1. For each of the following regular expressions, describe or draw two finite-state machines:

   • An NFA that accepts the same language, obtained using Thompson’s recursive algorithm (described in class and in the notes)
   • An equivalent DFA, obtained using the incremental subset construction described in class. For each state in your DFA, identify the corresponding subset of states in your NFA. Your DFA should have no unreachable states.

   a) \((01 + 10)^*(0 + 1 + \epsilon)\)
   b) \(1^* + (10)^* + (100)^*\)

2. Prove that for any regular language \(L\), the following languages are also regular:

   a) \(\text{SUBSTRINGS}(L) := \{x \mid wxy \in L \text{ for some } w, y \in \Sigma^*\}\)
   b) \(\text{HALF}(L) := \{w \mid ww \in L\}\)

   \[\text{Hint: Describe how to transform a DFA for } L \text{ into NFAs for SUBSTRINGS}(L) \text{ and HALF}(L). \text{ What do your NFAs have to guess? Don’t forget to explain in English how your NFAs work.}\]

3. Which of the following languages over the alphabet \(\Sigma = \{0, 1\}\) are regular and which are not? Prove your answers are correct. Recall that \(\Sigma^+\) denotes the set of all nonempty strings over \(\Sigma\).

   a) \(\{wxw \mid w, x \in \Sigma^+\}\)
   b) \(\{wxw \mid w, x \in \Sigma^+\}\)
   c) \(\{wxwy \mid w, x, y \in \Sigma^+\}\)
   d) \(\{wxxy \mid w, x, y \in \Sigma^+\}\)

*4. [Extra credit; held over from HW2] Suppose \(L\) is a regular language that contains at least one palindrome. Prove that if \(L\) is accepted by a DFA with \(n\) states, then \(L\) contains a palindrome of length polynomial in \(n\). What polynomial bound do you get?
For each of the following regular expressions, describe or draw first the NFA obtained from
Thompson's algorithm, and then the equivalent DFA obtained from the incremental subset
construction.

(a) $(01 + 10)^*(0 + 1 + \epsilon)$
(b) $1^* + (10)^* + (100)^*$
Prove that for any regular language $L$, the following languages are also regular:

(a) $\text{SUBSTRINGS}(L) := \{x \mid wxy \in L \text{ for some } w, y \in \Sigma^* \}$

(b) $\text{HALF}(L) := \{w \mid ww \in L \}$
Which of the following languages over the alphabet $\Sigma = \{0, 1\}$ are regular and which are not? Prove your answers are correct.

(a) $\{wxw \mid w, x \in \Sigma^+\}$
(b) $\{wx x \mid w, x \in \Sigma^+\}$
(c) $\{wxwy \mid w, x, y \in \Sigma^+\}$
(d) $\{wxxy \mid w, x, y \in \Sigma^+\}$
Suppose $L$ is a regular language that contains at least one palindrome. Prove that if $L$ is accepted by a DFA with $n$ states, then $L$ contains a palindrome of length polynomial in $n$. What polynomial bound do you get?