{halt}} 0^{2x}$). Could this be done more easily with a two-tape TM?
2. Give a reasonably detailed description of a TM that computes the function $f(n) = 2^n$. Again, obey the starting/ending conventions: $q_0 0^n \Rightarrow^* q_{\text{halt}} 0^{2^n}$. You don’t have to completely design the TM; just provide enough detail that a TM programmer would know what states and transitions to use. Multiple tapes will be convenient.
3. Think about at home... How would a (multi-tape) TM compute $\lceil \log n \rceil$? That is, if the initial ID is $q_0 0^x$, then the final ID should be $q_{\text{halt}} 0^{\lceil \log n \rceil}$