This lab gives practice at constructing DFAs.

1. Design a DFA that accepts all strings over the alphabet \{\$,\epsilon,0,1,2,3,4,5,6,7,8,9,\} that correspond to valid currency amounts. A valid string is either a dollar sign followed by a number which has no leading 0's, and may have a decimal point in which case it must be followed by exactly two decimal digits, OR a one or two-digit amount followed by the cent sign \epsilon. The only exceptions to this rule are amounts involving 0 dollars. The following forms are all acceptable: $0, 0c, and $0.ab where a and b are numerals 0 to 9. Thus, $432.63, $1, $0.29, 47\epsilon, 2\epsilon$ are all accepted, but $021, $4.3, $8.63\epsilon, $0.0 are not accepted.

2. Design the following DFAs assuming that the alphabet is \{0, 1\}.
   (a) A DFA for \{w \mid \text{\mid w\mid is odd}\}.
   (b) A DFA for \{w \mid \text{every prefix } x \text{ of } w \text{ has } |\#_0(x) - \#_1(x)| \leq 2\}. Here, \#_0(y) and \#_1(y) are the number of 0's and 1's respectively in the string y.
   (c) A DFA \(M = (Q, \Sigma, \delta, q_0, F)\) for the intersection of the previous two languages. Specify each element of the tuple precisely. Do not draw any pictures. Label the states reasonably.

3. To think at home: Given two regular expressions \(r\) and \(s\) we write \(r = s\) if \(L(r) = L(s)\). Prove or disprove the following.
   • \((r + s)^* = r^* + s^*
   • \((rs + r)^*r = r(sr + r)^*
   • s(rs + s)^*r = rr^*s(rr^*s)^*