1. Recall that $L_u = \{ \langle M \rangle \# w \mid M(w) \text{ accepts} \}$ is not decidable. In class we showed reductions from $L_u$ to various languages $L$ to show that $L$ was undecidable. One of the languages shown undecidable was the “halting language” $L_{\text{halt}} = \{ \langle M \rangle \mid M \text{ halts on blank input} \}$.

In order to show a language $L$ is undecidable, it is often just as easy, or even easier, to show a reduction from $L_{\text{halt}}$ to $L$.

**Example:** We show that $L_{1^*} = \{ \langle M \rangle \mid L(M) = 1^* \}$ is not decidable by showing $L_{\text{halt}} \leq L_{1^*}$.

**Reduction:** We show how a decider for $L_{1^*}$ could be used to decide $L_{\text{halt}}$. The reduction takes an instance of $L_{\text{halt}}$ (i.e., a TM $M$ that we’d like to know if it halts on blank input) and outputs an instance of $M'$ for $L_{1^*}$.

We need the following to be true: $M$ halts on blank input if and only if $L(M') = 1^*$.

Here is the (partial) code for $M'$. Fill in the blank, and then answer parts (a), (b), and (c).

```plaintext
M'(x: string)
    run M until, if ever, it halts
    if M halted, then accept x iff x   
```

(a) If $M$ doesn’t halt when run on blank input, what is $L(M')$? __________

(b) If $M$ halts when run on blank input, what is $L(M')$? __________

(c) Briefly argue that no decider for $L_{1^*}$ can exist.

2. Let $L_{\text{even}} = \{ \langle M \rangle \mid L(M) = \{ w : |w| \text{ is even} \} \}$.

Prove that $L_{\text{even}}$ is not decidable by showing that $L_{\text{halt}} \leq L_{\text{even}}$.

3. Let $L_h = \{ \langle M \rangle \# w \mid M(w) \text{ halts} \}$. Show how to use a decider for $L_h$ to build a decider for $L_u$. 