This homework contains three problems. Read the instructions for submitting homework on the course webpage.

Collaboration Policy: For this homework, each student should work independently and write up their own solutions and submit them.

1. Solve the following recurrences in the sense of giving an asymptotically tight bound of the form $\Theta(f(n))$ where $f(n)$ is a standard and well-known function. No proof necessary for the first four parts; simply state the bound.
   
   (a) $A(n) = n^{1/3}A(n^{2/3}) + n$, $A(n) = 1$ for $1 \leq n \leq 8$.
   
   (b) $B(n) = B(n/2) + n$, $B(1) = 1$.
   
   (c) $C(n) = 2C(n - 1) + 1$, $C(1) = 1$.
   
   (d) $D(n) = 3D(n/3) + 4D(n/4) + n^3$, $D(n) = 1$ for $n \leq 4$.
   
   (e) Prove by induction that the $T(n)$ defined by the recurrence
   \[
   T(n) = 2T(\sqrt{n}) + \log n
   \]
   if $n \geq 4$, and $T(n) = 3$ if $n < 4$ satisfies the bound $T(n) = O(\log n \log \log n)$.


3. Consider the standard balls and bins process. A collection of $m$ identical balls are thrown into $n$ bins: each ball is thrown independently into a bin chosen uniformly at random.
   
   (a) What is the (precise) probability that a particular bin $i$ contains exactly $k$ balls at the end of the experiment?
   
   (b) Suppose $m = n$. Let $Y$ be the number of bins that has exactly one ball. What is the expectation of $Y$?
   
   (c) What is the variance of $Y$?

   Explain your calculations when you derive the bounds.