This lab gives practice at constructing DFAs.

1. Design a DFA that accepts all strings over the alphabet {$,¢,0,1,2,3,4,5,6,7,8,9,.} that correspond to valid currency amounts. A valid string is either a dollar sign followed by a number which has no leading 0's, and may have a decimal point in which case it must be followed by exactly two decimal digits, OR a one or two-digit amount followed by the cent sign ¢. The only exceptions to this rule are amounts involving 0 dollars. The following forms are all acceptable: $0, 0c, and $0.ab where a and b are numerals 0 to 9. Thus, $432.63, $1, $0.29, 47¢, 2¢ are all accepted, but $021, $4.3, $8.63¢, $0.0 are not accepted.

2. Design the following DFAs assuming that the alphabet is $\{0, 1\}$.

   (a) A DFA for $\{w \mid |w| \text{ is odd}\}$.

   (b) A DFA for $\{w \mid \text{every prefix } x \text{ of } w \text{ has } |\#_0(x) - \#_1(x)| \leq 2\}$. Here, $\#_0(y)$ and $\#_1(y)$ are the number of 0's and 1's respectively in the string y.

   (c) A DFA $M = (Q, \Sigma, \delta, q_0, F)$ for the intersection of the previous two languages. Specify each element of the tuple precisely. Do not draw any pictures. Label the states reasonably.

3. **To think at home:** Given two regular expressions r and s we write $r = s$ if $L(r) = L(s)$. Prove or disprove the following.

   - $(r + s)^* = r^* + s^*$
   - $(rs + r)^* r = r(sr + r)^*$
   - $s(rs + s)^* r = rr^* s(rr^* s)^*$