CS 473 ✧ Spring 2020

Homework 2

Due Wednesday, February 12, 2020 at 9pm

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1. Let $D[1..n]$ be an array of digits, each an integer between 0 and 9. A ***digital subsequence*** of D is a sequence of positive integers composed in the usual way from disjoint intervals of D. For example, the sequence $3, 4, 5, 6, 8, 9, 32, 38, 46, 64, 83, 279$ is a digital subsequence of the first several digits of π:

$$\underline{3}, 1, \underline{4}, 1, \underline{5}, 9, 2, \underline{6}, 5, 3, 5, \underline{8}, \underline{9}, 7, 9, \underline{3, 2}, \underline{3, 8}, \underline{4, 6}, 2, \underline{6, 4}, 3, 3, \underline{8, 3}, \underline{2, 7, 9}$$

The *length* of a digital subsequence is the number of integers it contains, *not* the number of digits; the preceding example has length 12. As usual, a digital subsequence is ***increasing*** if each number is larger than its predecessor.

Describe and analyze an efficient algorithm to compute the longest increasing digital subsequence of D. *[Hint: First consider the special case where none of the digits is* 0*. Be careful about your computational assumptions. How long does it take to compare two* k*-digit integers?]*

2. The (Eleventh) Doctor and River Song decide to play a game on a directed acyclic graph G, which has one source vertex s and one sink vertex t.[1]

Each player has a token on one of the vertices of G. At the start of the game, The Doctor's token is on the source s, and River's token is on the sink t. The players alternate turns, with The Doctor moving first. On each of his turns, the Doctor moves his token forward along a directed edge; on each of her turns, River moves her token *backward* along a directed edge.

If the two tokens ever meet on the same vertex, River wins the game. ("Hello, Sweetie!") If the Doctor's token reaches t or River's token reaches s before the two tokens meet, then the Doctor wins the game.

Describe and analyze an algorithm to determine who wins this game, assuming both players play perfectly. That is, if the Doctor can win *no matter how River moves*, then your algorithm should output "Doctor", and if River can win *no matter how the Doctor moves*, your algorithm should output "River". (Why are these the only two possibilities?) The input to your algorithm is the graph G.

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[1]The labels s and t are abbreviations for the Untempered **S**chism and the **T**ime Vortex, or the Shining World of the Seven Systems (also known as Gallifrey) and Trenzalore, or Skaro and Telos, or Something else Timey-wimey. It's all very complicated, never mind.

3. ***[Extra credit[2]]*** After seeing your expertise in solving problem 2 in Homework 1, the Order of the Library of the Neitherlands immediately recruited you to tackle a significantly larger shelving project. Their problem is exactly the same as the one you already solved—they need an algorithm to put a sequence of n books on shelves using as little *vertical* space as possible—but the Neitherlands Library contains *far* too many books for an $O(n^2)$-time algorithm to be useful.

   Describe and analyze an algorithm to compute the minimum total height required to shelve a sequence of n books ***in*** $\boldsymbol{O(n \log n)}$ ***time***. As in Homework 1 problem 2, the input consists of two arrays $H[1..n]$ and $W[1..n]$, specifying the height and width of each book, and a number L, which is the common length of every shelf. Heights, widths, and lengths are not necessarily integers.

   *[I have no reason to believe that $O(n \log n)$ is the best possible running time. For the special case where heights, widths, and lengths **are** integers, $O(n \log n)$ is definitely **not** the best possible running time.]*

[2]Extra credit problems are ignored when we compute the curve at the end of the semester. In particular, they do not count toward the top 24 homework scores we will use to compute your raw homework average.