"CS 374" Fall 2014 — Homework 3

Due Tuesday, September 23, 2014 at noon

- As usual, groups of up to three students may submit common solutions for this assignment. Each group should submit exactly ***one*** solution for each problem. Please clearly print the names and NetIDs of each of your group members at the top of each submitted solution, along with ***one*** discussion section where we should return your graded work. If you submit hand-written solutions, please use the last three pages of this handout as templates.
- In any question that asks you to construct an NFA, you are welcome to use ε-transitions.

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1. For each of the following regular expressions, describe or draw two finite-state machines:
   - An NFA that accepts the same language, obtained using Thompson's recursive algorithm (described in class and in the notes)
   - An equivalent DFA, obtained using the incremental subset construction described in class. For each state in your DFA, identify the corresponding subset of states in your NFA. Your DFA should have no unreachable states.

   (a) $(\texttt{01} + \texttt{10})^*(\texttt{0} + \texttt{1} + \varepsilon)$

   (b) $\texttt{1}^* + (\texttt{10})^* + (\texttt{100})^*$

2. Prove that for any regular language L, the following languages are also regular:

   (a) $\textsc{Substrings}(L) := \{x \mid wxy \in L \text{ for some } w, y \in \Sigma^*\}$

   (b) $\textsc{Half}(L) := \{w \mid ww \in L\}$

   [Hint: Describe how to transform a DFA for L into NFAs for $\textsc{Substrings}(L)$ *and* $\textsc{Half}(L)$. *What do your NFAs have to guess? Don't forget to explain* ***in English*** *how your NFAs work.]*

3. Which of the following languages over the alphabet $\Sigma = \{\texttt{0}, \texttt{1}\}$ are regular and which are not? Prove your answers are correct. Recall that Σ^+ denotes the set of all *nonempty* strings over Σ.

   (a) $\{wxw \mid w, x \in \Sigma^+\}$

   (b) $\{wxx \mid w, x \in \Sigma^+\}$

   (c) $\{wxwy \mid w, x, y \in \Sigma^+\}$

   (d) $\{wxxy \mid w, x, y \in \Sigma^+\}$

*4. **[Extra credit; held over from HW2]** Suppose L is a regular language that contains at least one palindrome. Prove that if L is accepted by a DFA with n states, then L contains a palindrome of length polynomial in n. What polynomial bound do you get?

CS 374 Fall 2014 — Homework 3 Problem 1

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For each of the following regular expressions, describe or draw first the NFA obtained from Thompson's algorithm, and then the equivalent DFA obtained from the incremental subset construction.

(a) $(01 + 10)^*(0 + 1 + \varepsilon)$

(b) $1^* + (10)^* + (100)^*$

CS 374 Fall 2014 — Homework 3 Problem 2

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Prove that for any regular language L, the following languages are also regular:

(a) $\textsc{Substrings}(L) := \{x \mid wxy \in L \text{ for some } w, y \in \Sigma^*\}$

(b) $\textsc{Half}(L) := \{w \mid ww \in L\}$

CS 374 Fall 2014 — Homework 3 Problem 3

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Which of the following languages over the alphabet $\Sigma = \{0, 1\}$ are regular and which are not? Prove your answers are correct.

(a) $\{wxw \mid w, x \in \Sigma^+\}$

(b) $\{wxx \mid w, x \in \Sigma^+\}$

(c) $\{wxwy \mid w, x, y \in \Sigma^+\}$

(d) $\{wxxy \mid w, x, y \in \Sigma^+\}$

CS 374 Fall 2014 — Homework 3 Problem 4

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Extra credit. Submit answers in the drop box for problem 1 (but don't staple problems 1 and 4 together.)

Suppose L is a regular language that contains at least one palindrome. Prove that if L is accepted by a DFA with n states, then L contains a palindrome of length polynomial in n. What polynomial bound do you get?
