1. **C comments** are the set of strings over alphabet $\Sigma = \{*, /, A, \diamond, \downarrow\}$ that form a proper comment in the C program language and its descendants, like C++, Java. Here $\downarrow$ represents the newline character, $\diamond$ represents any other whitespace character (like the space and tab characters), and $A$ represents any non-whitespace character other than $*$ or $/$. There are two types of C comments:

- Line comments: Strings of the form $// \ldots \downarrow$.
- Block comments: Strings of the form $/* \ldots */$.

Following the C99 standard, we explicitly disallow nesting comments of the same type. A line comment starts with $//$ and ends at the first $\downarrow$ after the opening $//$. A block comment starts with $/*$ and ends at the first $*/$ completely after the opening $/$; in particular, every block comment has at least two $*$s. For example, the following strings are all valid C comments:

- $/**$/
- $//\diamond//\downarrow$
- $/*///\diamond\downarrow///**$/
- $/*/\downarrow/\diamond/*/$

On the other hand, the following strings are not valid C comments:

- $/*$
- $//\diamond//\downarrow\diamond$
- $/*//\diamond\downarrow/*/$

(a) Describe a DFA that accepts the set of all C comments.

(b) Describe a DFA that accepts the set of all strings composed entirely of blanks($\diamond$), newlines($\downarrow$), and C comments.

**You must explain in English how your DFAs work.** Drawings or formal descriptions without English explanations will receive no credit, even if they are correct.
2. Construct a DFA for the following language over alphabet \{0, 1\}:

\[
L = \left\{ w \in \{0, 1\}^* \mid \text{the number represented by binary string } w \text{ is divisible by 19, but the length of } w \text{ is not a multiple of 23} \right\}.
\]

**You must explain in English how your DFA works.** A formal description without an English explanation will receive no credit, even if it is correct. Don’t even try to draw the DFA.

3. Prove that each of the following languages is not regular.

(a) \( \{ w \in \{0\}^* \mid \text{length of } w \text{ is a perfect square; that is, } |w| = k^2 \text{ for some integer } k \} \).

(b) \( \{ w \in \{0, 1\}^* \mid \text{the number represented by } w \text{ as a binary string is a perfect square} \} \).

4. [Extra credit] Suppose \( L \) is a regular language which guarantees to contain at least one palindrome. Prove that if an \( n \)-state DFA \( M \) accepts \( L \), then \( L \) contains a palindrome of length polynomial in \( n \). What is the polynomial bound you get?

---

\(^1\)The actual C commenting syntax is considerably more complex than described here, because of character and string literals.

- The opening /* or // of a comment must not be inside a string literal ("...") or a (multi-)character literal (‘...’).
- The opening double-quote of a string literal must not be inside a character literal ("'") or a comment.
- The closing double-quote of a string literal must not be escaped ("")
- The opening single-quote of a character literal must not be inside a string literal ("...") or a comment.
- The closing single-quote of a character literal must not be escaped (\')
- A backslash escapes the next symbol if and only if it is not itself escaped (\") or inside a comment.

For example, the string "/\*\*\*\*\*/\*\*\*\*/\*\*\*\*\/*\*/\*\*\*\*/\*\*\*\*\*/ is a valid string literal (representing the 5-character string */\*\*/\*\*\*/, which is itself a valid block comment!) followed immediately by a valid block comment. For this homework question, just pretend that the characters ‘’, “, and \ don’t exist.

The C++ commenting is even more complicated, thanks to the addition of raw string literals. Don’t ask. Some C and C++ compilers do support nested block comments, in violation of the language specification. A few other languages, like OCaml, explicitly allow nesting comments.
1. Describe a DFA that accepts the set of all C comments.
2. Describe a DFA that accepts the set of all strings composed entirely of blanks(\(\diamond\)), newlines(\(\hat{e}\)), and C comments.
Construct a DFA for the following language over alphabet \{0, 1\}:

\[ L = \{ w \in \{0, 1\}^* \mid \text{the number represented by binary string } w \text{ is divisible by 19, but the length of } w \text{ is not a multiple of 23} \}. \]
Prove that each of the following languages is not regular.

1. \( \{ w \in \{0\}^* \mid \text{length of } w \text{ is a perfect square; that is, } |w| = k^2 \text{ for some integer } k \} \).
2. \( \{ w \in \{0,1\}^* \mid \text{the number represented by } w \text{ as a binary string is a perfect square} \} \).
Suppose $L$ is a regular language which guarantees to contain at least one palindrome. Prove that if an $n$-state DFA $M$ accepts $L$, then $L$ contains a palindrome of length polynomial in $n$. What is the polynomial bound you get?