

Suppose we are given both an undirected graph G with weighted edges and a minimum spanning tree T of G .

1. Describe an efficient algorithm to update the minimum spanning tree when the weight of one edge $e \in T$ is decreased.
2. Describe an efficient algorithm to update the minimum spanning tree when the weight of one edge $e \notin T$ is increased.
3. Describe an efficient algorithm to update the minimum spanning tree when the weight of one edge $e \in T$ is increased.
4. Describe an efficient algorithm to update the minimum spanning tree when the weight of one edge $e \notin T$ is decreased.

In all cases, the input to your algorithm is the edge e and its new weight; your algorithms should modify T so that it is still a minimum spanning tree. Of course, we could just recompute the minimum spanning tree from scratch in $O(E \log V)$ time, but you can do better.