“CS 374” Spring 2015 — Homework 10

Due Thursday, April 30th, 2015 at 10am

• • • Some important course policies • • •

- **You may work in groups of up to three people.** However, each problem should be submitted by exactly one person, and the beginning of the homework should clearly state the names and NetIDs of each person contributing.
- **You may use any source at your disposal**—paper, electronic, or human—but you ***must*** cite ***every*** source that you use. See the academic integrity policies on the course web site for more details.
- **Submit your pdf solutions in Moodle.** See instructions on the course website and submit a separate pdf for each problem. Ideally, your solutions should be typeset in LaTex. If you hand write your homework make sure that the pdf scan is easy to read. Illegible scans will receive no points.
- **Avoid the Three Deadly Sins!** There are a few dangerous writing (and thinking) habits that will trigger an automatic zero on any homework or exam problem. Yes, we are completely serious.
  - Give complete solutions, not just examples.
  - Declare all your variables.
  - Never use weak induction.
- Unlike previous editions of this and other theory courses we are not using the “I don’t know” policy.

See the course web site for more information.

If you have any questions about these policies,
please don’t hesitate to ask in class, in office hours, or on Piazza.

You can assume NP-Completeness of 3-SAT, Independent Set, Clique, Vertex Cover, Hamilton Cycle, and 3-Coloring.

1. A kite is a graph on an even number of nodes, say $2n$, in which n of the nodes form a clique and the remaining n vertices are connected in a "tail" that consists of a path joined to one of the nodes in the clique. Given a graph G and an integer k, the KITE problem asks for a subgraph which is a kite that contains $2k$ nodes. Prove that KITE is NP-Complete.

2. Given an undirected graph $G = (V, E)$, a partition of V into $V_1, V_2, \ldots, V_k$ is said to be a clique cover of size k if each V_i is a clique in G. Prove that the problem of deciding whether G has a clique cover of size at most k is NP-Complete. *Hint:* Consider the complement of G.

3. (a) Consider two boolean variables x and y. Write a 2CNF formula that computes the function $\neg(x \wedge y)$.

   (b) Given a connected graph $G = (V, E)$ with n vertices and m edges, we want to compute its maximum size independent set. To this end, we define a boolean variable for every vertex of V. Describe how to write a 2CNF formula that is true if and only if the vertices that are assigned value 1 are all independent.

   (c) Given a graph G and an integer k, describe how to compute a 2CNF formula φ and a value Δ such that at least Δ clauses of φ can be satisfied if and only if there is an independent set in G of size k (or larger). To make things easy, you are allowed to duplicate the same clause in your formula as many times as you want. Naturally, the algorithm for computing this formula from G, k should work in polynomial time (and of course, you need to describe this algorithm). What is the value of Δ as a function of n, m and k?

   (d) Using the above, prove that MAX 2SAT (i.e., given a 2CNF formula, compute the assignment that maximizes the number of clauses that are satisfied) is an NP-Hard *optimization* problem. That is, show that if one can solve MAX 2SAT in polynomial time then one can solve 3SAT in polynomial time.