

1. There are n light bulbs in a garden. These bulbs can be turned on manually by flipping on the switches at the light posts. Also, each light post can broadcast turn-on signals to some other pre-defined light posts in the garden, turning them on. When a light post is turned on, it will automatically broadcast a turn-on signal to its pre-defined light posts.

This signal broadcasting is directional. If a broadcasts to b , it is not necessarily true that b also broadcasts to a .

So one can manually flip on some of the switches to the light posts, and those light posts will broadcast a turn-on signal to other light posts. These will in turn be switched on and broadcast signals to their own pre-defined set of light posts, and so on.

Given each light post in the garden and the respective light posts to which they broadcast, derive a linear time algorithm for finding the minimum number of switches needed to be flipped to light up the whole garden. (Linear time means $O(n + m)$ where n is the number of light posts and m is the number of broadcast associations between them).

Source: ACM ICPC 2010 World Finals Warmup 2

Example Case: Number of lights : 5 , Number of broadcast associations: 4

Associations : $1 \Rightarrow 2$, $1 \Rightarrow 3$ $3 \Rightarrow 4$, $5 \Rightarrow 3$

Answer: Minimum number of flips required : 2 ,

Turning on switches 1 and 5 should light up the whole garden

Hints:

- (a) Model the problem using directed graphs.
 - (b) What is the solution if the graph in question is strongly connected?
 - (c) What is the solution if the graph in question is a DAG?
 - (d) What is the solution in general?
2. Let G be a directed acyclic graph. Prove that G has a unique topological sort if and only if it has a Hamiltonian Path.