1. Design a one-tape TM that computes the function \( f(x) = 2x \). More specifically, when started in the initial state scanning the first 0 in a block of \( x \) consecutive 0s (i.e., the “Instantaneous Description” (or “ID”) at the beginning is just \( q_00^x \)) your TM should halt scanning the first 0 in a block of \( 2x \) consecutive 0s (i.e. the ID at the end is just \( q_{\text{halt}}0^{2x} \)). Could this be done more easily with a two-tape TM?

2. Give a reasonably detailed description of a TM that computes the function \( f(n) = 2^n \). Again, obey the starting/ending conventions: \( q_00^n \Rightarrow^* q_{\text{halt}}0^{2^n} \). You don't have to completely design the TM; just provide enough detail that a TM programmer would know what states and transitions to use. Multiple tapes will be convenient.

3. Think about at home... How would a (multi-tape) TM compute \( \lceil \log n \rceil \)? That is, if the initial ID is \( q_00^x \), then the final ID should be \( q_{\text{halt}}0^{\lceil \log n \rceil} \).